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Section B.

Subject Differential equation

Qno1:

$$W = \sin(x+ct) + \cos(2x+2ct).$$

Solution:

$$\frac{\partial w}{\partial t} = \cos(x+ct).c - \sin(2x+2ct).2c$$

$$\frac{\partial w}{\partial t} = \cos(x+ct) - 2c\sin(2x+ct)$$

$$\frac{\partial^2 w}{\partial t^3} = -c^2\sin(x+ct) - 2\cos(2x+2ct).2c$$

$$\frac{\partial^2 w}{\partial t^2} = -c^2\sin(x+ct) - 4c^2\cos(2x+2ct)$$

equ (a).

$$\frac{\partial w}{x} = \cos(x+ct) - \sin(2x+2ct).2$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$$c^2\frac{\partial^2 w}{\partial x^2} = -c^2\sin(x+ct) - 4c^2\cos(2x+2ct)$$

equ (b).

$$\frac{\partial^2 w}{\partial t^2} = c^2\frac{\partial^2 w}{\partial x^2}.$$

Q.No: (b).

$$\frac{\partial^2 w}{\partial t^2} = \frac{c^2 \partial^2 w}{\partial x^2}$$

Solution:

$$\frac{\partial w}{\partial t} = \sec^2(2x+ct)\frac{\partial}{\partial t}(2x+ct)$$

$$\Rightarrow c\sec^2(2x+ct).$$

$$\frac{\partial^2}{\partial t^2} = c2\sec(2x+ct)\frac{\partial}{\partial t}\sec(2x+ct).$$

$$= 2c^2\sec(2x+ct)\sec(2x+ct)\tan(2x+ct)$$

$$\frac{\partial^2 w}{\partial t^2} = 2c^2\sec^2(2x+ct)\tan(2x+ct)$$

$$\frac{\partial w}{\partial x} = 2\sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial w^2} = 2.2\sec(2x+ct).\sec(2x+ct)\tan(2x+ct).2$$

$$= 8\sec^2(2x+ct).\tan(2x+ct)$$

$$= 2c^2\sec^2(2x+ct).\tan(2x+ct) \neq c^2 8\sec^2(2x+ct)\tan(2x+ct)$$

Not satisfied - Answer.

Ans 2:

Given:

$$f(x) = \begin{cases} x \; ; & -\pi < x \le 0 \\ 2x \; ; & 0 \le x \le \pi \end{cases}$$

Solution:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = 1/\pi\int_{-\pi}^{0} x\,dx + 1/\pi\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= 1/\pi\left[0 - \pi^2/2\right] + \left\{\frac{2}{\pi}\left[\pi^2/2 - 0\right]\right.$$

$$a_0 = -\pi/2 + \pi = \pi/2$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx.$$

$$= 1/\pi\int_{-\pi}^{0} (x\cos nx\,dx + 1/\pi\int_{0}^{\pi} (2x\cos nx)\,dx$$

$$= 1/\pi\left[x\left(\frac{\sin nx}{n}\right) - \left(\frac{-\cos nx}{n^2}\right)\right]_{-\pi}^{0} + \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

$$= 1/\pi\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

$$a_n = \begin{cases} \frac{-2}{\pi n^2}, & (n \text{ is odd}) \\ 0, & (n \text{ is even}) \end{cases}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi}$$

$$\hookrightarrow x \sin nx.$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(\frac{-\sin nx}{n^2}\right)\right]_{-\pi}^{0} + \frac{2}{\pi}\left[x\left(\frac{-\cos nx}{n}\right)\right.$$

$$\left. - \left(\frac{-\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$b_n = \frac{1}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right]$$

$$= \frac{-3\cos n\pi}{n}$$

$$= \frac{3(-1)^{n+1}}{n}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx).$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2x-1)}{(2n-1)} + 3\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin nx}{n}$$

Qno3:

Given:

$$y'' - 4y' + 13y = 8\sin 3x$$

Required:

$$y = y_c + y_p$$

Solution:-

$$m^2 - 4m + 13 = 0$$

$$m = \frac{4 \pm \sqrt{16-52}}{2} = \frac{4 + 6i}{2}$$

$$2 \pm 3i.$$

$$y_c = e^{2x}\{C_1 \cos 3x + C_2 \sin 3x\}$$

$$y_p = \text{Imag}\left(\frac{1}{m^2-4m+13} 8e^{3ix}\right)$$

$$8\,\text{Imag}\left(\frac{1}{(3i)^2-4(3i)+13}\right.$$

$$= 8\,\text{Imag}\,\frac{e^{3ix}}{-9-12i+13}$$

$$2\ \mathrm{Imag}\ \frac{e^{3ix}}{(1-3i)} \times \frac{1+3i}{1+3i} \Rightarrow 2.\ \mathrm{Imp}\ \frac{(1+3i)e^{3ix}}{(1)^2-(3i)^2}$$

$$\frac{2}{10}\ \mathrm{Imp}\ (1+3i)(\cos 3ix + 2\sin 3x)$$

$$\frac{1}{5}(i\sin 3x + 3\cos 3x)$$

$$y = y_c + y_p$$

$$C_1 e^{2x}\cos 2x + C_2 e^{2x}\sin 3x + 1/5(\sin 3x + 3\cos 3x)$$

$$y(0) = C_1 + 1/5(3)$$

$$1 = C_1 + 3/5\ ,\ C_1 = 1 - 3/5 = 2/5$$

$$C_1 = 2/5$$

$$y'(0) = 2$$

$$y' = C_1 2e^{2x}\cos 3x + C_1 e^{2x}(-3\sin 3x) + 2e^{2x}\sin 3x\ C_2$$

$$+ C_2 e^{2x}(3\cos 3x) + \frac{2}{10}(\cos 3x - \sin 3x).$$

$$= 2C_1 + 0 + 0 + C_2 3(1) + \frac{2}{10}(1 - 3(0))$$

$$2 = 2C_1 + 3C_2 + 2/10$$

$$2 = 4/5 + 3C_2 + 1/5$$

$$2 = 9/5 + 1/5 + 3C_2$$

$$2 = 1 + 3C_2$$

$$2 - 1 = 3C_2$$

$$C_2 = 1/3$$

$$y = 9/5\, e^{2x} \cos 3x + 1/3\, e^{2x} \sin 3x + 1/3\,(\sin 3x + 2\cos 3x)$$

Q no 4:

Given:

$$(D^2 - DD') = \cos x \cos 2y.$$

Solution:

$$D(D - D')u = \cos x \cos 2y$$

$$CF = \phi_1(y) + \phi_2(y+x)$$

$$PI = \frac{1}{(D^2 - DD')} \cdot \frac{1}{2}\left[\cos(x+2y) + \cos(x-2y)\right]$$

$$= \frac{1}{2}\left[\frac{1}{(-1+2)}\cos(x+2y) + \frac{1}{(-1-2)}\cos(x-2y)\right]$$

$$= \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$

$$u = \phi_1(y) + \phi_2(y+x) + \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$