Department of Electrical Engineering
Sessional Assignment
Date: 01/06/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: Sir Pir Mehr **Total Marks:** 20

Student Details

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Q1.	(a)	Determine the response y(n), n ≥ 0, of the system described by the second order difference equation $$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$ To the input $x(n) = 4^n u(n)$.	Marks 6
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$$	
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	Marks 6
	(b)	Determine the partial fraction expansion of the following proper function $$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$	
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response H(ω) satisfies the condition H(0) = 1 and $\left|H(\frac{\pi}{4})\right|^2 = \frac{1}{2}$.	Marks 4

	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.	
Q 4	(c)	A finite duration sequence of Length L is given as $$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & otherwise \end{cases}$$ Determine the N- point DFT of this sequence for $N \ge L$	Marks 4
	(d)	Compute the DFT of the four-point sequence $$x(n) = (0\ \ 1\ \ 2\ \ 3)$$	

IQRA NATIONAL UNIVERSITY PESHAWAR

Sessional Assignment

Digital Signal Processing

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Q 1:- (Part - a)

★ Solution:-

Consider the difference equation

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

The homogenous equation of the system is; $y(n) - 3y(n-1) - 4y(n-2) = 0$

The characteristic equation of the system is;

$$\lambda - 3\lambda^{-1} - 4\lambda^{-2} = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

Determine the root of the characteristic equation

$$\lambda^{-2} - 4\lambda + \lambda - 4 = 0$$

$$\lambda(\lambda - 4) + 1(\lambda - 4) = 0$$

$$(\lambda - 4)(\lambda + 1) = 0$$

$$\lambda_1 = 4, \quad \lambda_2 = -1$$

The homogenous solution is;

$$y_h(n) = C_1(-1)^n u(n) + C_2(4)^n u(n)$$

Since 4 is a characteristic root & the excitation is;

$$x(n) = 4^n u(n)$$

we assume a particular solution of the form

$$y_p(n) = Kn4^n u(n)$$

Then,

$$Kn4^n u(n) - 3K(n-1)4^{n-1}u(n-1) - 4K(n-2)4^{n-2}u(n-2)$$
$$= 4^n u(n) + 2(4)^{n-1}u(n-1)$$

For $n = 2$

$$K(32-12) = 4^2+8 = 24$$
$$\Rightarrow K = 6/5$$

The total solution is;

$$y(n) = y_p(n) + y_h(n)$$
$$= \left[\tfrac{6}{5} n4^n + C_1 4 + C_2(-1)^n\right]u(n)$$

To solve for C_1 & C_2 we assume that

$y(-1) = y(-2) = 0$ then,

$y(0) = 1$ &

$y(1) = 3y(0) + 4 + 2 = 9$

Hence, $C_1 + C_2 = 1$ &

$$\frac{24}{5} + 4C_1 - C_2 = 9$$

$$4C_1 - C_2 = \frac{21}{5}$$

Therefore,

$$C_1 = \frac{26}{25} \text{ \& } C_2 = -\frac{1}{25}$$

The total solution is;

$$y(n) = \left[\frac{6}{5} n4^n + \frac{26}{25}4^n - \frac{1}{25}(-1)^n\right]u(n)$$

Q 1:- (Part-b):-

★ Solution:-

$$y(n) = 0.6y(n-1) - 0.8(n-2) + x(n)$$

Consider the different equation.

$$y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$$

~~y(n) = 0~~

$$y(n) - 0.6y(n-1) + 0.8y(n-2) = x(n)$$

To obtain the homogeneous equation set input $x(n) = 0$ then,

$$y(n) - 0.6y(n-1) + 0.8y(n-2) = 0$$

Determine the solution to the homogeneous equation

$$y_h(n) = \lambda^n$$

$$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 0.6\lambda + 0.08) = 0$$

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$$(\lambda - 0.2)(\lambda - 0.4) = 0$$

So, the roots are

$$\lambda_1 = 0.2 \ \& \ \lambda_2 = 0.4$$

Thus, the general form of solution to the homogeneous equation is

$$y_h(n) = C_1(\lambda_1)^n + C_2(\lambda_2)^n$$

$$y(n) = C_1(0.2)^n + C_2(0.4)^n$$

$$\lambda = 0.2, \ \lambda = 0.4$$

Hence, $y_h(n) = C_1\left(\frac{1}{5}\right)^n + C_2\left(\frac{2}{5}\right)^n$

with $x(n)=\delta(n)$, the initial condition are;

$$y(0)=1$$

$$y(1)-0.6y(0)=0$$

$$y(1)=0.6$$

Hence $C_1+C_2=1$ &

$$\frac{1}{5}C_1+\frac{2}{5}=0.6$$

So, $C_1=-1$, $C_2=3$

Therefore, $h(n)=\left[-\left(\frac{1}{5}\right)^n+2\left(\frac{2}{5}\right)^n\right]u(n)$

The step response is;

$$s(n)=\sum_{k=0}^{n} h(n-k) \quad , \quad n>0$$

$$=\sum_{k=0}^{n}\left[2\left(\frac{2}{5}\right)^{n-k}-\left(\frac{1}{5}\right)^{n-k}\right]$$

$$=\left\{\frac{1}{0.12}\left[\frac{2}{5}^{n+1}-1\right]-\frac{1}{0.16}\left[\frac{1}{5}^{n+1}-1\right]\right\}u(n)$$

Q2:- (Part-a)

* Solution:-

The z-transform is;

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

The expression can be written as;

$$\frac{1}{\left(1-\frac{2}{z}\right)\left(1-\frac{1}{z}\right)^2}$$

$$= \frac{1}{\left(\frac{z-2}{z}\right)\left(\frac{z-1}{z}\right)^2}$$

$$= \frac{1}{\frac{(z-2)(z-1)^2}{z^3}}$$

$$= \frac{z^3}{(z-2)(z-1)^2} \longrightarrow (1)$$

$X(z)$ has a simple pole at $P_1 = 2$ & a double $P_2 = P_3 = 1$. In such a case the appropriate Partial fraction expression is

$$X(z) = \frac{z^3}{(z-2)(z-1)^2} = \frac{A_1}{(z-2)} + \frac{A_2}{(z-1)} + \frac{A_3}{(z-1)^2}$$

Now determine the coefficient A_1, A_2, A_3 we proved that as in the case of distinct Pole to determine A_1, we multiply both sides of above eq by $(z-2)$

$$(z-2)\times X(z) = A_1 + \frac{z-2}{z-1}A_2 + \frac{z-2}{(z-1)^2}A_3$$

which evaluated at $z=2$

$$A_1 = \frac{(z-2)X(z)}{z}\Big|_{z=2} \quad \because z=2$$

$$A_1 = 4$$

$$A_2 = A_1 + \frac{z-2}{z-1}$$

$$A_2 = -3$$

$$A_3 = A_1 + \frac{z-2}{z-1}A_2$$

$$A_3 = -1$$

Hence $x(n) = [4(2)^n - 3 - n]u(n)$

Q2:- (Part-b)

* Solution:- First we eleminate the negative power by multiplying both numerator & denominator by z^2 then,

$$X(z) = \frac{z^2}{z^2 - 1.5z + 0.5}$$

The poles $X(z)$ are $P_1 = 1$ & $P_2 = 0.5$
Consequently, the expansion of the form

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A_1}{z-1} + \frac{A_2}{z-0.5}$$

To determine A_1 & A_2 then, multiply the equation by the denominator term $(z-1)(z-0.5)$ thus, we obtain

$$z = (z-0.5)A_1 + (z-1)A_2 \longrightarrow ①$$

Now if we set $z = P_1 = 1$ in eq ①
& we eleminate the term involving A_2
hence, $1 = (1-0.5)A_1$

Thus, we obtain the result $A_1 = 2$
& $z = P_2 = 0.5$ then, eleminating the term involving A_1 so we have

$$0.5 = (0.5-1)A_2$$

& hence $A_2 = -1$

Therefore, the result of the partial fraction expansion is;

$$\frac{X(z)}{(z)} = \frac{z}{z-1} - \frac{1}{z-0.5}$$

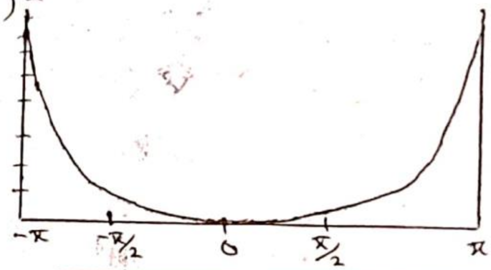
Q 3:- (Part - a)

Solution:-

At $\omega = 0$ we have

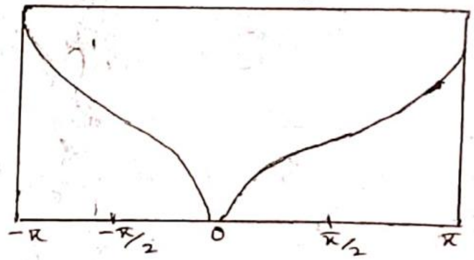
$$H(0) = \frac{b_0}{(1-P)^2} = 1$$

Hence $b_0 = (1-P)^2$



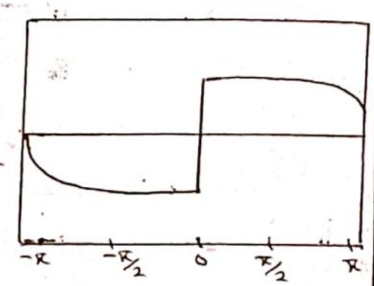
At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$



$$= \frac{(1-P)^2}{(1-P\cos(\pi/4)+(jP\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2}\; jP/\sqrt{2})^2}$$



Hence $\dfrac{(1-P)^4}{[(1-P/\sqrt{2})^2+P^2/2]^2} = 1/2$

Or equivalently

$$\sqrt{2}\,(1-P)^2 = 1 + P^2 - \sqrt{2}P$$

The value of $P = 0.32$ satisfies this equation. Consequently, the system function for the desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

The same principles can be applied for the design of bandpass filters.

Q 3:- (Part-b):-

★ Solution:-

Clearly, the filter must have poles at

$$P_{1,2} = re^{\pm j\pi/2}$$

& zeros at $z=1$ & $z=-1$

Consequently the system function is

$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G \frac{z^2-1}{z^2+r^2}$$

The gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$

Thus we have

$$H(\pi/2) = G \frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

~~The~~

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = \frac{1}{2}$$

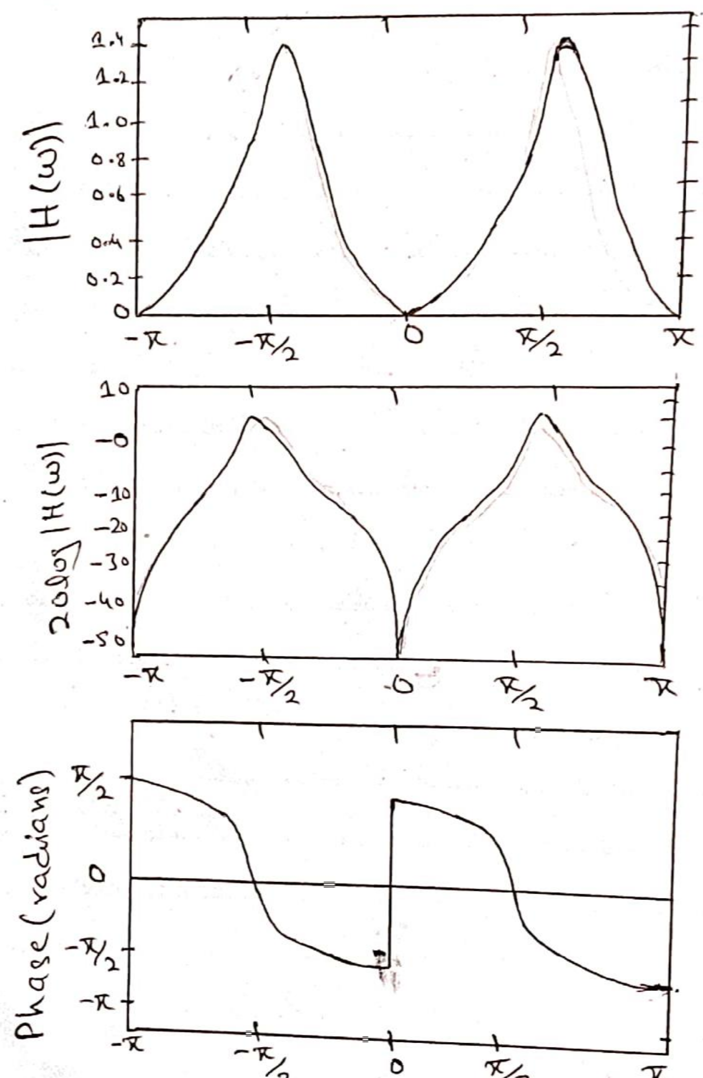
or equivalently,

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

The value of $r^2 = 0.7$ satisfies this equation. Therefore, the system function for the desired filter is

$$H(z) = 0.15 \frac{1 - z^{-2}}{1 + 0.7z^{-2}}$$

Its frequency response is illustrated in below figure.



Q 4:- (Part-a)

★ Solution:-

The Fourier transform of this sequence ~~For~~ is;

$$X(w) = \sum_{n=0}^{L-1} x(n) e^{-jwn}$$

$$= \sum_{n=0}^{L-1} e^{-jwn} = \frac{1-e^{-jwL}}{1-e^{-jw}}$$

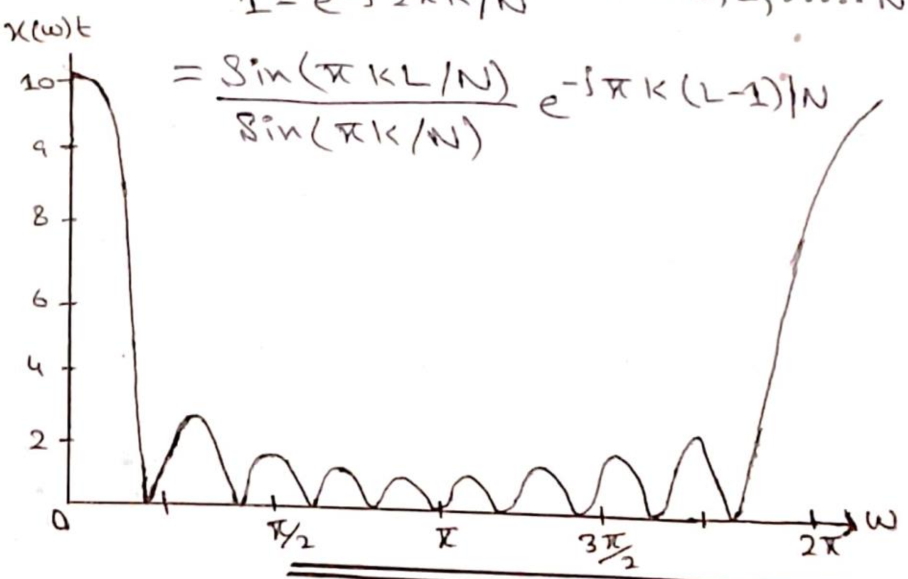
$$= \frac{\sin(wL/2)}{\sin(w/2)} e^{-jw(L-1)/2}$$

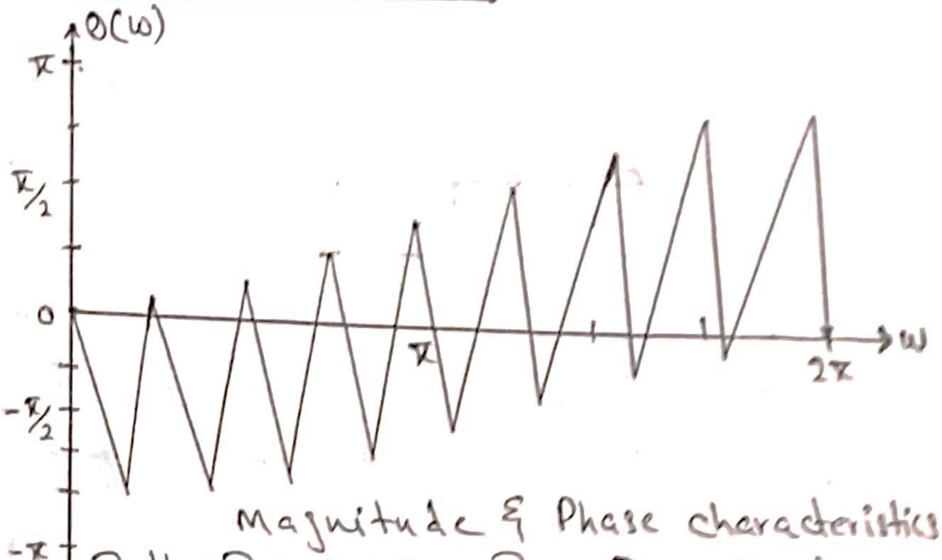
The magnitude & Phase of $X(w)$ are illustrated in below figure for $L=10$. The N-Point DFT of $x(n)$ is simply $X(w)$ evaluated at the set of N equally spaced frequencies

$W_K = 2\pi K/N$, $K = 0, 1, \ldots\ldots N-1$ Hence

$$X(K) = \frac{1-e^{-j2\pi KL/N}}{1-e^{-j2\pi K/N}} \quad K = 0, 1, \ldots\ldots N-1$$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$





Magnitude & Phase characteristics of the Fourier transform for signal.

If N is selected such that $N=L$ then the DFT becomes

$$X(K) \begin{cases} L, & K=0 \\ 0, & K=1,2,\ldots L-1 \end{cases}$$

Thus there is only Non-zero value in DFT. This is apparent from observation of $X(\omega)$, Since $X(\omega)=0$ at the frequencies $\omega_K = 2\pi K/L$ $K \neq 0$. The reader should verify that $x(n)$ can be recovered from $X(K)$ by performing an L-point IDFT.

Q 4:- (Part - b)

★ Solution:-

The first step is to determine the matrix W_4. By exploiting the periodicity property of W_4 & the symmetry property

$$W_N^{kn+2/2} = -W_N^k$$

the matrix W_4 may be expressed as;

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^2 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^4 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Then, $$Y_4 = W_4 y_4 \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

The DFT of Y_4 may be determined by conjugating the element in W_4 to obtain W_4^* & then applying the formula.