1) $x^3 y''' + 2x^2 y' + 2y = 10x + \frac{10}{x}$

Sol:-

$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{dy}{dx} + 2y = 10x + 10x^{-1}$

$x^3 D^3y + 2x^2 D^2 + 2y = 10x + 10x^{-1}$

let

$x = e^t \Rightarrow t = \ln x.$

$xD = D.$

$x^2 D^2 = D(D-1) = D^2 - D$

$x^3 D^3 = D(D-1)(D-2).$

substituting

$(D - 3D^2 + 2D + \cancel{2}(D^2 - D) + 2)\, y = 10x - 10x^{-1}$

$(D^3 - D^2 + 2)y = 10x + 10x^{-1}$

$(m^3 - m^2 + 2)y = 10e^t + \frac{10}{e^t}$

using synthetic division

-1	1	-1	0	2
		-1	2	-2
	1	-2	2	$\boxed{0}$

using quadritic formula

$a = 1,\ b = -2,\ c = 2$

$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$

2

$D = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2a}$

$D = \frac{2 \pm \sqrt{4-8}}{2}$

$D = \frac{2 \pm \sqrt{-4}}{2} \Rightarrow D = \frac{2 \pm 2i}{2}$

$\Rightarrow \frac{\cancel{2}(1 \pm i)}{\cancel{2}} \Rightarrow \boxed{1 \pm i}$

roots are complex

$y_c = e^{x}(c_1 \cos t + c_2 \sin t)$

Now

$y_p = \frac{1}{D^3 - D^2 + 2} \cdot 10e^{t} + \frac{1}{D^3 - D^2 + 2} \cdot 10/e^{t}$

$= \frac{10e^{t}}{(1)^3 - (1)^2 + 2} + \frac{10e^{t}}{(1)^3 - (1)^2 + 2}$

$\frac{\overset{5}{\cancel{10}}e^{t}}{\cancel{2}} + \frac{\overset{5}{\cancel{10}}e^{-t}}{\cancel{2}}$

$y_p = 5e^{t} + 5e^{-t}$

General sol:-

$y = y_c + y_p$

$y = e^{t}(c_1 \cos t + c_2 \sin t) + 5e^{t} + 5e^{-t}$

put $e^{t} = x$ and $t = \ln x$

$y = e^{x}(c_1 \ln x + c_2 \sin \ln x) + 5e^{x} + 5e^{-x}$

Ans

Q2:

(2) $x^3 \frac{d^3y}{dx} + 4x^2 \frac{d^2y}{dx} - 5x\frac{dy}{dx} - 15y = x^4$

Let $\frac{d}{dx} = D$.

$x^3 D^3 y + 4x^2 D^2 y - 5xDy - 15y = x^4$

$(x^3 D^3 + 4x^2 D^2 - 5x D - 15) y = x^4$

Let

$x = e^t \Rightarrow t = \ln x$

$xD = D$

$x^2 D^2 = D(D-1) = D^2 - D$

$x^3 D^3 = D(D-1)(D-2) = D^3 - 3D^2 + 2D$

New substituting.

$(x^3 D^3 + 4x^2 D^2 - 5xD - 15)y = x^4$

$(D^3 - 3D^2 + 2D + 4(D^2 - D) - 5(D) - 15)y = e^{4t}$

$(D^3 + D^2 - 7D - 15)y = ex$

$$\begin{array}{c|cccc} 5 & 1 & 1 & -7 & 15 \\ & & 3 & 12 & 15 \\ \hline & 1 & 4 & 5 & \boxed{0} \end{array}$$

$D^2+4D+5=0$

By quadratic formula

$$D = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2-4(1)(5)}}{2}$$

$$= \frac{-4 \pm \sqrt{16-20}}{2}$$

$$= \frac{-4 \pm 2i}{2}$$

$$= \frac{2(-2 \pm i)}{2}$$

$$= -2 \pm i$$

$y_c = e^{-2x}(c_1 \cos t + c_2 \sin t)$

For y_p?

$$y_p = \frac{1}{D^3+D^2-7D-15} \cdot e^x$$

$$= \frac{1}{(4)^3+(4^2)-7(4)-5} e^{4t}$$

$$= \frac{1}{64+16-28-15}$$

$$= \frac{1}{80-43} e^{4t} \Rightarrow y_p = \frac{1}{37} e^{4t}$$

~~Here~~

$$y = y_c + y_p$$

$$y = c_1 \cos t + c_2 \sin t + \frac{1}{37} e^{4t}$$

again put $t = \ln x$ and $x = \ln x$

$$y = e^{2x} (c_1 \cos \ln x + c_2 \sin \log x) + \frac{1}{37} e^{4x \ln x}$$

Q3:-

$x^2 y'' + 2xy - 6y = 10x^2$

Sol:-

$y(1) = 1$ and $y'(1) = -6$

$$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 6y = 10x^2$$

$$\Rightarrow (x^2 \frac{d^2}{dx^2} + 2x \frac{d}{dx} - 6) y = 10x^2$$

put $xD = D \Rightarrow x^2D^2 = D(D-1) = D^2 - D$

(3)

$x = e^t$ and $\log x = t$

$(D^2 - D + 2D - 6)y = 10e^{2t}$

$(D^2 + D - 6)y = 10e^{2t}$

The characteristic eq

$D^2 + D - 6 = 0$

$D^2 + 3D - 2D - 6 = 0$

$\Rightarrow D(D+3) - 2(D+3) = 0$

$\Rightarrow (D+3)(D-2) = 0$

$D + 3 = 0, \; D - 2 = 0$

$D = -3 \;\&\; D = 2$

since root are real & distinct

For y_c :-

$y_c = c_1 e^{-3t} + c_2 e^{2t}$

For y_p :-

$$y_p = \frac{1}{D^2 - D - 6} \cdot 10e^{2t}$$

$$= \frac{10}{D^2 - D - 6} e^{2t} \Rightarrow 10 \frac{1}{0} e^{2t}$$

Now

$$10 \frac{1}{D^2 (D^2+D-6)} e^{2t}$$

$$\Rightarrow 10 \frac{t}{2D+1} e^{2t}$$

$$= 10 \frac{1 \cdot t}{4+1} e^{2t}$$

$$\Rightarrow y_p = 2t e^{2t}$$

∴ General sol:-

$$y = y_c + y_p$$
$$= C_1 e^{-3t} + C_2 e^{2t} + 2te^{2t}$$ Ans

$$y = C_1 x^{-3} + C_2 x^2 + 2(\log x) x^2 \quad \text{—} (i)$$

Put $y(1) = 1$, $\therefore x=1, y=1$ in —(i)

$$1 = C_1 (1)^{-3} + C_2 (1)^2 + 2\log(1)$$
$$1 = C_1 + C_2 \quad \text{—} (A)$$

Now differentiating eq(B) w.r.t x.

$y_1 = -3C_1 x^4 + 2(C_2 x + 2/x (x^2) + C_2 x \log x)$

Now put $y(1) = -6$ i-e $y = -6$ and $x = -6$.

$-6 = -3C_1 + 2(C_2 + 2 + 0)$

~~$-6 = -3(3)(4)$~~

$-6 = -3C_1 + 2C_2 - 2$

$-8 = -3C_1 + 2C_2 \rightarrow (D)$

multip eq(C) with (2) and Subtract from (D)

$2C_1 + 2C_2 = 2$

$\mp 8C_1 \pm 2C_2 = \mp 8$

$5C_1 = 10$

$C_1 = \frac{10}{5} \Rightarrow \boxed{C_1 = 2}$

$-8 = -3(2) + 2C_2$

$-8 = -3(2) + 2C_2$

$-8 = -6 + 2C_2$

$-8 + 6 = 2C_2$

$2C_2 = -2$

$C_2 = -1$

Now putting values

$y = 2x^3 - x^2 + 2\ln x \; x(x^2)$

$y = \frac{2}{x^3} - x^2 + 2x^2 \log x$ Ans.

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Q4:-

$x^2y'' + 7xy' + 5y = x^5$

$y(0) = 1$ and $y(1) = 2$.

Sol:-

$$x^2\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 5y = x^5.$$

$$\Rightarrow \left(x^2\frac{d^2}{dx^2} + 7x\frac{d}{dx} + 5\right)y = x^5 \rightarrow (A).$$

put

$xD = D \Rightarrow x^2D^2 = D(D-1) = D^2 - D$

$x = e^t \Rightarrow \log x = t$ in eq(A)

~~put~~

$\cancel{xD = D} \Rightarrow$

$\Rightarrow (D^2 - D + 7D + 5)y = e^{5t}$

$\Rightarrow (D^2 + 6D + 5)y = e^{5t}$

By formula

$$D = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36-20}}{2}$$

$$= \frac{-6 \pm \sqrt{16}}{2}$$

$$= \frac{-6 \pm 4}{2}$$

$$= \frac{\cancel{2}(-3 \pm 2)}{\cancel{2}}$$

$$= (-3 \pm 2.$$

tho NOW

$$y_c = c_1 e^{5t} + c_2 e^{-t}$$

FOR $y_p = ?$

$$y_p = \frac{1}{D^2 + 6(D) + 5}$$

$$y_p = \frac{1}{25 + 6(5) + 5} e^{5t}$$

$$= \frac{1}{60} e^{5t}$$

Now General eq sol

$y = y_c + y_p$

$y = c_1 e^{-5t} + c_2 e^{-t} + \frac{1}{60} e^{5t}$.

$y = c_1 x^{-5} + c_2 x^{-1} + \frac{1}{60} x^5 \rightarrow (B)$

$x = 0$ put in this eq

Now

put $y(0) = 2$ ie $y = 2$ and $x = 2$

$2 = c_1 (2)^{-5} + c_2 (2)^{-1} + \frac{1}{60} (2)^5$

$2 = -32 c_1 - 2 c_2 + \frac{1}{60} (32)$

$2 = -32 c_1 - 2 c_2 + \frac{8}{15}$.

$2 - \frac{8}{15} = -32 c_1 - 2 c_2$

$\frac{22}{15} = -32 c_1 - 2 c_2 \rightarrow (C)$.

Now diff eq (B) w r t (x)

$y' = -5 c_1 x^{-6} - c_2 x^{-2} + \frac{1}{12} x^4$

Put $y'(1) = 2$ i.e $y' = 2$ and $x = 2$

$2 = -5 c_1 c_2^{-6} \; c_2 (2)^{-2} + \frac{1}{12} (2)^4$.

$2 = -5 c_1 (-64) - 2 c_2 (4) + \frac{1}{12} (16)$

$2 = 32c_1 + 4c_2 + \frac{4}{3}$

$\Rightarrow 2 - \frac{4}{3} = 320c_1 + 4c_2$

$\Rightarrow \frac{2}{3} = 320c_1 + 4c_2 \rightarrow (D)$

Xing eq (C) with ~~on~~ 2 and then subtract eq(C) from D

$-\frac{44}{15} = 64c_1 + 4c_2$

$-\frac{44}{15} = 64c_1 + 4c_1$

$+\frac{2}{3} = \pm 320c_1 \pm 4c_2$

$\frac{34}{15} = -256$

$c_1 = \frac{34}{15} \times 256 \Rightarrow c_1 = 580$

put c_1 ind (C)

$\frac{22}{15} = -32(580) - 2c_2$

$\frac{22}{15} = -18560 - 2c_2$

$\Rightarrow \frac{15}{15} + 18560 = -2c_2$

$\Rightarrow \frac{18561}{-2} = c_2 \quad \Rightarrow c_2 = -9280.$

Now put c_1 and c_2 in eq(5).

$y = 580x^{-5} - 9280x^{-1} + \frac{1}{80}x^5.$

$y = \frac{580}{x^5} - \frac{9280}{x} + \frac{1}{80}x^5$ Ans

Q 5:-

$(x+1)^2y'' - 3(x+1)'y' + 4y = x^2$

Sol:-

$(x+1)^2\frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 4y = x^2$

$\Rightarrow \left((x+1)^2\frac{d^2}{dx^2} - 3(x+1)\frac{d}{dx} + 4\right)y = x^2$

$\Rightarrow \left((x+1)^2D^2 - 3(x+1)D + 4\right)y = x^2 \rightarrow (i)$

put $(x+1)D = \mathcal{D} \Rightarrow (x+1)^2D^2 = \mathcal{D}(\mathcal{D}-1)$ $\mathcal{D}-\mathcal{D}$

$x = e^t$ in eq(A)

$\Rightarrow (D^2 - D - 3D + 4)y = e^{2t}$

$\Rightarrow (D^2 - 4D + 4)y = e^{2t}$

$\Rightarrow (D^2 - 4D + 4)z = e^{2t}$

For y_c

$D^2 - 4D + 4 = 0$

$D^2 - 2D - 2D + 4 = 0$

$D(D-2) - 2(D-2) = 0$

$(D-2)(D-2) = 0$

$D = 2$, $D = 2$

Now

$y = (c_1 + c_2 x)e^{2x}$

$y = (c_1 + c_2 x)e^{2x}$

for $y_p = ?$

$y_p = \frac{1}{D^2 - 4D + 4}$ $(D)^2 - 4(2) + 4$

$y_p = \frac{2}{2D - 4} e^{2t}$

put
$D=2$

$2D-4 \Rightarrow 2(2)-4=0$

$y_p = \frac{2}{2} \cdot e^{2x}$

$y = (c_1 + c_2 x)e^{2x} + e^{2x}$ Ans