



Mid Exam Summer

Submitted By:

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BS (SE-8) Section: A

Submitted To:

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Q1

Answer:

The image shows a handwritten solution for a system of linear equations using row reduction. The steps are as follows:

$$= \left[\begin{array}{ccc|c} 1 & -3 & 4 & -3 \\ -3 & -7 & 7 & -3 \\ -4 & 6 & 1 & 0 \end{array} \right] \quad -26$$
$$= \left[\begin{array}{ccc|c} -3 & -7 & 7 & -3 \\ 1 & -3 & 4 & -3 \\ -4 & 6 & 1 & 0 \end{array} \right] \quad R_1 \leftrightarrow R_2$$
$$= \left[\begin{array}{ccc|c} -1 & -13 & 15 & -9 \\ 1 & -3 & 4 & -3 \\ -4 & 6 & 1 & 0 \end{array} \right] \quad R_1 + 2R_2$$
$$= \left[\begin{array}{ccc|c} 1 & 13 & -15 & 9 \\ 1 & -3 & 4 & -3 \\ -4 & 6 & 1 & 0 \end{array} \right] \quad -R_1 \quad \&$$
$$= \left[\begin{array}{ccc|c} 1 & 13 & -15 & 9 \\ 0 & -3 & 4 & -3 \\ -4 & 6 & 1 & 0 \end{array} \right] \quad \left. \begin{array}{l} \frac{1}{2} R_3 \\ \frac{1}{2} R_2 \end{array} \right\}$$
$$= \left[\begin{array}{ccc|c} 1 & 13 & -15 & 9 \\ 0 & -6 & 8 & -6 \\ -4 & 6 & 1 & 0 \end{array} \right] \quad R_3 - R_1$$
$$= \left[\begin{array}{ccc|c} 1 & 13 & -15 & 9 \\ 4 & -12 & 16 & -12 \\ -4 & 6 & 1 & 0 \end{array} \right] \quad 4R_2$$
$$= \left[\begin{array}{ccc|c} 1 & 13 & -15 & 9 \\ 0 & -6 & 15 & -12 \\ -4 & 6 & 1 & 0 \end{array} \right] \quad R_2 + R_3 \quad \times$$
$$= \left[\begin{array}{ccc|c} 1 & 13 & -15 & 9 \\ 4 & -12 & 16 & -12 \\ 0 & -6 & 17 & -12 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 13 & -15 & 9 \\ 0 & -64 & 26 & -48 \\ 0 & -6 & 17 & -12 \end{array} \right]$$

$$R_2 - 4R_1$$

-12-36

From the above Augmented matrix -

$$\begin{array}{l} x = \frac{11}{10} \\ y = \frac{9}{15} \\ z = \frac{13}{6} \end{array}$$

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Q2 Part A

Answer:

Question # 2 (a) ¹

Find Inverse of Matrix

$$\begin{bmatrix} 1D3 & -1 & 0 \\ 0 & 1 & 1D3 \\ 1 & 1 & 0 \end{bmatrix}$$

Solution:

My id is 13033

So $1D3 = 0$

Putting value in matrix

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Finding determinant.

$$|A| = \begin{vmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix}$$
$$= 0 \times \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} + 1 \times \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} + 0 \times \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}$$

$$0 \times (1 \times 0 - 0 \times 1) + 1 (0 \times 0 - 0 \times 1) + 0 (0 \times 1) - 1$$

$$= 0 (0 - 0) + 1 (0 - 0) + 0 (0 - 1)$$

$$= 0 (-0) + 1 (-0) + 0 (-1)$$

$$= -0 - 0 + 0$$

$$= 0$$

Now Taking adj :-

$$\text{Adj}(A) = \text{Adj} \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\left[\begin{array}{l} + \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} \\ - \begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix} \\ + \begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & -1 \\ 0 & 1 \end{vmatrix} \end{array} \right]$$

$$= \begin{bmatrix} + (1 \times 0 - 0 \times 1) - (0 \times 0 - 0 \times 1) + (0 \times 1 - 1 \times 1) \\ - (-1 \times 0 - 0 \times 1) + (0 \times 0 - 0 \times 1) - (0 \times 1 - (-1) \times 1) \\ + (1 \times 0 - 0 \times 1) - (0 \times 0 - 0 \times 0) + (0 \times 1 - (-1) \times 1) \end{bmatrix}$$

$$= \begin{bmatrix} + (0 - 0) - (0 - 0) + (0 - 1) \\ - (0 + 0) + (0 + 0) - (0 + 1) \\ + (-0 + 0) - (0 + 0) + (0 + 0) \end{bmatrix}$$

$$= \begin{bmatrix} -0 & 0 & -1 \\ 0 & 0 & -1 \\ -0 & -0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -0 & 0 & -0 \\ 0 & 0 & -0 \\ -1 & -1 & 0 \end{bmatrix}$$

Now taking, $A^{-1} = \frac{1}{|A|} \times \text{Adj}(A)$

$$= \left(\frac{1}{0} \right) \begin{bmatrix} -0 & 0 & -0 \\ 0 & 0 & -0 \\ -1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Answer,

Question 2 Part B

Answer

Question No 2 (b)

Find cell echelon form for the below matrix using row operation.

$$\begin{bmatrix} 1 & 103 & 8 \\ 2 & 104 & -1 \\ -3 & 0 & 0 \\ 1 & -03 & 16 \end{bmatrix}$$

Solution:

$$\text{My ID} = 13033$$

Solution let $A = \begin{bmatrix} 1 & 0 & 8 \\ 2 & 3 & -1 \\ -3 & 0 & 0 \\ 1 & -0 & 16 \end{bmatrix}$

by Row operation:

$$A = \begin{bmatrix} 1 & 0 & 8 \\ 0 & 3 & -33 \\ -3 & 0 & 0 \\ 1 & 0 & 16 \end{bmatrix} \quad R_2 - 2R_4$$

$$= \begin{bmatrix} 1 & 0 & 8 \\ 0 & 3 & -33 \\ 0 & 0 & -48 \\ -1 & 0 & 16 \end{bmatrix} \quad R_3 - 3R_4$$

$$= \begin{bmatrix} 1 & 0 & 8 \\ 0 & 3 & -33 \\ 0 & 0 & -48 \\ 0 & 0 & 8 \end{bmatrix} \quad R_4 - R_1$$

$$= \begin{bmatrix} 1 & 0 & 8 \\ 0 & 3 & -33 \\ 0 & 0 & -8 \\ 0 & 0 & 8 \end{bmatrix} \quad \frac{R_3}{6}$$

$$= \begin{bmatrix} 1 & 0 & 8 \\ 0 & 3 & -33 \\ 0 & 0 & -8 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 + R_4$$

Required echelon form

Question 3

Answer

$$(3) \Rightarrow A = \begin{bmatrix} 0 & -6 & 2 \\ -6 & 3 & -4 \\ 2 & -4 & 3 \end{bmatrix} \text{ for eigen value.}$$
$$|(A - \lambda I)| = 0$$
$$= \left| \begin{pmatrix} 0 & -6 & 2 \\ -6 & 3 & -4 \\ 2 & -4 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$
$$= \begin{vmatrix} -\lambda & -6 & 2 \\ -6 & 3-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$
$$= -\lambda \left((3-\lambda)(3-\lambda) - 16 \right) + 6 \left((3-\lambda)(-6) + 8 \right) + 2 \left(24 - (2)(3-\lambda) \right) = 0$$
$$= -\lambda \left(9 - 3\lambda - 3\lambda + \lambda^2 - 16 \right) + 6 \left(-18 + 6\lambda + 8 \right) + 2 \left(48 - 4(3-\lambda) \right) = 0$$
$$= -9\lambda + 3\lambda^2 + 3\lambda^2 - \lambda^3 + 16\lambda - 108 + 36\lambda + 48 + 48 - 12 + 4\lambda = 0$$
$$= \lambda^3 + 6\lambda^2 + 47\lambda - 24 = 0$$
$$= \lambda \left(\lambda^2 + 6\lambda + 47 \right) = 24$$
$$\boxed{\lambda_1 = 24}$$

again

$$\lambda^2 + 6\lambda + 47 = 24$$
$$\lambda^2 + 6\lambda + 23 = 0$$

Here $a = 1, b = 6, c = 23$

$$\Rightarrow -b \pm \frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{using quadratic equation}$$

$$= \frac{-b \pm \sqrt{36 - 92}}{2}$$

$$= k_2 = \frac{-6 + \sqrt{128}}{2}$$

$$= k_3 = \frac{-6 - \sqrt{128}}{2}$$

$$k_2 = \frac{-6 + 11}{2} = \frac{5}{2}$$

$$k_3 = \frac{-6 - 11}{2} = \frac{-17}{2}$$

Eigen values is

$$\begin{cases} k_1 = 24 \\ k_2 = \frac{5}{2} \\ k_3 = \frac{-17}{2} \end{cases}$$

