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Section B

Assignment of ~~the~~ Differential  
equation.

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Q 10

$$x^3 y''' + 2x^2 y' + 2y = 10x + \frac{10}{x}$$

Sol:

Here we need some correction because  $2x^2 y'$  are not same we should try to solve for  $2x^2 y''$

$$x^3 y''' + 2x^2 y'' + 2y = 10x + 10/x$$

$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10x + 10x^{-1}$$

$$(x^3 D^3 + 2x^2 D^2 + 2)y = 10x + 10x^{-1} \rightarrow \textcircled{1}$$

let  $x = e^t \Rightarrow t = \ln x$

$$xD = D$$

$$x^2 D^2 = D(D-1) = D^2 - D$$

$$x^3 D^3 = D(D-1)(D-2) = D^3 - 3D^2 + 2D$$

substituting in eq  $\textcircled{1}$

$$(D^3 - 3D^2 + 2D + D^2 - 2D)y = 10x + 10x^{-1}$$

$$(D^3 - D^2 + 2)y = 10x + 10x^{-1}$$

$$(m^3 - m^2 + 2)y = \frac{10}{e^{-t}} + \frac{10}{e^t}$$

(2)

using synthetic division

$$\begin{array}{r|rrrr} & 1 & -1 & 0 & 2 \\ -1 & & -1 & 2 & -2 \\ \hline & 1 & -2 & 2 & \underline{0} \end{array}$$

now using quadratic equation

$$a=1, b=-2, c=2$$

$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$\Delta = \frac{2 \pm \sqrt{4 - 8}}{2}$$

$$\Delta = \frac{2 \pm \sqrt{-4}}{2} \Rightarrow \frac{2 \pm \sqrt{(-2)^2}}{2}$$

$$\Delta = \frac{2 \pm 2i}{2} = \frac{2(1+i)}{2}$$

$$\frac{2(1-i)}{2}$$

$$y_c = e^{-x} (c_1 \cos x + 2 \sin x)$$

(3)

Now for Particular solution

$$y_p = \frac{1}{D^3 - D^2 + 2} \cdot 10e^t + \frac{1}{D^3 - D^2 + 2} \cdot 10e^{-1}$$

$$= \frac{10e^t}{(1)^3 - (1)^2 + 2} + \frac{10e^{-1}}{(1)^3 - (1)^2 + 2}$$

$$= 5e^t + 5e^{-1}$$

General sol:

$$y = y_c + y_p$$

$$y = e^{-x} (c_1 \cos t + c_2 \sin t) + 5e^t + 5e^{-1}$$

put  $e^t = x$  &  $t = \ln x$ .

$$y = e^{-x} (c_1 \ln x + c_2 \sin x) + 5e^x + 5e^{-x}$$

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2.2  
→

$$x^3 y''' + 4x^2 y'' - 5xy' - 15y = x^4.$$

$$x^3 \frac{d^3 y}{dx^3} + 4x^2 \frac{d^2 y}{dx^2} - 5x \frac{dy}{dx} - 15y = x^4.$$

$$\left( x^3 \frac{d^3}{dx^3} + 4x^2 \frac{d^2}{dx^2} - 5x \frac{d}{dx} - 15 \right) y = x^4.$$

$$(x^3 D^3 + 4x^2 D^2 - 5x D - 15) y = x^4 \rightarrow \textcircled{1}$$

$$\text{let } x = e^t \Rightarrow t = \ln x.$$

$$xD = \Delta.$$

$$x^2 D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta.$$

$$\Delta^3 D^3 = \Delta(\Delta - 1)(\Delta - 2) = \Delta^3 - 3\Delta^2 + 2\Delta.$$

substituting into eq  $\textcircled{1}$ .

$$\Delta^3 - 3\Delta^2 + 2\Delta + 4\Delta^2 - 4\Delta - 5\Delta - 15 = x^4.$$

$$\Rightarrow \Delta^3 + \Delta^2 - 7\Delta - 15 = e^{4t}.$$

Complementary set:  $y_c =$

solve by synthetic eq.

	1	1	-7	-15
3		3	12	15
	1	4	+5	0

(5)

$$\Delta^2 + 4\Delta + 5 = 0$$

$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2} \Rightarrow \frac{-4 \pm 2i}{2}$$

$$\Delta = \frac{-2 \pm i}{1} \Rightarrow$$

$$y_c = e^{4t} (c_1 \cos t + c_2 \sin t)$$

Particular integral

$$y_p = \frac{1}{F(\Delta)} e^{4t}$$

$$= \frac{1}{\Delta^3 + \Delta^2 - 7\Delta - 15} e^{4t}$$

$$= \frac{1}{4^3 + 4^2 - 7(4) - 15} e^{4t}$$

$$y_p = \frac{1}{37} e^{4t}$$

The general sol. is

$$y = y_c + y_p$$

$$y = e^{4t} (c_1 \cos t + c_2 \sin t) + \frac{1}{37} e^{4t}$$

Again put  $t = \ln x$  &  $u = \ln x$

$$y = e^{4u} (c_1 \cos \ln x + c_2 \sin \ln x) + \frac{1}{37} e^{4u}$$

Q3:

$$n^2 y'' + 2ny' - 6y = 10n^2$$

Sol

$$y(1) = 1 \quad \& \quad y'(1) = -6.$$

$$n^2 \frac{d^2 y}{dn^2} + 2n \frac{dy}{dn} - 6y = 10n^2$$

$$\left( n^2 \frac{d^2}{dn^2} + 2n \frac{d}{dn} - 6 \right) y = 10n^2$$

Put  $nD = \Delta$

$$n^2 D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta.$$

$$n = et \quad \& \quad \log n = t.$$

$$(\Delta^2 - \Delta + 2\Delta - 6) y = 10e^{2t}.$$

$$(\Delta^2 + \Delta - 6) y = 10e^{2t}$$

The characteristic equation

$$\Delta^2 + \Delta - 6 = 0$$

$$\Delta^2 + 3\Delta - 2\Delta - 6 = 0$$

$$\Delta(\Delta+3) - 2(\Delta+3) = 0$$

$$(\Delta+3)(\Delta-2)$$

$$\Delta = -3, \quad \Delta = 2.$$

$$\Delta = 2.$$

(7)

Since Root are <sup>Real.</sup> general & disti

For  $y_c = ?$

$$y_c = c_1 e^{-3t} + c_2 e^{2t}$$

For  $y_p = ?$

$$y_p = \frac{1}{\Delta^2 - \Delta - 6} 10e^{2t}$$

$$= \frac{10 e^{2t}}{\Delta^2 - \Delta - 6}$$

$$= \frac{10 \cdot 1}{0} e^{2t} \rightarrow \text{fail.}$$

now.

$$10 \frac{1}{\frac{d}{dn} (\Delta^2 + \Delta - 6)} e^{2t}$$

$$y_p = 2te^{2t}$$

General solution.

$$y = y_c + y_p \\ = c_1 e^{-3t} + c_2 e^{2t} + 2te^{2t}$$

$$y = c_1 n^{-3} + c_2 n^2 + 2(\log n) n^2 \rightarrow B.$$

Put  $y(1) = 1$  i.e.  $n=1, y=1$  in eq (1)

$$1 = c_1 + c_2 \rightarrow C$$



now differentiate eq B wrt  $x$ .

$$y' = -3c_1 x^{-4} + 2c_2 x + \frac{2}{x} (x^2 + 4x \log x)$$

$$-6 = -3c_1 + 2c_2 + 2 + 0$$

$$-8 = -3c_1 + 2c_2 \rightarrow$$

Multiplying eq (1) with (2) & subtracting from (2)

$$2c_1 + 2c_2 = 2$$

$$-8c_1 + 2c_2 = -8$$

$$5c_1 = 10$$

$$\boxed{c_1 = 2}$$

$$-8 = -3(2) + 2c_2$$

$$\boxed{c_2 = -1}$$

now put the value of  $c_1$  &  $c_2$  in eq B

$$y = 2x^{-3} - x^2 + 2(x \cdot x \log x)$$

$$y = \frac{2}{x^3} - x^2 + 2x^2 \log x$$

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Q 50

$$x^2 y'' + 7xy' + 5y = x^5$$

Solve

$$x^2 \frac{d^2 y}{dx^2} + 7x \frac{dy}{dx} + 5y = x^5$$

$$\left( x^2 \frac{d^2}{dx^2} + 7x \frac{d}{dx} + 5 \right) y = x^5$$

$$\left( x^2 D^2 + 7xD + 5 \right) y = x^5 \rightarrow \textcircled{1}$$

let  $x = et \Rightarrow t = \ln x$

$$xD = D$$

$$x^2 D^2 = D(D-1) = D^2 - D$$

Substituting into eq  $\textcircled{1}$

$$D^2 - D + 7D + 5 = 0$$

$$\Rightarrow (D^2 + 6D + 5) y = e^{5t}$$

Complementary sol:  $y_c$

$$D^2 + D + 5D + 5 = 0$$

$$D(D+1) + 5(D+1)$$

$$(D+1)(D+5)$$

$$D = -1, D = -5$$

$$y_c = C_1 e^{-x} + C_2 e^{-5x}$$

$$y_c = c_1 e^{-x} + c_2 e^{-5x}$$

Particular integral:

$$y_p = \frac{1}{\Delta^2 + 6\Delta + 5} e^{5t}$$

$$y_p = \frac{1}{25 + 30 + 5} e^{5t}$$

$$= \frac{1}{60} e^{5t}$$

General sol

$$y = c_1 e^t + c_2 e^{-5t} + \frac{1}{60} e^{5t}$$

Replace  $x = e^t \Rightarrow t = \ln x$

$$y = c_1 e^{\ln x} + c_2 e^{-5 \ln x} + \frac{1}{60} e^{5 \ln x} \rightarrow \textcircled{B}$$

$x=0$  Put in this eqn  
no in eq (B)  $e^0 = 1$

Put  $y(0) = 2$  i.e  $y = 2$  &  $x = 2$

$$2 = c_1 (2)^5 + c_2 (2)^{-5} + \frac{1}{60} (2)$$

$$2 = -32c_1 - 2c_2 + \frac{1}{60} (38)$$

$$2 = -32c_1 - 2c_2 + 8/15$$

$$\frac{22}{15} = -32c_1 - 2c_2 \rightarrow \textcircled{C}$$

now differentiate eq (B) wrt (x)

$$y' = -5c_1 x^{-6} - c_2 x^{-2} + \frac{1}{12} x^4 \rightarrow$$

Put  $y'(2) = 2$  i.e.  $y' = 2$  at  $x = 2$  in above eqn

$$2 = -5c_1 (2)^{-6} - c_2 (2)^{-2} + \frac{1}{12} (2)^4$$

$$2 = -5c_1 (-64) - c_2 (4) + \frac{1}{12} (16)$$

$$2 = 320c_1 + 4c_2 + \frac{4}{3}$$

$$\frac{2}{3} = 320c_1 + 4c_2 \rightarrow \textcircled{D}$$

ming eq (C) with 2 and then in eq (C)  
from (D)

$$-\frac{44}{15} = 64c_1 + 4c_2$$

$$-\frac{44}{15} = 64c_1 + 4c_1$$

$$\frac{34}{15} = -256c_1$$

$$c_1 = \frac{34}{15} \times 256$$

$$\boxed{c_1 = 580}$$

Put the value of  $c_1$  in eq (C)

$$\frac{22}{15} = -32(580) - 2c_2$$

$$\frac{22}{15} + 18560 = -2c_2$$

$$c_2 = -9280$$

Now put the value of  $c_1$  &  $c_2$  in eq (B).

$$y = 580x^{-5} - 9280x^{-1} + \frac{1}{60}x^5 \quad \text{Ans}$$

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Q. 58  $(x+1)^2 y'' - 3(x+1)y' + 4y = x^2$

let  $(x+1)^2 = z$

$= (x+1)^2 = z^2$

$x = e^t \Rightarrow \ln x = t$

$\Rightarrow z^2 y'' - 3zy' + 4y = x^2$

$zy' = \Delta$

$zy'' = \Delta(\Delta-1) = \Delta^2 - \Delta$

~~$\Delta^2 - \Delta - 3\Delta + 4$~~

$(\Delta^2 - \Delta - 3\Delta + 4)y = x^2$

$(\Delta^2 - 4\Delta + 4)y = x^2 e^{2t}$

complementary set  $y_c =$

$\Delta^2 - 2\Delta - 2\Delta + 4 = 0$

$\Delta(\Delta-2) - 2(\Delta-2)$

$(\Delta-2)(\Delta-2) = 0$

$\Delta = 2, 2$

$$y_c = (C_1 + C_2 n + \dots) e^{mn}$$

$$= (C_1 + C_2 n + \dots) e^{2n}$$

Particular Integral :-

$$y_p = \frac{1}{F(\Delta)} e^{2t}$$

$$= \frac{1}{\Delta^2 - 4\Delta + 4} e^{2t}$$

$$= \frac{1}{(2)^2 - 4(2) + 4} e^{2t}$$

$$\frac{1}{0} e^{2t}$$

$F(\Delta)$  should not equal to zero

$$\Rightarrow \Delta^2 - 4\Delta + 4$$

$$2\Delta - 4 \rightarrow (2)$$

If we put 2 in eq (2) ~~it~~  
 again give zero

take 2nd time derivatives

$$= \frac{dy}{d\Delta} (2\Delta - 4)$$

$$= 2 \Rightarrow y_p = \frac{1}{2} e^{2t}$$

$$\Rightarrow y = (C_1 + C_2 n)^{2t} + \frac{1}{2} e^{2t}$$

$$y_p = \frac{1}{2} e^{2t}$$

The general sol is

$$y = y_c + y_p$$

$$y = (c_1 + c_2 n + \dots) e^{2n} + \frac{1}{2} e^{2t} \frac{dus}{\cancel{dus}}$$

~~Replace  $n = et$   $\Rightarrow$   $y = (c_1 + c_2 e^t)$~~

~~$$y = (c_1 + c_2 e^t)$$~~