

Department of Electrical Engineering
Assignment
Date: 14/04/2020

Course Details

Course Title:	<u>Signals and Systems</u>	Module:	<u>6th</u>
Instructor:	<u>Engr; Amir Aman</u>	Total Marks:	<u>30</u>

Student Details

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1



$$y[n] + 0.567 y[n-1] + 33.3 y[n-2] + y[n-4] = x[n]$$

For unit step $x[n] = 10 u[n]$
with $y[-1] = 1, y[-2] = -1$

part - (1) Homogeneous and particular solution

As we know that the total solution is equal to the sum of homogeneous & particular sol.

$$y[n] = y_h[n] + y_p[n]$$

Homogeneous solution:

the characteristic equation is

Replace $y[n]$ with d^n so

$$d^n + 0.567 d^{n-1} + 33.3 d^{n-2} + d^{n-4} = 0$$
$$d^{n-4} (d^4 + 0.567 d^3 + 33.3 d^2 + 1) = 0$$

either $(d^4 + 0.567 d^3 + 33.3 d^2 + 1) = 0$, $d^{n-4} = 0$.

Solve the roots with the help of calculator by himself.
The find (if the roots are real & not equal)
 $y_h[n] = C_1 d_1^n + C_2 d_2^n + C_3 d_3^n + C_4 d_4^n$
or if the roots are real & repeated then
 $y_h[n] = C_1 d_1^n + C_2 n d_1^n + C_3 n^2 d_1^n + C_4 n^3 d_1^n$

particular sol:

suppose $y_p[n] = 10 k u[n]$

$$\Rightarrow 10 k u[n] + 0.567 (10 k u[n-1]) + 33.3 * 10 k u[n-2] + 10 k u[n-4] = 10 u[n]$$

$$\Rightarrow 10 k + 5.67 k + 333 k + 10 k = 10$$
$$k (10 + 5.67 + 333 + 10) = 10$$

~~k = 10/358.67~~ $k = 10/358.67$

$$y_p[n] = \frac{10}{358.67} * 10 u[n] = \frac{20}{358.67} u[n] = \frac{20}{358.67}$$

$$d^2(d^2 + 0.6d + 33.4) = -1$$

So No w to finding the value

Using calculator to find the value

$$d^2 = -1$$

$$d^2 = 1$$

$$d = \pm 1$$

$$d^2 + 0.6d + 33.4 = -1$$

$$d^2 + 0.6d = -1 - 33.4$$

$$d = 34.4$$

$$d = 34.9$$

Now as we know we have 3 different roots: 1 imaginary & 2 real & non-repeated

* For imaginary roots:

$$y_h(n) = c_1 \cos \lambda_1^n + c_2 \sin \lambda_2^n$$

$$y_h(n) = c_1 \cos(1)^n$$

* For real & non-repeated roots.

$$y_h(n) = c_1 \lambda_1^n + c_2 \lambda_2^n + c_3 \lambda_3^n$$

\therefore as we have λ_2 & λ_3 , so

$$= c_1 \lambda_1^n + c_2 (-34.4)^n$$

$$+ c_3 (-35.04)^n$$

Putting value of $c_1 \lambda_1^n = c_1 \cos \lambda_1^n$

$$\Rightarrow \boxed{y_h(n) = c_1 \cos(1)^n + c_2 (-34.4)^n + c_3 (-35.04)^n}$$

Homogeneous

Solution

* Particular Solution:-

As we know that

$$y_p(n) = 10 u(n)$$

$$\text{So } \Rightarrow 10u(n) + 0.6(10)ku(n-1) + 33.4(10)ku(n-2) + (1)(10)ku(n-4) = 10u(n)$$

* Now for unit step = 1 = $u(n)$

$$\Rightarrow 10k + 6.04k + 333k + 10k = 10$$

$$\Rightarrow k(10 + 6.04 + 333k + 10) = 10$$

$$\Rightarrow k (359.04) = 10$$

Dividing by "359.04"

$$k = \frac{10}{359.04}$$

$$\Rightarrow \boxed{k = 0.0278}$$

Now

$$\Rightarrow y_p(n) = 10 k u(n)$$

$$= 10 \times \frac{10}{359.67} u(n)$$

$$= 10 \times 0.0278 u(n)$$

$$\Rightarrow y_p(n) = 2.8 u(n)$$

$$\boxed{\Rightarrow y_p(n) = 2.8}$$

Now for total solution

$$y(n) = y_h(n) + y_p(n)$$

$$= c_1 \cos(1)^n + c_2 (-34.4)^n + (-35.04)^n + 2.8$$

$$\Rightarrow y(n) = c_1 \cos(1)^n + c_2 (-34.4)^n + c_3 (-35.04)^n + 2.8$$

Total solution

Now Applying initial conditions

$$\therefore c_1 \cos(1)^{-1} = 0$$

$$\Rightarrow c_1 \cos(-1) = 0$$

$$\Rightarrow c_1 = 0 / \cos(-1) = 0$$

$$\boxed{c_1 = 0}$$

$$\circ y(-1) = 1$$

$$c_1 \cos(1)^{-1} + c_2 (-34.4)^{-1} + c_3 (-35.04)^{-1} = 1$$

$$= -c_1 + \left(\frac{-1}{34.4}\right) c_2 + \left(\frac{-1}{35.04}\right) c_3 = 1$$

$$= 0 \cdot 0 \cdot 0.02 c_2 - 0.029 c_3 = 1 = \textcircled{1}$$

$$\Rightarrow y(-1) = +0.02 c_2 - 0.029 c_3 = 1 = \textcircled{1}$$

Now Applying 2nd condition:

$$y(-2) = -1$$

$$= c_1 \cos(1)^{-2} + c_2 (-34.4)^{-2}$$

$$+ c_3 (-35.04)^{-2} = -1$$

$$= 0 + \left(\frac{-2}{34.04}\right) c_2 + \left(\frac{-2}{35.04}\right) c_3 = -1$$

$$= 0.05 c_2 - 0.067 c_3 = -1 = \textcircled{2}$$

Now xing eq (1) with "5"

$$\Rightarrow -5(-0.02 c_2 - 0.029 c_3) = 1(-5)$$

$$\Rightarrow 0.1 c_2 + 0.015 c_3 = -5 \Rightarrow \textcircled{3}$$

Also xing eq (2) with "2"

$$2(-0.05C_2 - 0.057C_3) = -2 \rightarrow (4)$$

Applying eq (3) & eq (4)

$$0.1C_2 + 0.014C_3 = -5$$

$$\underline{-0.1C_2 - 0.114C_3 = -2}$$

$$-0.1C_3 = -5 - 2$$

$$-0.1C_3 = -7$$

$$C_3 = \frac{-7}{-0.1}$$

$$\boxed{C_3 = 70}$$

Now putting value of c_3 into eq (3)

$$\Rightarrow 0.1c_2 + 0.014(70) = -5$$

$$\Rightarrow 0.1c_2 + 0.98 = -5$$

$$\Rightarrow 0.1c_2 = -5 - 0.98$$

$$\Rightarrow \frac{0.1c_2}{0.1} = \frac{-5.98}{0.1}$$

$$\boxed{\Rightarrow c_2 = -59.8}$$

Part b :-

Zero input & zero state :-

So the Homogeneous & Particular and the zero input zero state solution answer are same

So for zero input:

$$y_h(n) = c_1 \cos(17^n) + c_2 (-34.04)^n + c_3 (-35.04)^n$$

and for zero state:-

$$y_p[n] = \cos 4(n)$$

which will be

$$y_p(n) = 2.7$$

\Rightarrow total solution will be ; a

$$y(n) = y_h(n) + y_p(n)$$

$$\Rightarrow y(n) = c_1 \cos(1)^n + c_2 (-34.04)^n + c_3 (-35.04)^n + 2.7$$

Now putting the 4 random value is total solution;

Ex : $y(n) = n = 1, 2, 3, 4$

Now $y(n) = 1 = n \Leftarrow$

$$= c_1 \cos(1)' + c_2 (-34.04)' +$$

$$c_3 (-35.04)' + 2.7$$

$$\Rightarrow y(1) = c_1 (1) + c_2 (-34.04) + c_3 (-35.04) + 2.8$$

Now Applying initial conditions

$$\therefore c_1 \cos(1)^{-1} = 0$$

$$\Rightarrow c_1 \cos(-1) = 0$$

$$\Rightarrow c_1 = 0 / \cos(-1) = 0$$

$$\boxed{c_1 = 0}$$

$$\textcircled{1} \quad y(-1) = 1$$

$$c_1 \cos(1)^{-1} + c_2 (-34.4)^{-1} + c_3 (-35.04)^{-1} = 1$$

$$= -c_1 + \left(\frac{-1}{34.4}\right) c_2 + \left(\frac{-1}{35.04}\right) c_3 = 1$$

$$= 0.002 c_2 - 0.029 c_3 = 1 = \textcircled{1}$$

$$\Rightarrow y(-1) = 0.002 c_2 - 0.029 c_3 = 1 = \textcircled{1}$$

Now Applying 2nd condition:

$$y(-2) = -1$$

$$= c_1 \cos(1)^{-2} + c_2 (-34.4)^{-2}$$

$$+ c_3 (-35.04)^{-2} = -1$$

$$= 0 + \left(\frac{-2}{34.4}\right) c_2 + \left(\frac{-2}{35.04}\right) c_3 = -1$$

$$= 0.05 c_2 - 0.067 c_3 = -1 = \textcircled{2}$$

Now xing eq (1) with "5"

$$\Rightarrow -5(-0.002 c_2 - 0.029 c_3) = 1(-5)$$

$$\Rightarrow 0.01 c_2 + 0.015 c_3 = -5 \Rightarrow \textcircled{3}$$

* Now 2nd

$$y(2) = c_1 (\cos(1))^2 + c_2 (-34.04)^2 + c_3 (-35.04) + 2.7$$
$$= c_1 \cos(1) + c_2 (34.04)^2 + c_3 (35.04)^2 + 2.7$$

$$\Rightarrow y(2) = c_1 + 1186.4c_2 + 1221.04c_3 + 2.7$$

* Now 3rd

$$y(3) = c_1 \cos(1)^3 + c_2 (-34.04)^3 + c_3 (-35.04)^3 + 2.7$$

$$y(3) = c_1 \cos(1) + c_2 (-3944.8) + c_3 (-43022.16) + 2.7$$

$$= c_1 \cos(1) - 3944.8c_2 - 43022.16c_3 + 2.7$$

* Now 4th

$$y(4) = c_1 \cos(1)^4 + c_2 (-34.04)^4 + c_3 (-35.04)^4 + 2.7$$

$$= c_1 \cos(1) + c_2 (1342635.74) + c_3 (1507496.76) + 2.7$$

$$= c_1 + 1342635.74c_2 + 1507496.76c_3 + 2.7$$

$$c_1 = 1342635.74$$

$$c_2 = 1507496.76$$

$$c_3 = 2.7$$

Q-2. Part - (a)

$$x(t) = 5000 \cos 5.0\pi t + \sin 0.5\pi t + 5.89 \cos 10\pi t$$

For Sampling frequency,

$$f_s \geq 2 f_m$$

So

$$x(t) = 5000 \cos 5.0\pi t + \sin 0.5\pi t + 5.89 \left(\frac{1}{2} (\sin(10\pi + 0.5\pi)t + \sin(10\pi - 0.5\pi)t) \right) + \sin 100\pi t$$

$$\Rightarrow x(t) = 5000 \cos 5.0\pi t + \sin 0.5\pi t + 2.94 (\sin 10.5\pi t - \sin 9.5\pi t) + \sin 100\pi t$$

$$\Rightarrow x(t) = 5000 \cos 5.0\pi t + \sin 0.5\pi t + 2.94 \sin 10.5\pi t - 2.94 \sin 9.5\pi t + \sin 100\pi t$$

Now

$$\omega_1 = 5.0\pi \Rightarrow \omega_1 f_1 = 5.0\pi \Rightarrow f_1 = 2.5 \text{ Hz}$$

$$\omega_2 = 0.5\pi \Rightarrow \omega_2 f_2 = 0.5\pi \Rightarrow f_2 = 0.25 \text{ Hz}$$

$$\omega_3 = 10.5\pi \Rightarrow \omega_3 f_3 = 10.5\pi \Rightarrow f_3 = 5.25 \text{ Hz}$$

$$\omega_4 = 9.5\pi \Rightarrow \omega_4 f_4 = 9.5\pi \Rightarrow f_4 = 4.75 \text{ Hz}$$

$$\omega_5 = 100\pi \Rightarrow \omega_5 f_5 = 100\pi \Rightarrow f_5 = 25 \text{ Hz} \rightarrow f_{\text{maximum}}$$

So

$$f_s \geq 2 f_m$$

$$f_s \geq 2(25)$$

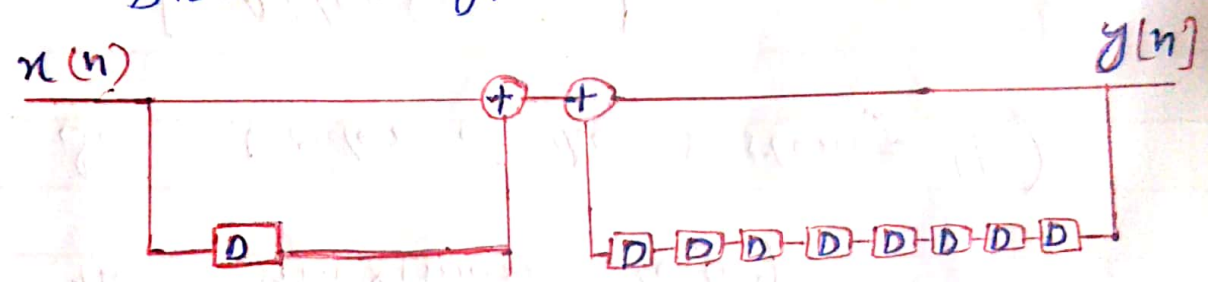
$$f_s \geq 50 \text{ Hz}$$

* ii) $y[n] - 10.3y[n-8] = x[n] + 3x[n-1]$

Sol

$\Rightarrow y[n] = 10.3y[n-8] + x[n] + 3x[n-1]$

Block diagram



(order = 8)

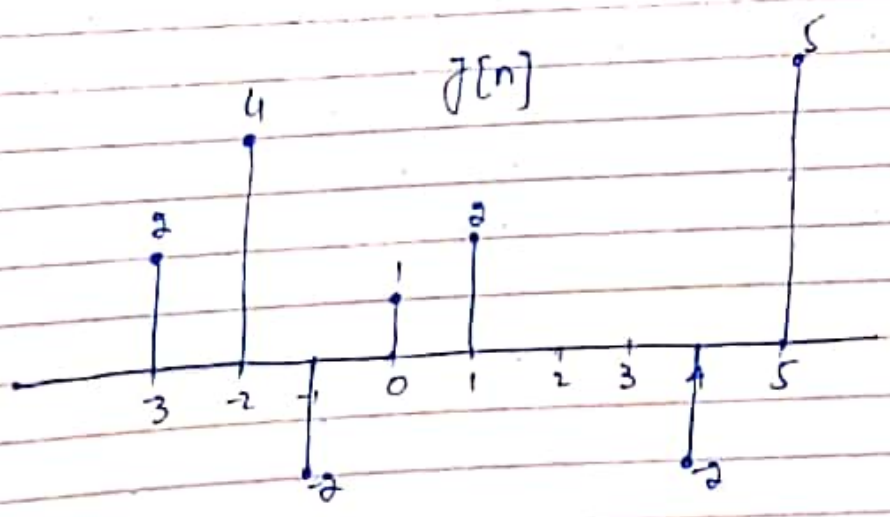
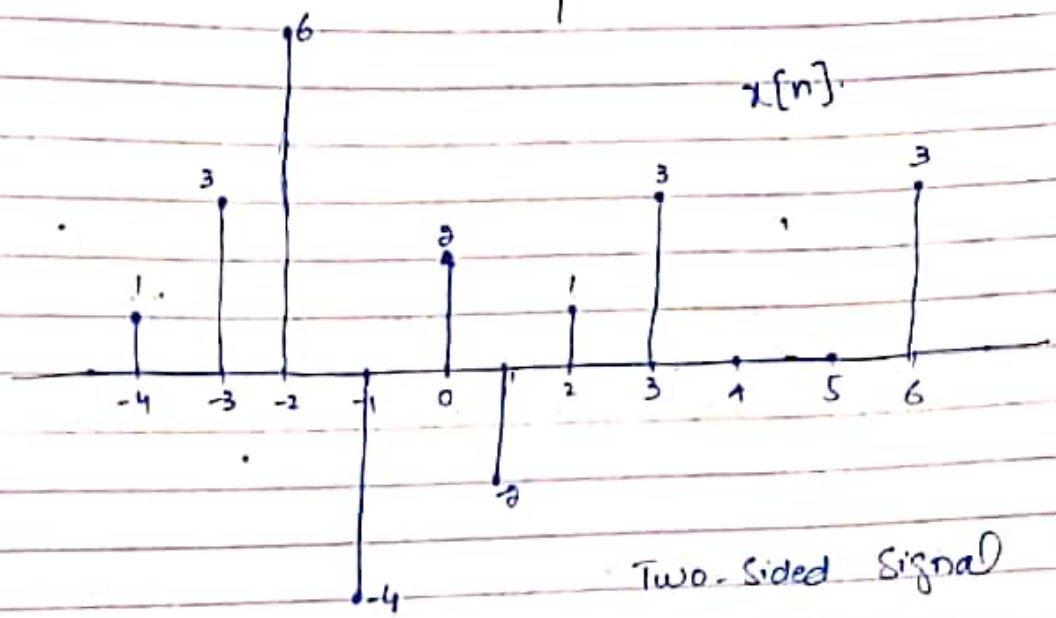
(adders = 2)

(scalars = 2)

Q.3:

x[n] = [1, 3, 6, -4, 5, -2, 1, 3, 0, 0, 3]

y[n] = [3, 4, -2, 1, 2, 0, 0, -2, 5]



Q No: 3

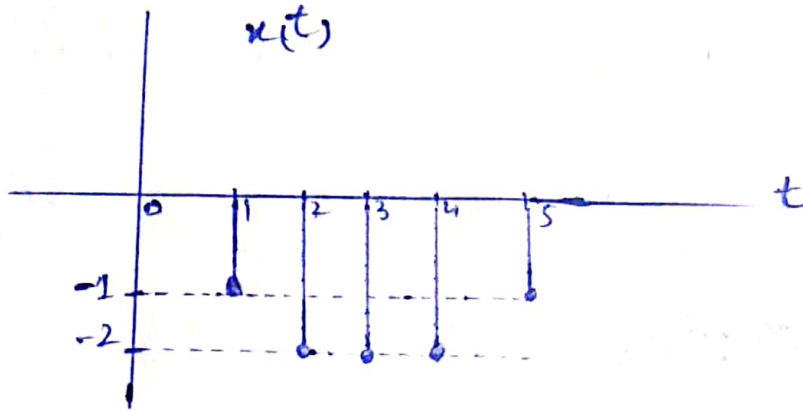
Part (b)

$$x(t) = (-1, -1, -2, -2, -2, -1)$$

0 1 2 3 4 5

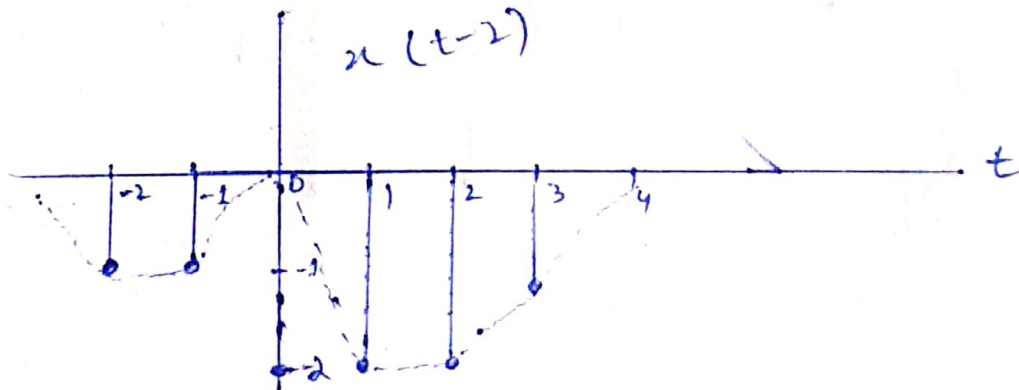
Plot

Sol



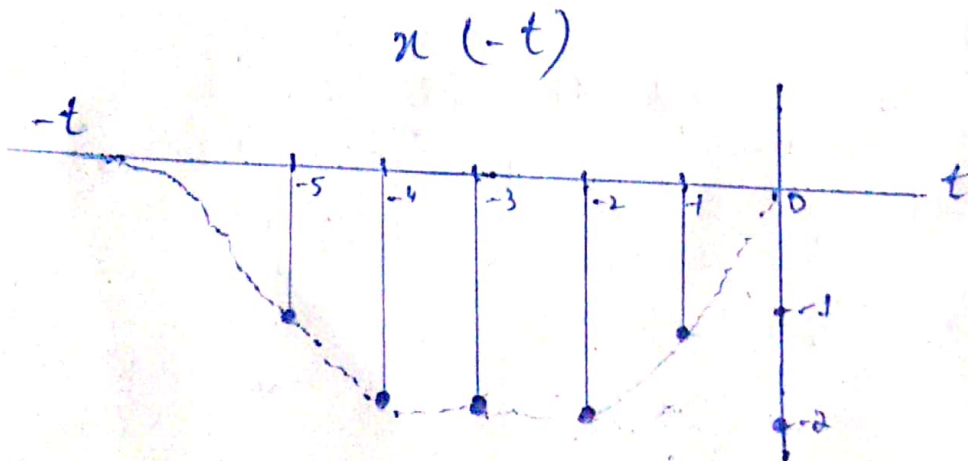
(i)

$$x(t-2)$$



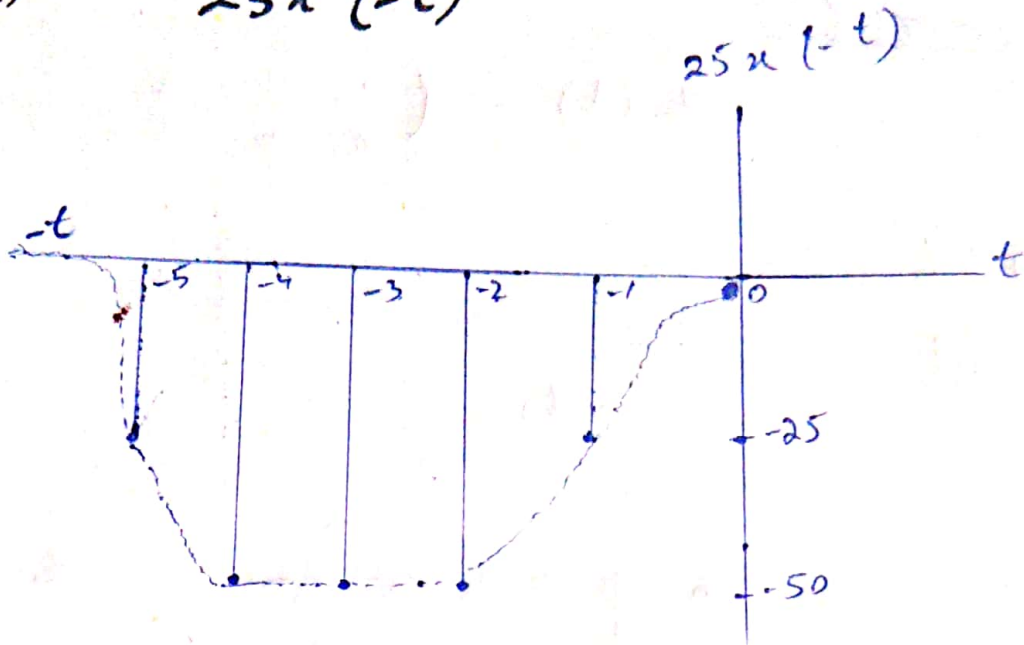
(ii)

$$x(-t)$$



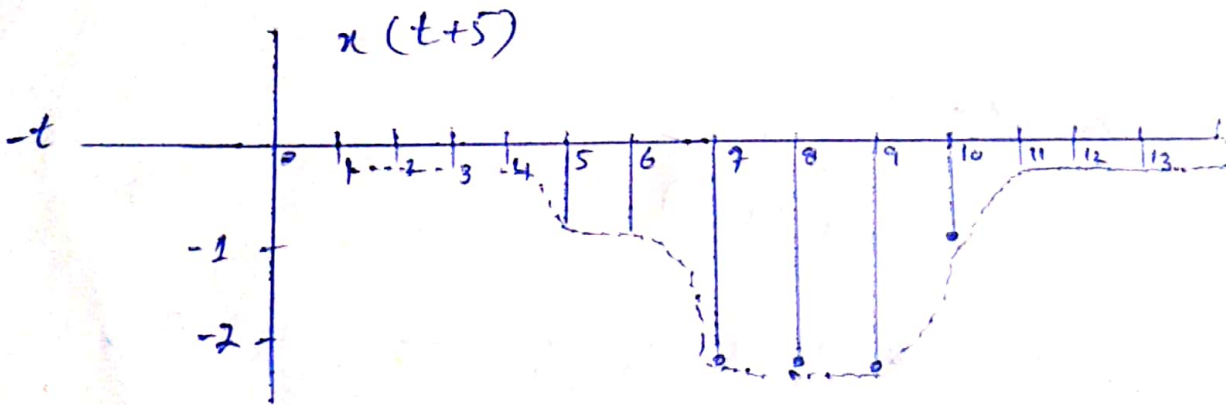
(iii)

$$25x(-t)$$



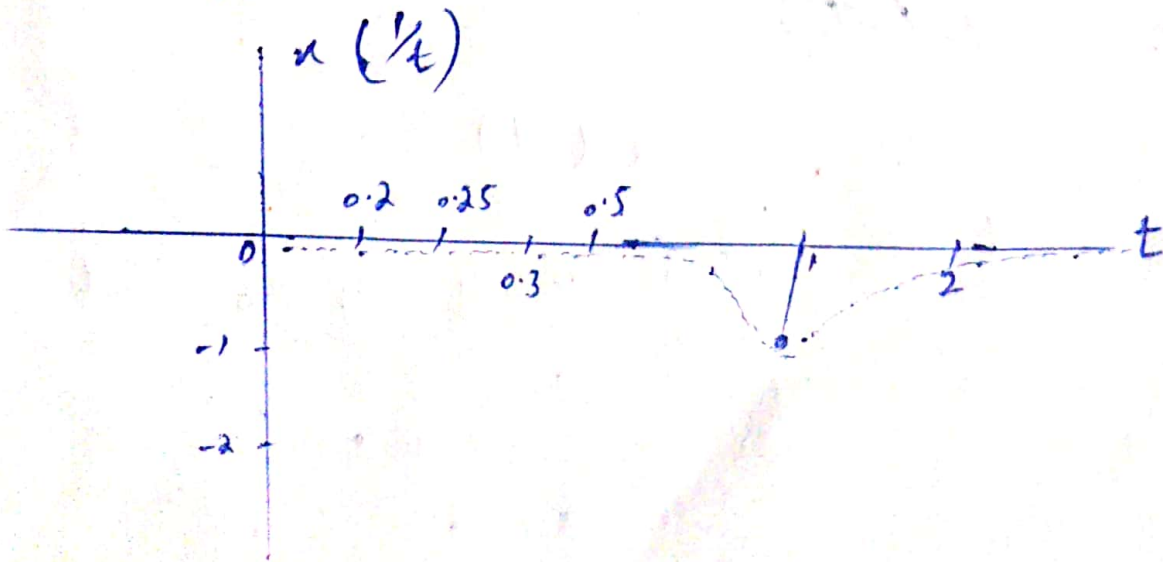
(iv)

$$x(t+5)$$



(v)

$$x(1/t)$$



So

$$\text{at } t=0 \quad u\left(\frac{1}{0}\right) \Rightarrow u(\alpha) \Rightarrow u(\alpha) = 0$$

$$\text{at } t=1 \quad u\left(\frac{1}{1}\right) \Rightarrow u(1) \Rightarrow u(1) = 0$$

$$\text{at } t=2 \quad u\left(\frac{1}{2}\right) \Rightarrow u(0.5) \Rightarrow u(0.5) = 0$$

$$\text{at } t=3 \quad u\left(\frac{1}{3}\right) \Rightarrow u(0.33) \Rightarrow u(0.33) = 0$$

$$\text{at } t=4 \quad u\left(\frac{1}{4}\right) \Rightarrow u(0.25) \Rightarrow u(0.25) = 0$$