STRUCTURAL ANALYSIS 2

FINAL TERM

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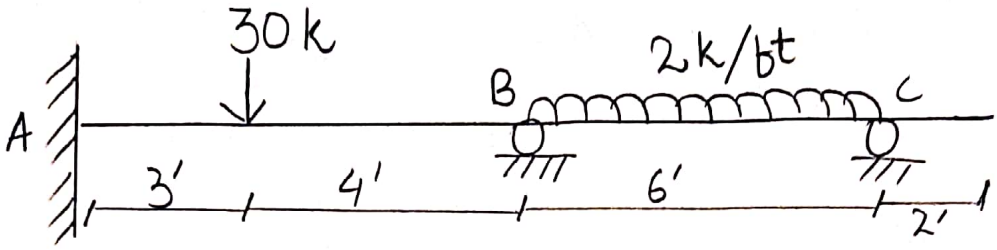
Question # 01



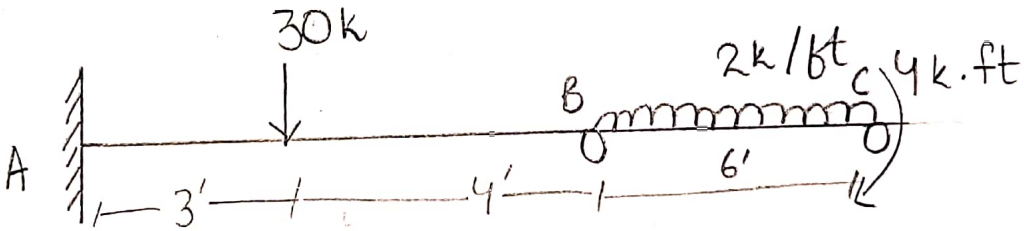
Solution

Step# 01

Determining Kinematic Indeterminacy

$$K.I = 5^\circ$$

So we have to reduce the extended portion

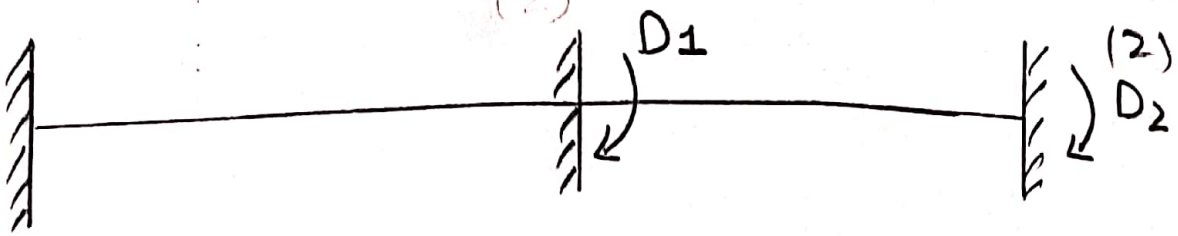


$$\frac{2(2)}{1} = 4k.ft$$

Now $K.I = 2^\circ$

Step# 02

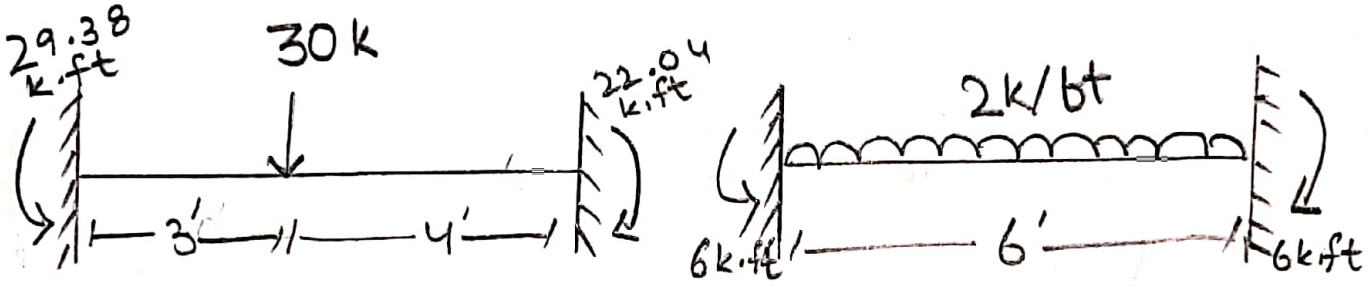
Determine unknown joint Displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step#03

Compute [ADL] Matrix



→ For point load (not at mid)

→ For left end:

$$\frac{Pab^2}{L^2} = \frac{(30)(3)(4)^2}{(7)^2} = 29.38 \text{ k.ft}$$

→ For right end:

$$\frac{Pa^2b}{L^2} = \frac{(30)(3)^2(4)}{(7)^2} = 22.04 \text{ k.ft}$$

→ For Uniformly Distributed Load

$$\frac{WL^2}{12} \Rightarrow \frac{(2)(6)^2}{12} = 6\ k.ft$$

$$ADL_1 = +22.04 - 6 = 16.04\ k.ft$$

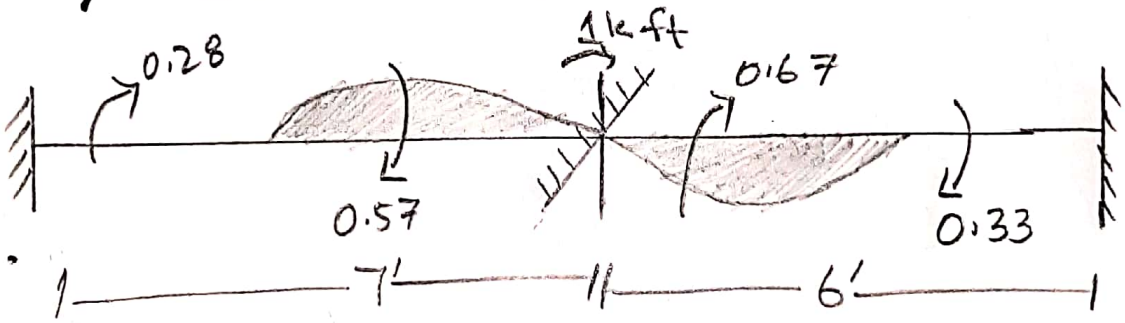
$$ADL_2 = 6\ k.ft$$

Step#04

Now compute [S] Matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

a) $D_1 = 1k$, $D_2 = 0$



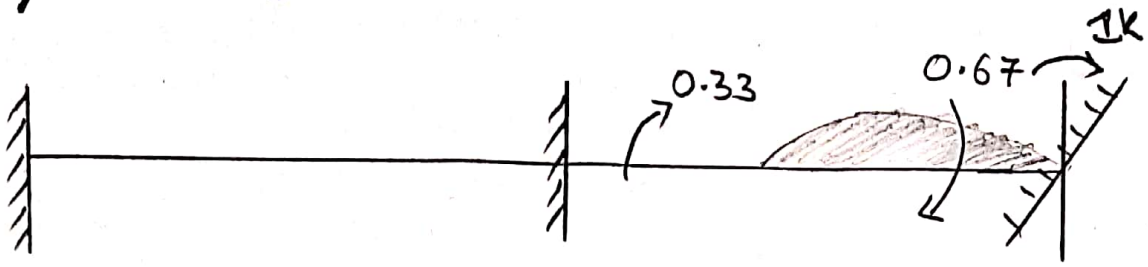
$$\frac{4EI}{7} = 0.57 \qquad \frac{2EI}{6} = 0.33$$

$$\frac{4EI}{6} = 0.67 \qquad \frac{2EI}{7} = 0.28$$

$$S_{11} = 0.57 + 0.67$$

$$\boxed{S_{11} = 1.24\ EA} \qquad \boxed{S_{21} = 0.33\ EA}$$

b) $D_1 = 0$ $D_2 = 1k$



$$\frac{4EI}{6} = 0.67$$

$$\frac{2EI}{6} = 0.33$$

$$\boxed{S_{12} = 0.33} \qquad \boxed{S_{22} = 0.67}$$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

Step# 05

Now Compute [D] matrix

$$\begin{bmatrix} D1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$= \frac{1}{\begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}} \times \text{Adj A} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$$

$$= 0.8308 - 0.1089$$

$$|S| = 0.7219$$

$$\text{Adj } A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

Now

$$\begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix} = \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} = \begin{bmatrix} -16.04 \\ -2 \end{bmatrix} E$$

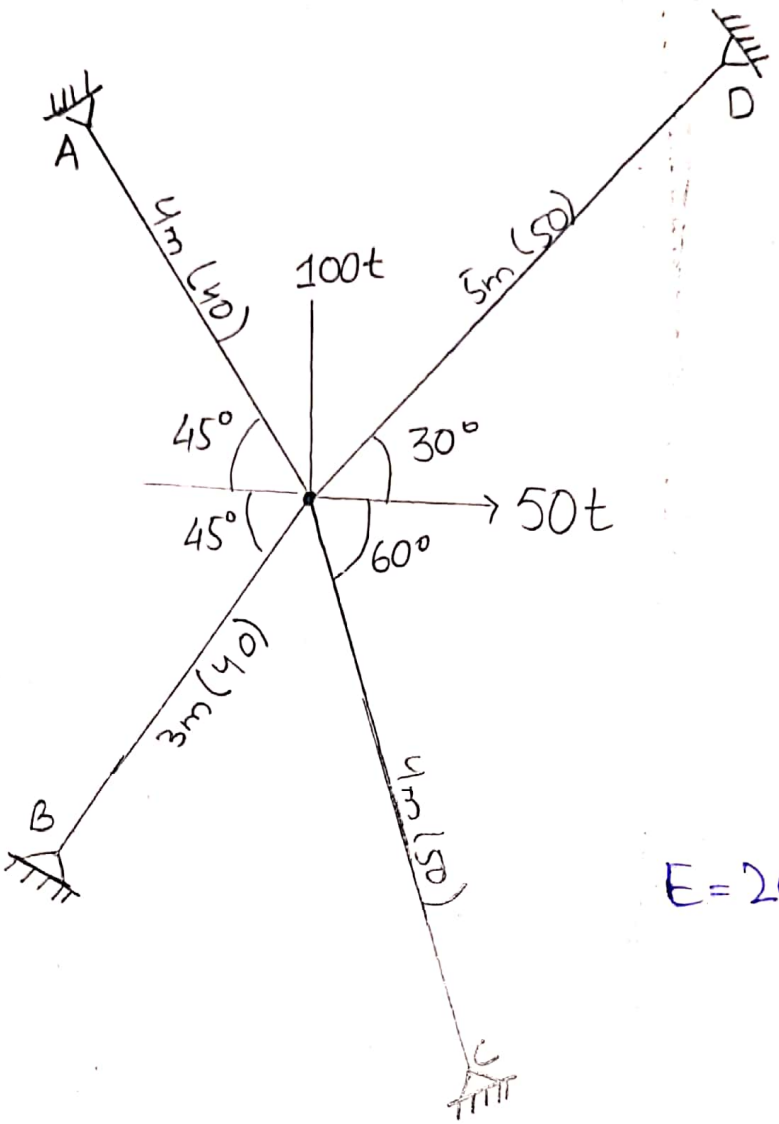
$$\rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{|S|} \times \text{Adj } A \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}}{0.729} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.919 & -0.452 \\ -0.452 & 1.70 \end{bmatrix} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} -13.83 \\ 3.85 \end{bmatrix}$$

Question #02



$E = 2000\ t/cm^2$

Solution

For A

$$\sin 45 = \frac{P}{H} = \frac{P}{4}$$

$$\rightarrow P = 2.828\,m$$

$$\cos 45^\circ = \frac{b}{H} = \frac{b}{4}$$

$$\rightarrow b = 2.828\,m$$

For B

$$\sin 45 = \frac{P}{H} = \frac{P}{3}$$

$$\rightarrow P = 2.12m$$

$$\cos 45 = \frac{b}{H} = \frac{b}{3}$$

$$\rightarrow b = 2.12m$$

For C

$$\sin 60 = \frac{P}{H} = \frac{P}{4}$$

$$(\sin 60)(4) = P$$

$$\rightarrow P = 3.46$$

$$\cos 60 = \frac{b}{H} = \frac{b}{4}$$

$$\cos 60 \times 4 = b$$

$$\rightarrow b = 2$$

For D

$$\sin 30 = \frac{P}{5}$$

$$\rightarrow P = 2.5m$$

$$\cos 30 = \frac{b}{5}$$

$$b = 4.33m$$

Now

$EA_{(A)} = 2000 \times 40 = 80,000t$

$EA_{(B)} = 2000 \times 40 = 80,000t$

$EA_{(C)} = 2000 \times 50 = 100,000t$

$EA_{(D)} = 2000 \times 50 = 100,000t$

Step# 01 KI

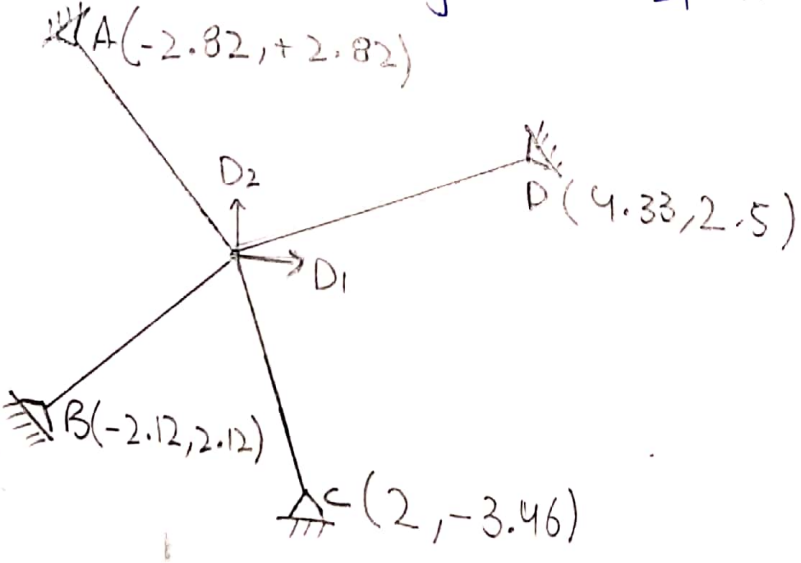
$KI = 2j - r$

$= 2(5) - 8$

$KI = 2^{o}$

Step# 02

Select unknown joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step#03

$$[AMD]_{4\times 2} \qquad [S]_{2\times 2}$$

i) $D_1 = 1k \quad , D_2 = 0$

$$AMD = \frac{EA}{L^2}(X_k - X_j)$$

$$AMD_{11} = \frac{80000}{(400)^2} \times (0+282) = 141$$

$$AMD_{21} = \frac{80,000}{(300)^2} \times (0+212) = 188.44$$

$$AMD_{31} = \frac{120,000}{(500)^2} \times (0-433) = -173.2$$

$$AMD_{41} = \frac{100,000}{(400)^2} \times (0-200) = -125$$

Now,

$$S_{11} = \sum_{i=1}^{m} \frac{EA}{L^3}(X_k - X_j)^2$$

$$= \frac{80,000}{(400)^3}(282)^2 + \frac{80,000}{(300)^3} \times (212)^2$$

$$+ \frac{100,000}{(500)^3} \times (-433)^2 + \frac{100,000}{(400)^3} \times (-200)^2$$

$$S_{11} = 99.405 + 133.107 + 149.991 + 62.5$$

$$S_{11} = 445.063$$

$$\Rightarrow S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3} \times (X_k - X_j)(Y_k - Y_j)$$

$$= \frac{80,000}{(400)^3}(282)(-282) + \frac{80,000}{(300)^3}(212)(212)$$

$$+ \frac{100,000}{(500)^3}(-433)(0-250) + \frac{100000}{(400)^3}(-200)(0-346)$$

$$S_{12} = S_{21} = 12.237$$

ii) $D_1 = 0 \qquad D_2 = 1k'$

$$AMD = \frac{EA}{L^2}(Y_k - Y_j)$$

$$AMD_{12} = \frac{80,000}{(400)^2}(-282) = -141$$

$$AMD_{22} = \frac{80,000}{(300)^2}(212) = 188.44$$

$$AMD_{32} = \frac{100,000}{(500)^2}(-250) = -100$$

$$AMD_{42} = \frac{100,000}{(400)^3}(346) = 216.25$$

$$\text{NOW, } S_{22} = \sum_{i=1}^{m} \frac{EA}{L^3}(Y_k - Y_j)^2$$

$$= \frac{80,000}{400^3}(-282)^2 + \frac{80,000}{300^3}(212^2$$

$$+ \frac{100,000}{500^3}(-250)^2 + \frac{100,000}{400^3}(346)^2$$

$$S_{22} = 469.628$$

Step # 04

$$[D] = [S]^{-1} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.003 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

Step # 05

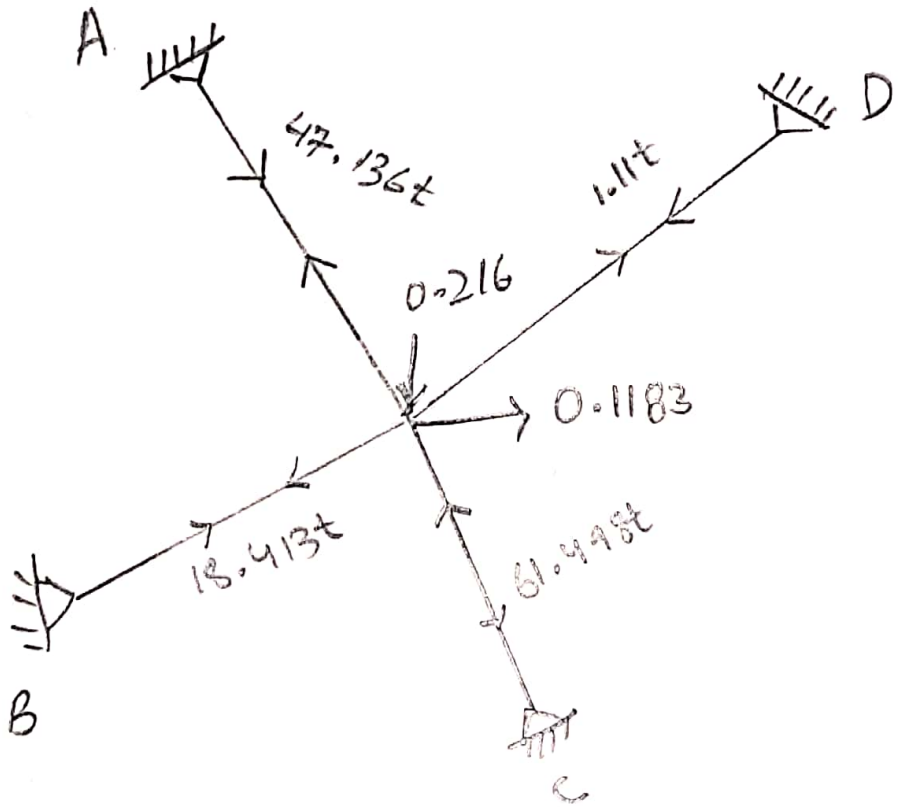
$$[AM]$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

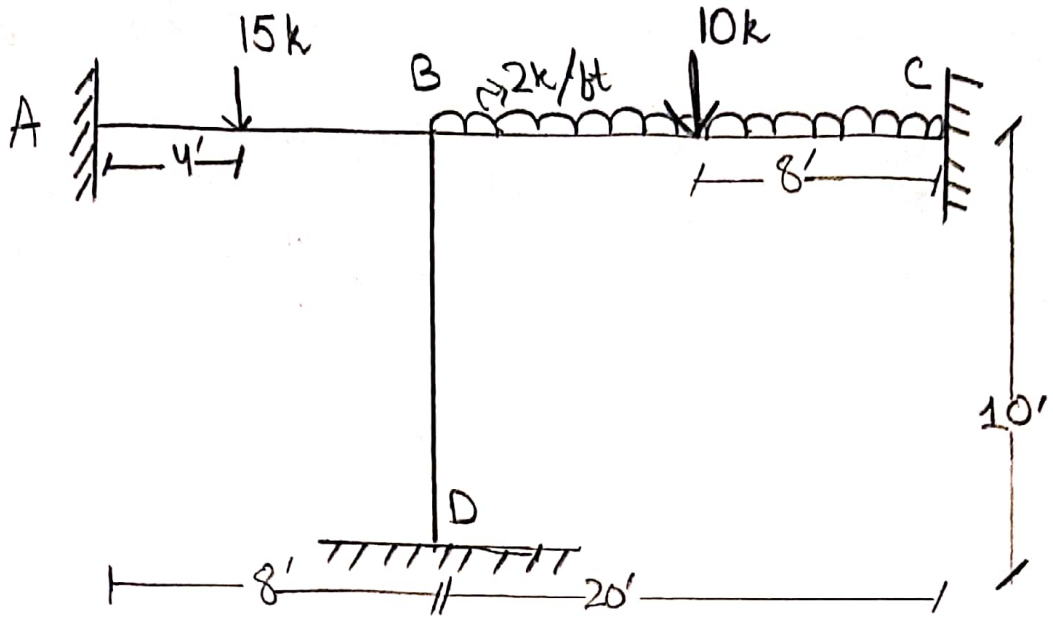
$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + (188.44) \times (-0.216) \\ -173.2 \times 0.1183 + (-100)(-0.216) \\ -125 \times 0.1183 + 216.25 \times (-0.1216) \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 22.29 + -40.70 \\ -20.49 + 21.6 \\ -14.79 + 46.71 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix}$$



Question #03



Solution

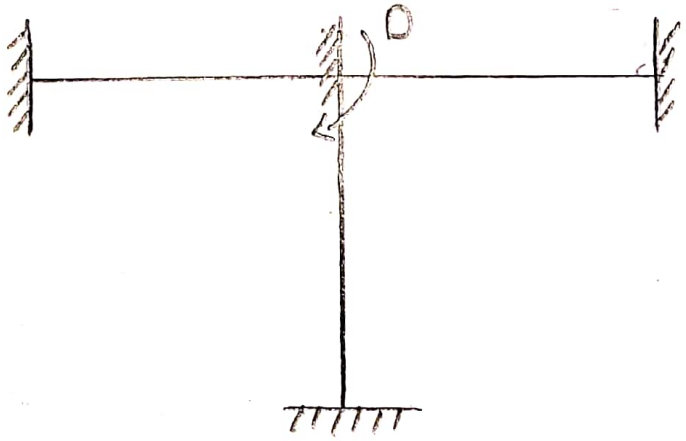
Step#01

Determine Kinematic Indeterminacy

$K.I = 1^{\circ}$

Step#02

Determine Unknown Joint Displacement

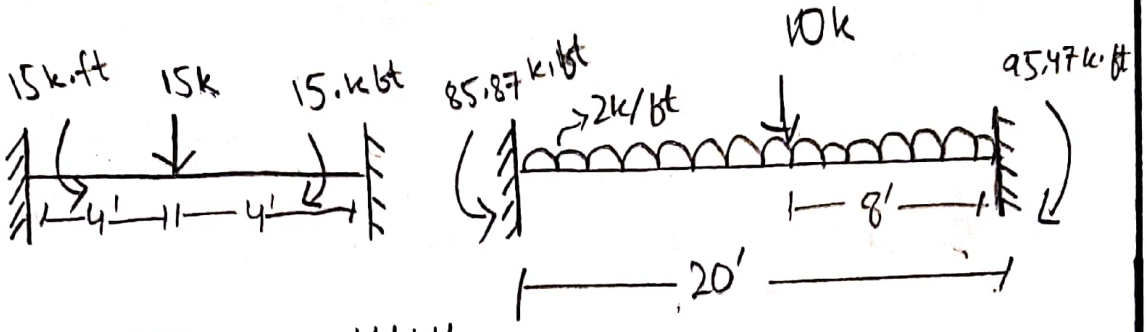


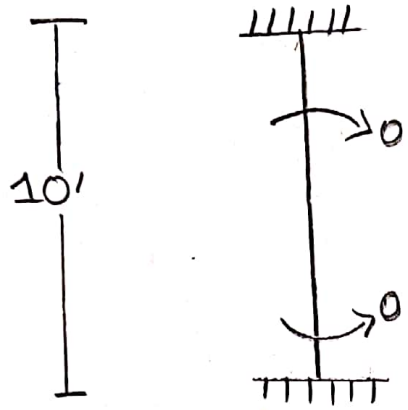
$[D] = [?]$

$[AD] = [0]$

Step#3

Compute [ADL] Matrix





→ Point load at center

$$\frac{PL}{8} \Rightarrow \frac{(15)(8)}{8} \Rightarrow 15 \text{ k.ft}$$

→ Uniformly distributed load

$$\frac{WL^2}{12} \Rightarrow \frac{(2)(20)^2}{12} \Rightarrow 66.67 \text{ ft}$$

→ Point load (not at mid)

let's say

for left end

$$\frac{Pab^2}{l^2} = \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ k.ft}$$

for right end

$$\frac{Pa^2b}{l^2} = \frac{(10)(12^2)(8)}{(20)^2} = 28.8\ k.ft$$

→ So total moment at left end

$$19.2 + 66.67 = 85.87\ k.ft$$

→ Similarly at right end

$$28.8 + 66.67 = 95.47\ k.ft$$

Now

$$[AD_1] = -85.87 + 15 = -70.87\ k.ft$$

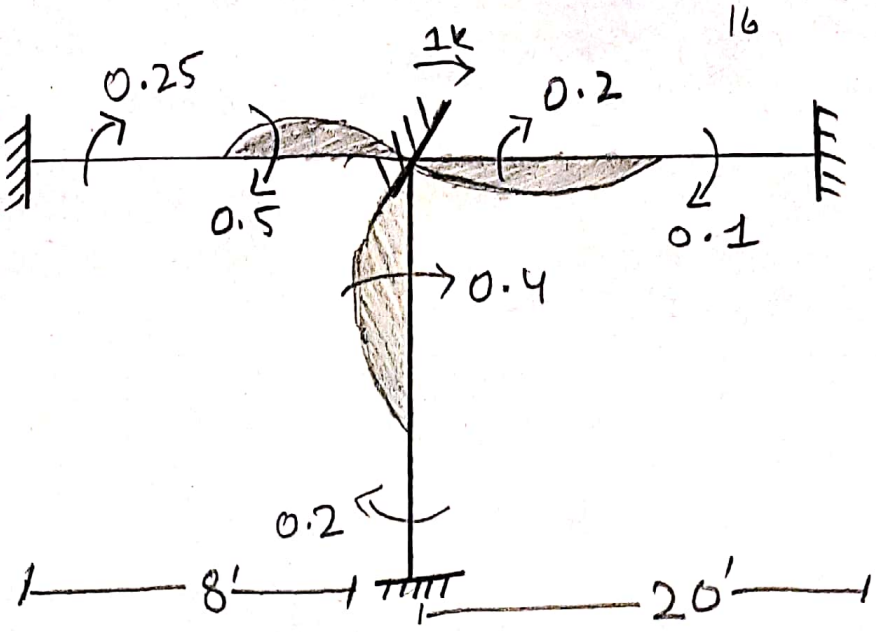
Step # 4

Determine [S] matrix

$$[S] = [S_{11}]$$

Now

$$D = \underline{1}k$$



$\rightarrow \frac{4EI}{8} = 0.5$ $\quad \frac{2EI}{8} = 0.25$

$\rightarrow \frac{4EI}{20} = 0.2$ $\quad \frac{2EI}{20} = 0.1$

$\rightarrow \frac{4EI}{10} = 0.4$ $\quad \frac{2EI}{10} = 0.2$

$$[S] = (0.5 + 0.4 + 0.2)EI$$

$$[S] = 1.1EI$$

Step#5

Compute [D] matrix

$$[D] = [S]^{-1} \times [AD] - [ADL]$$

$$[D] = \frac{1}{1.1} \times [0] - [-70.87]$$

$$[D] = 70.87/1.1$$

$$[D] = [64.42] \times 1/EI$$