Topic

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Subject :- Differential equation.

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Question No 1 :-

Estimate the general solution of $4y'' - 20y + 25y = 0$

Solution:-

As the characteristics equation is given by:-

$$\lambda^2 + a\lambda + b = 0 \quad - (1)$$

For a general form of:

$$y'' + ay' + by = 0 \quad - (2)$$

So first to modify the given eq (A) by ÷ing it by 4

$$\frac{4y''}{4} - \frac{20y'}{4} + \frac{25y}{4} = \frac{0}{4}$$

$$\Rightarrow y'' - 5y' + \frac{25}{4} = \frac{0}{4}$$

from 2 & 3

$$a = -5 \quad \& \quad b = \frac{25}{4}$$

Putting a & b in eq (1)

$$\lambda^2 - 5\lambda + \frac{25}{4} = 0$$

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(2)

$4\lambda^2 - 20\lambda + 25 = 0$

using quadratic formula:

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \begin{aligned} a &= 4 \\ b &= -20 \\ c &= 25 \end{aligned}$$

$$= \frac{-(-20) \pm \sqrt{(-20)^2 - 4(4)(25)}}{2(4)}$$

$$\lambda_{1,2} = 20 \pm \sqrt{400 - 400}$$

$$\lambda_{1,2} = \frac{20 \pm 0}{8} = \frac{\cancel{20}^{5}}{\cancel{8}_{2}}$$

$$\lambda_{1,2} = \frac{5}{2} \Rightarrow \lambda_1 = \frac{5}{2} \;\&\; \lambda_2 = \frac{5}{2}$$

The roots are real & repeated equal:

So the general solution will be:

~~$y = C$~~ $y = (C_1 + C_2 x)\, e^{\lambda x}$

$\Rightarrow y = (C_1 + C_2 x)\, e^{5x/2}$ is the required estimated general solution.

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Question No 2:-

Calculate the initial value problem

$$y'' + 2y + y = 0 \quad , y(0) = 4 , \; y'(0) = -6$$

Solution:-

we can write the given eq as

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

we can write it as differential operator as:-

$$D^2y + 2D + y = 0$$

its auxillary equation will be:

$$D^2 + 2D + 1 = 0$$

$$D^2 + 1D + 1D + 1 = 0$$

$$D(D+1) + 1(D+1) = 0$$

$$(D+1)(D+1) = 0$$

$$\Rightarrow D = 1, 1$$

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④

So the roots are real & Some equal. So the general solution y_c will be.

$$y = y_c = (c_1 + c_2 x) e^x \quad - ①$$

Now using the initial condition $y(0) = 4$

$$\Rightarrow 4 = (c_1 + c_2(0)) e^{(0)} \Rightarrow \boxed{4 = c_1} \quad - Ⓐ$$

taking derivative of eq ①

$$\frac{dy}{dx} = \frac{dy_c}{dx} = \frac{d}{dx}\left\{(c_1 + c_2 \cdot x) e^x\right\}$$

$$\Rightarrow y' = y_c' = \frac{d}{dx}(c_1 e^x + c_2 x e^x)$$

$$= c_1 \frac{d}{dx} e^x + \frac{c_2 d}{dx} x e^x$$

$$c_1 e^x + c_2 (x e^x + e^x)$$

$$y' = y_c' = c_1 e^x + c_2 e^x + c_2 x e^x$$

Now using the second initial condition:

$$y'(0) = -6$$

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$\Rightarrow -6 = c_1 e^{(0)} + c_2 e^{(0)} + c_2 \cancel{(0)}^0 e^{(0)}$

$\Rightarrow -6 = c_1 + c_2$ — (B)

putting (A) in eq (B)

$-6 = 4 + c_2 \Rightarrow c_2 = -6 - 4$

$\boxed{c_2 = -10}$

putting values of c_1 & c_2 in eq (1)

$y = y_c = (4 - 10x) e^{x}$

is the required solution to the given initial value problem.

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For $K = 0$

$\Rightarrow y_p = x^0(Ax^3 + Bx^2 + Cx + D)$

$y_p = Ax^3 + Bx^2 + Cx + D \quad — ①$

To find co-efficients.

$y_p' = 3Ax^2 + 2Bx + C \quad — ②$

$y_p'' = 6Ax + 2B \quad — ③$

putting ①, ②, ③ in main eq.

$y'' + y' - 6y = 6x^3 - 3x^2 + 12x + 0$

$\Rightarrow (6Ax + 2B) + (3Ax^2 + 2Bx + C) + Ax^3 + Bx^2 + Cx + D$
$= 6x^3 - 3x^2 + 12x + 0$

$Ax^3 + 3Ax^2 + Bx^2 + 6Ax + 2Bx + Cx + C + D$
$= 6x^3 - 3x^2 + 12x + 0$

Now By comparing co-efficients on B.H.S,

$A = 6, \quad 3A + B = -3, \quad 6A + 2B + C = 12, \quad C + D = 0$

as $\boxed{A = 6} \Rightarrow 3(6) + B = -3 \Rightarrow B = -3 - 18$

$\Rightarrow \boxed{B = -21} \Rightarrow 6(6) + 2(-21) + C = 12$

$\Rightarrow C = 12 - 36 + 42 \Rightarrow \boxed{C = 18}$

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Question 3:-

Examine the method of undetermined co-efficient method for

$$y'' + y - 6y = 6x^3 - 3x^2 + 12x$$

Solution:-

The equation given is a a non-homogeneous equation & its solution can be given as...

$$y = y_c + y_p \quad -① $$

First to find y_c:

we can write the ~~giver~~ given eq as only L.H.S in differential operator forms

$$Dy^2 + D - 6 = 0$$

$$\Rightarrow D^2 - 3D - 2D - 6 = 0$$

$$D(D-3) + 2(D-3) = 0$$

$$\Rightarrow D = 3 \ \& \ D = -2$$

roots are real & unique so

$$\left(y_c = c_1 e^{3x} + c_2 e^{-2x}\right)$$

Now for y_p:

As, $f(x) = 6x^3 - 3x^2 + 12x$ (is polynomial)

so

$$y_p = x^k (Ax^3 + Bx^2 + Cx + D)$$

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