Department of Electrical Engineering
Final Exam Assignment
Date: 27/06/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: **Sir Pir Meher Ali Shah** **Total Marks:** 50

Student Details

Name: Talha Khan **Student ID:** 13845

Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$ To the input $(n) = (-1)^n u(n)$. And the initial conditions are y (-1) = y (-2) = 0.	**Marks 7** **CLO 2**
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$	**Marks 7** **CLO 2**
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	**Marks 6** **CLO 2**
	(b)	Evaluate the inverse z- transform using the complex inversion integral $$X(z) = \frac{1}{1-az^{-1}} \qquad \lvert z\rvert > \lvert a\rvert$$	**Marks 6** **CLO 2**
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response H(ω) satisfies the condition H(0) = 1 and $\lvert H(\frac{\pi}{4})\rvert^2 = \frac{1}{2}$.	**Marks 6** **CLO 3**

	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.	**Marks 6** **CLO 3**
Q 4	(a)	A finite duration sequence of Length L is given as $$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & otherwise \end{cases}$$ Determine the N- point DFT of this sequence for $N \ge L$	**Marks 6** **CLO 2**
	(b)	Perform the circular convolution of the following two sequences. Solve the problem step by step $$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$ $$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$	**Marks 6** **CLO 2**

Q 1: (a)

Sol:- $y(n)-4y(n-1)+4y(n-2)$

$= x(n) - x(n-1)$

The characteristic equation is

$\lambda^2-4\lambda+4=0$

$\lambda = 2, 2$ Hence,

$y_h(n) = c_1 2^n + c_2 n 2^n$

$y_p(n) = k(-1)^n u(n).$

difference equation we obtain

$$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n=2$, $k(1+4+4) = 2 \Rightarrow K = \frac{2}{9}$

The total Solution is

$$y(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n \right] u(n)$$

from the initial condition, we obtain $y(0) = 1$, $y(1) = 2$ Then,

$$c_1 + \frac{2}{9} = 1$$

$$\Rightarrow c_1 = \frac{7}{9}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 2$$

$$\Rightarrow c_2 = 1/3.$$

Q1:(b)

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

Sol:- equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = \frac{1}{2}, \frac{1}{5}$ Hence,

~~y(n) (n) =~~

$$y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n$$

with $z(n) = \delta(n)$, we have

$$y(0) = 2,$$

$$y(1) - 0.7\,y(0) = 0 \Rightarrow y(1) = 1.4$$

$$c_1 + c_2 = 2 \quad \text{and}$$

$$\frac{1}{2} c_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$$

$$c_1 + \frac{2}{5} c_2 = \frac{14}{5}$$

These eq yield

$$C_1 = \frac{10}{3}, C_2 = \frac{-4}{3}.$$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right] u(n)$$

The step respone is

$$S(n) = \sum_{k=0}^{n} h(n-k),$$

$$= \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - 4/3 \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n$$

$$\sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left(\frac{1}{2}^{n}(2^{n+1}-1)u(n) - \frac{1}{3}\right.$$

$$\left(\frac{1}{5}(5^{-n+1}-1)u(n)\right.$$

Q 2: (a)

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$\underline{\text{Sol}}$:- By partial fraction methods.

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$$= \frac{A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1})}{(1-2z^{-1})(1-z^{-1})^2}$$

$$1 = A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1})$$
$$+ C z^{-1}(1-2z^{-1}) \text{ — } ①$$

Put $z=1$

$$1 = A(1-0)^2 + B(1-2)(1-1) + c(1)(1-2)$$
$$1 = 0+0-c$$
$$1 = -c$$
$$\boxed{C = -1}$$

Put $z=2$ in eq ①

$$1 = A\left(1-\frac{1}{2}\right)^2 + B\left(1-\frac{2}{2}\right)\left(1-\frac{1}{2}\right) +$$
$$c\left(\frac{1}{2}\right)\left(1-\frac{2}{2}\right)$$
$$1 = A\left(\frac{1}{2}\right)^4 + B(1-1)\left(\frac{1}{2}\right) + c\left(\frac{1}{2}\right)(1-1)$$
$$1 = \frac{A}{4} + B(0)\left(\frac{1}{2}\right) + c\left(\frac{1}{2}\right)(0)$$

So, $\cancel{1 = A\left(\frac{1}{2}\right)^4 + B(1-1)\left(\frac{1}{2}\right) +}$

$$1 = \frac{A}{4} + 0 + 0$$

$$\boxed{A = 4}$$

Put $z = 3$ in eq 1

$$1 = A\left(1-\frac{1}{3}\right)^2 + B\left(1-\frac{2}{3}\right)\left(1-\frac{1}{3}\right) + C\left(\frac{1}{3}\right)\left(1-\frac{2}{3}\right)$$

$$1 = A\left(\frac{4}{9}\right) + B\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + C\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$$

$$1 = \frac{4}{9}A + \frac{2}{9}B + \frac{1}{9}C$$

$$1 = \frac{4(4)}{9} + \frac{2}{9}B - \frac{1}{9}$$

$$1 + \frac{1}{9} - \frac{16}{9} = \frac{2}{9}B$$

$$-\frac{6}{9} \times \frac{9}{2} = B$$

$$\boxed{-3 = B}$$

Hence

$$x(n) = [4(2)^n - 3 - n]\, u(n).$$

Q 2: (b)

Sol:- $x(z) = \dfrac{1}{1 - az^{-1}}$ $|z| > |a|$

Solution:- ~~whe~~ we have

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{z^{n-1}}{1 - az^{-1}} dz = \frac{1}{2\pi j} \oint_c \frac{z^n dz}{z - a}$$

where c is a circle at radius greater that |a|. we shall evaluate this integral with $f(z) = z^n$ we distingusish two cases.

1. If $n \geq 0$, $f(z)$ has only zeros and hence no pole inside c. The only pole inside c is $z = a$ Hence

$$x(n) = f(z_0) = a^n, \quad n \geq 0$$

2. If $n < 0$, $f(z) = z^n$ has an nth - order pole at $z = 0$ which is also inside c. Thus there are contribution from both poles For $n = -1$ we have

$$x(-1) = \frac{1}{2\pi j}\oint_c \frac{1}{z(z-a)}\,dz = \frac{1}{z-a}$$

$$\Big|_{z=0} + \frac{1}{z}\Big|_{z=a} = 0$$

If $n=-2$, we have

$$x(-2) = \frac{1}{2\pi j}\oint_c \frac{1}{z^2(z-a)}\,dz = \frac{d}{dz}$$

$$\left(\frac{1}{z-a}\right)\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=a} = 0$$

By continuing in the same way we can show that $x(n)=0$ for $n<0$. Thus

$$x(n) = a^n u(n)$$

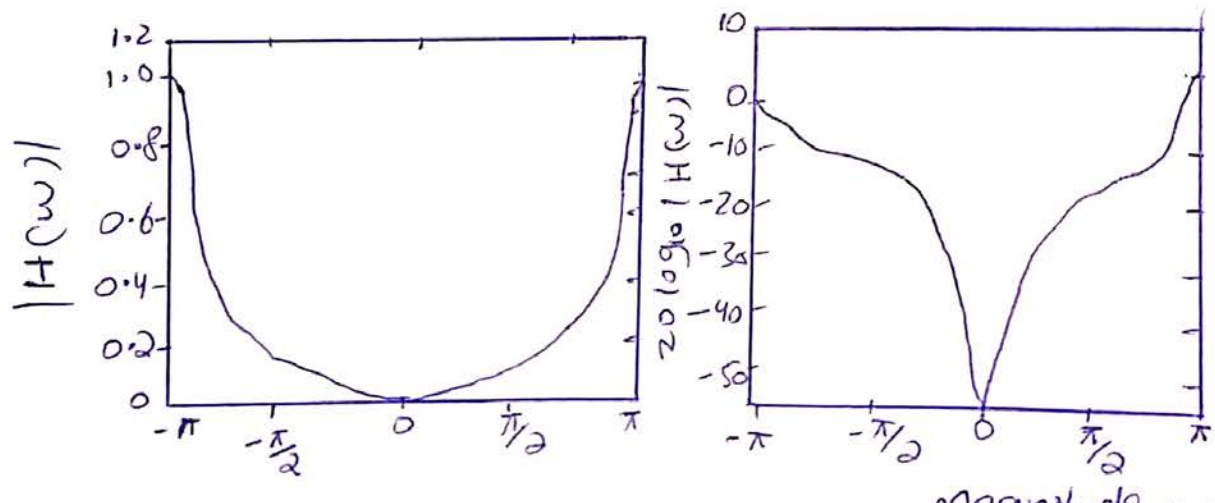
Q3: (a)

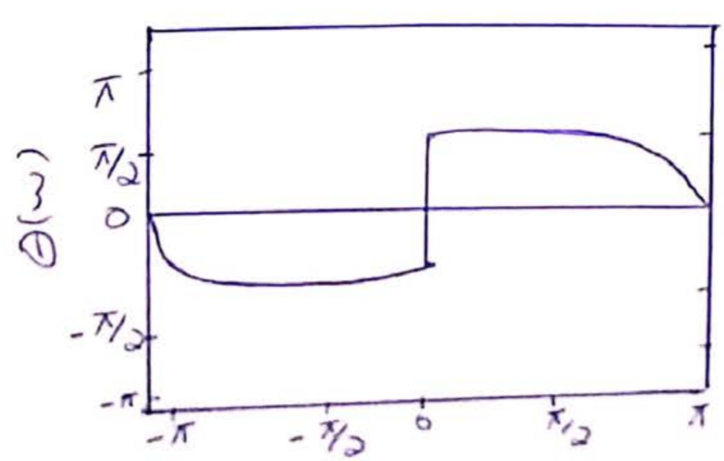
Sol:- At $\omega=0$ we have

$$H(0) = \frac{b_0}{1-p^2} = 1$$

Hence

$$b_0 = (1-p)^2$$





Magnitude and Phase a simple high pass filter: $H(z)=[(1-a)/2][(1-z^{-1})/(1+az^{-1})]$ with $a=0.9$.

At $w = \pi/4$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\pi/4)+jp\sin(\pi/4))^2}$$

Hence

$$= \frac{(1-p)^2}{(1-p/\sqrt{2}+jp/\sqrt{2})^2}$$

$$\frac{(1-p)^4}{(1-p/\sqrt{2})^2+p^2/\sqrt{2})^2} = \frac{1}{2}$$

or, equivalently.

$$\sqrt{2}\,(1-p)^2 = 1+p^2-\sqrt{2}p$$

The value of $p = 0.32$ Satisfies this equation. Consequently the System function for the desired Filter is

$$H(z) = \frac{0.46}{(1 - 0.32z^{-1})^2}$$

Q 3: (b)

<u>Sol</u>:- 8 Clearly the filter must have pole at

$$P_{1,2} = re^{\pm j\pi/2}$$

and zeros $z = 1$ and $z = -1$. Consequently the ~~same~~ System function is

$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G \frac{z^2-1}{z^2+r^2}$$

The gain factor is determined by evaluating the frequency responce $H(\omega)$ of the Filter at $\omega = \pi/2$.

$$H(\pi/2) = G \frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$.

Thus we have

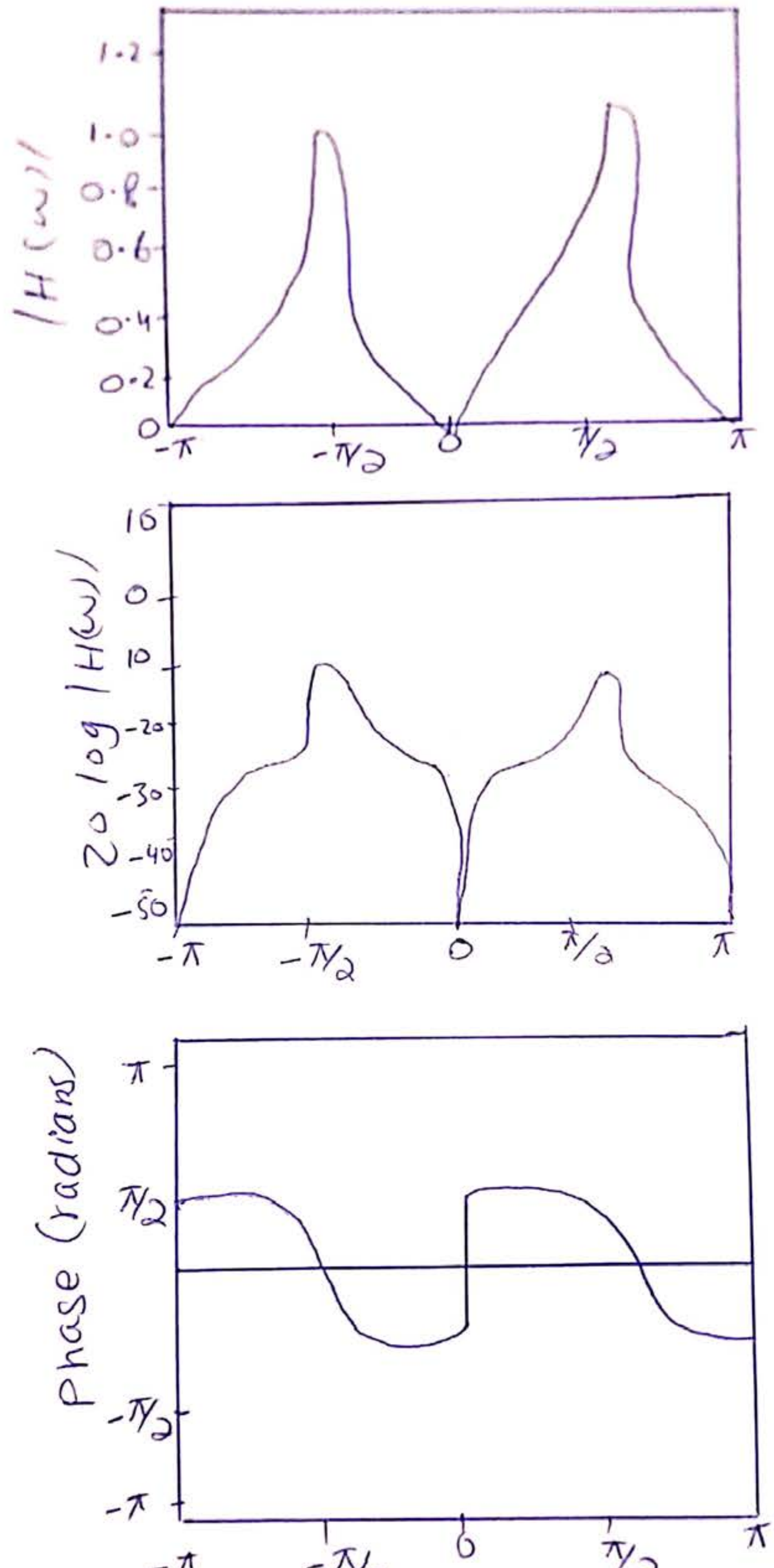
$$\left| H\left(\tfrac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = \frac{1}{2}$$

or. equivalently,

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

The value of $r^2 = 0.7$ satisfies this equation. Therefore the system function for the desired filter is

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$



Magnitude and phase response of a simple bandpass filter $H(z) = 0.15[(1-z^{-2}) / (1+0.7z^{-2})]$

Q 4: (a) A finite duration sequence
Sol:- of Length L is given as

$$x(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & \text{otherwise} \end{cases}$$

Determine the N- Point DFT of this sequence for $N \geq L$

Sol:- The Fourier transform of this sequence is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}}$$

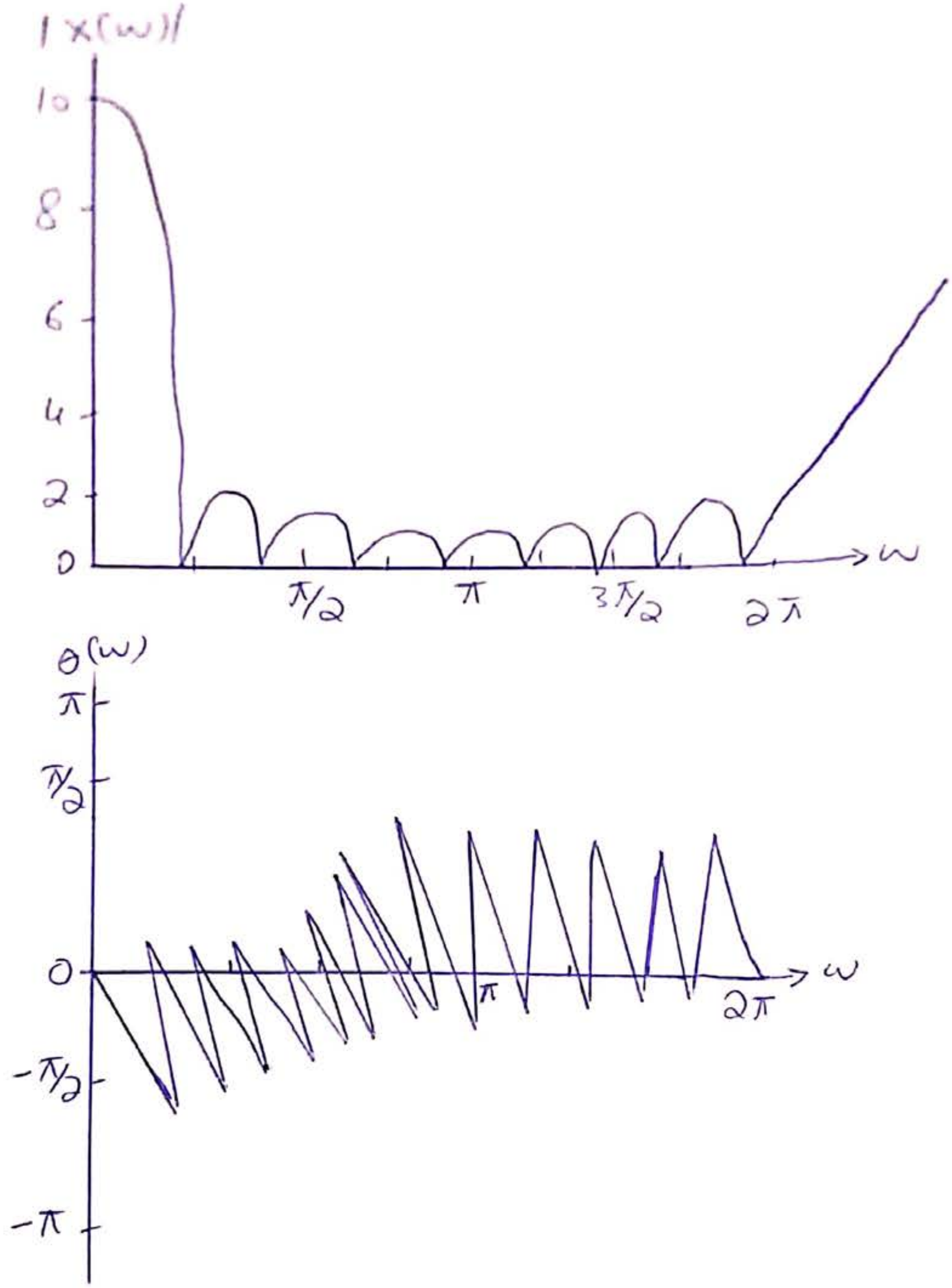
$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The magnitude and phase of $x(\omega)$ ~~are~~ ~~illus~~ $L=10$ The N-point DFT of $x(n)$ is simply $x(\omega)$ evaluated at the set of N equally spaced frequencies $\omega_k = 2\pi k/N$,

$k = 0, 1, \dots, N-1$, Hence

$$x(k) = \frac{1-e^{-j2\pi kL/N}}{1-e^{-j2\pi k/N}}, \quad k=0,1,\dots,N-1$$

$$= \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$



Magnitude and phase characteristics of the Fourier transform for signal.

If N is selected such that $N=L$ then the DFT becomes

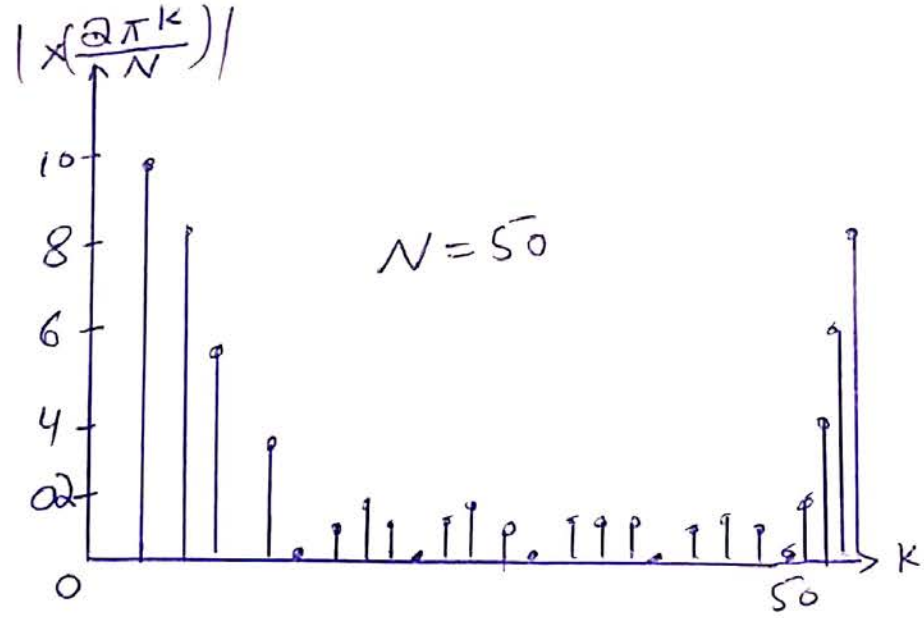
$$X(k)=\begin{cases} L, & k=0 \\ 0, & k=1,2,\ldots,L-1 \end{cases}$$

Thus there is only one nozero value in the DFT. This is apparent from observation of $X(\omega)$, since $X(\omega)=0$ at the freqvencies $\omega_k = 2\pi k/L$, $k \neq 0$ The reader should verify that $x(n)$ can be recovered from $X(k)$ by performing an L-point IDFT.

Figure provides a plot of the N-point DFT, in magnitude and

Phase, for $L = 10$, $N = 50$, and $N = 100$.

Now the spectral characteristics of the sequence are more clearly evident, as one will conclude by comparing these spectra with the continuous spectrum $X(\omega)$



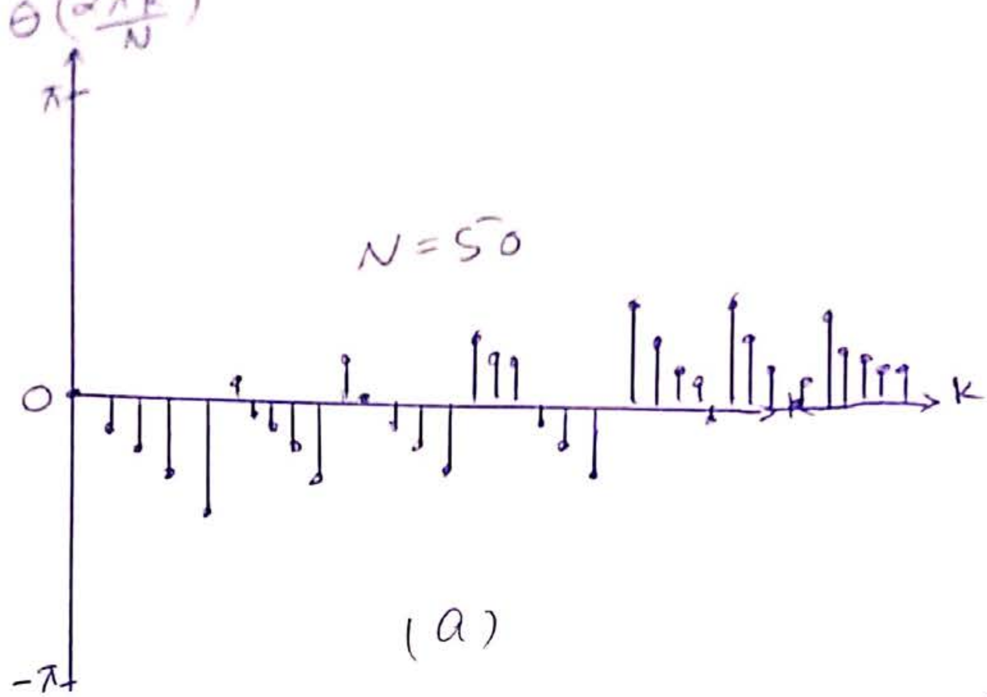
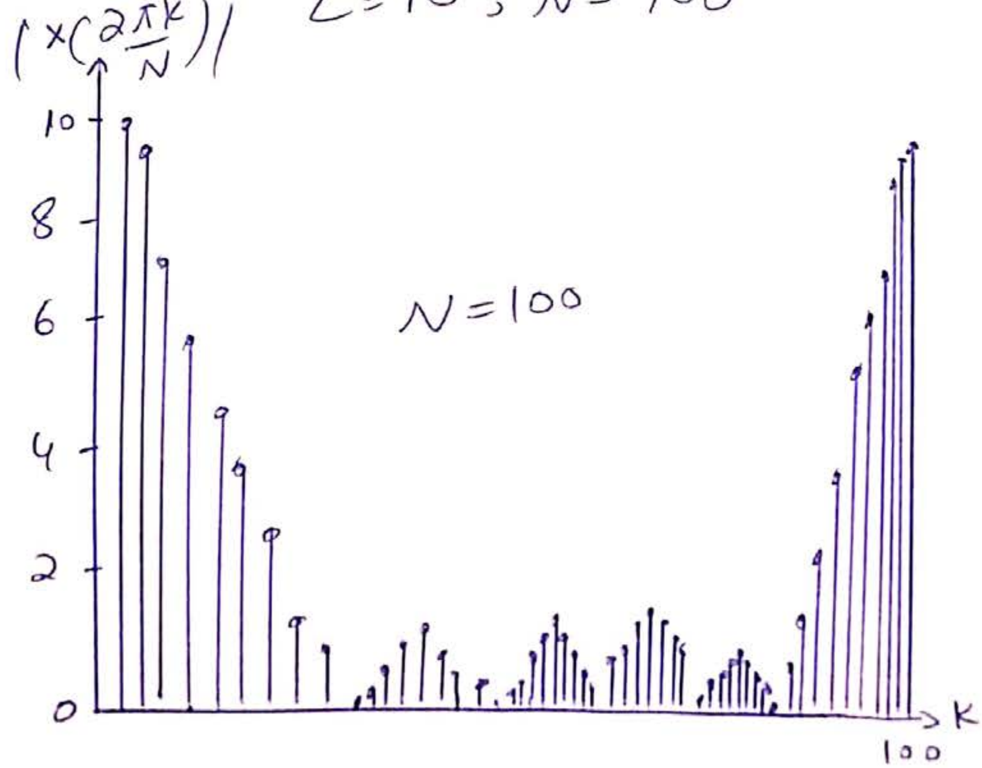
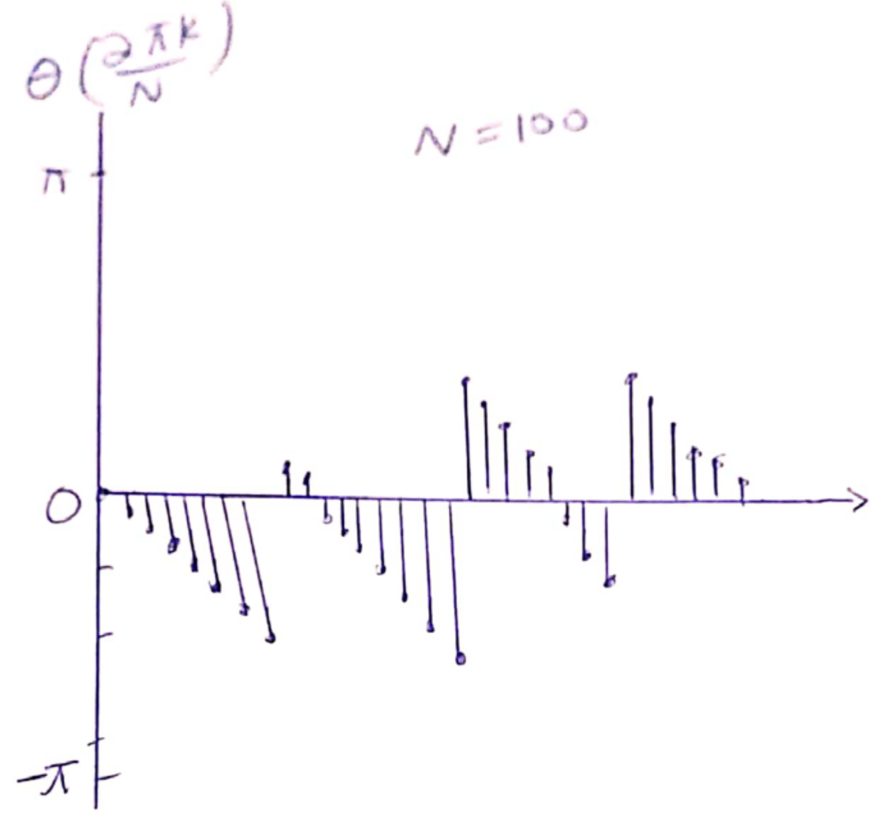


Figure Magnitude and Phase of an N-point DFT in (a) $L=10, N=50$; $L=10, N=100$





Q 4: (b) Perform the circular convolution of the following sequences:

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

Solution:- Thus the sequences $x_1(n)$ and $x_2(n)$ are graph ~~each~~ as illustrated in Fig 1. We note that the sequence are graphed in a counterclock-wise direction on a circle. This establishes the reference direction in rotating one of the sequence relative to the other.

Now $x_3(m)$ is obtained by circularly convolving $x_1(n)$ with $x_2(n)$.

Beginning with $m=0$ we have

$$x_3(0) = \sum_{n=0}^{3} x_1(n)\, x_2((-n))_N$$

The product sequence is obtained by multiplying $x_1(n)$ with $x_2((-n))_4$, point by point.

$$x_3(0) = 14$$

For $m=1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

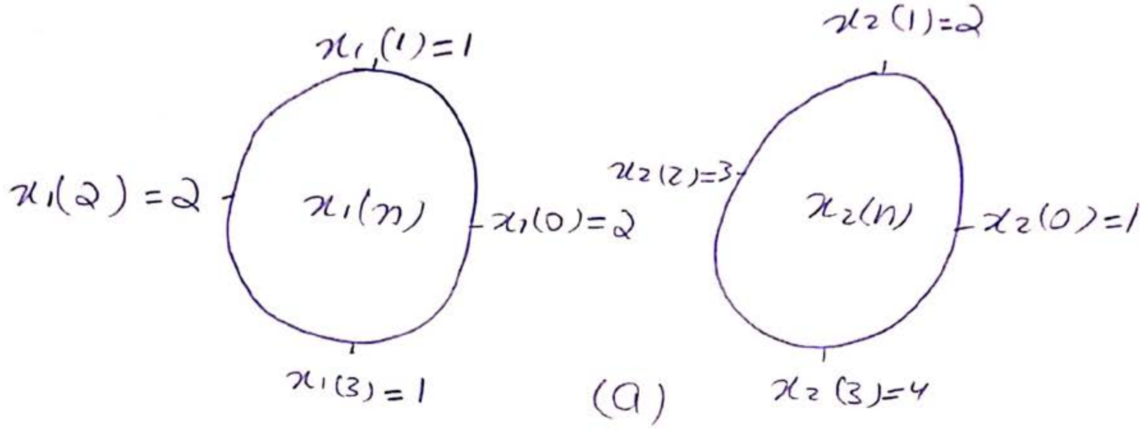
It is easily verified that $x_2((1-n))_4$ is simply the sequence $x_2((-n))_4$ rotate counterclock-wise by one unit in time as in Fig(4) This rotated sequence multiplies $x_1(n)$ to yield the product sequence also in fig (4) Finally we sum the values in the product sequence to obtain $x_3(1)$. Thus

$$x_3(1) = 16$$

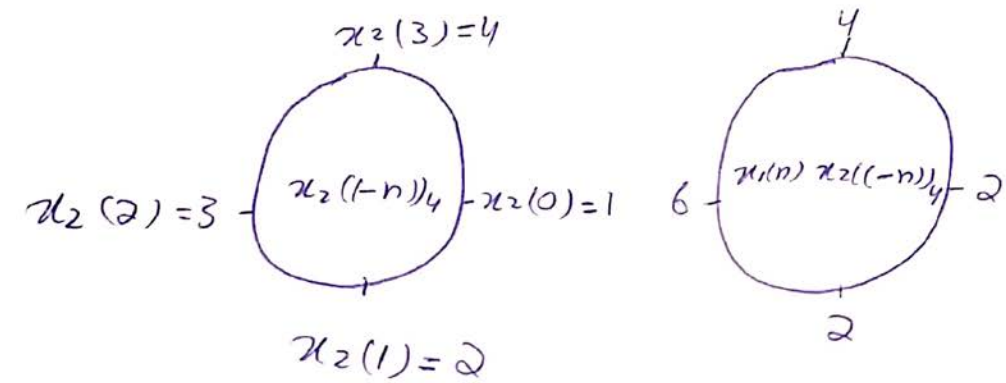
For $m=2$ we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$

Now $x_2((2-n))_4$ is the folded sequence in fig (↓) rotated two units of time in the counterclockwise direction.



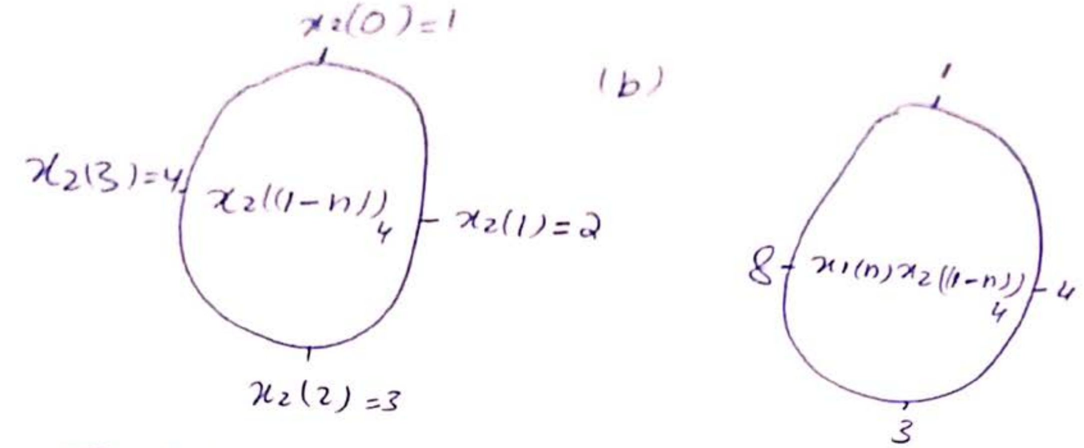
(a)



Folded sequence

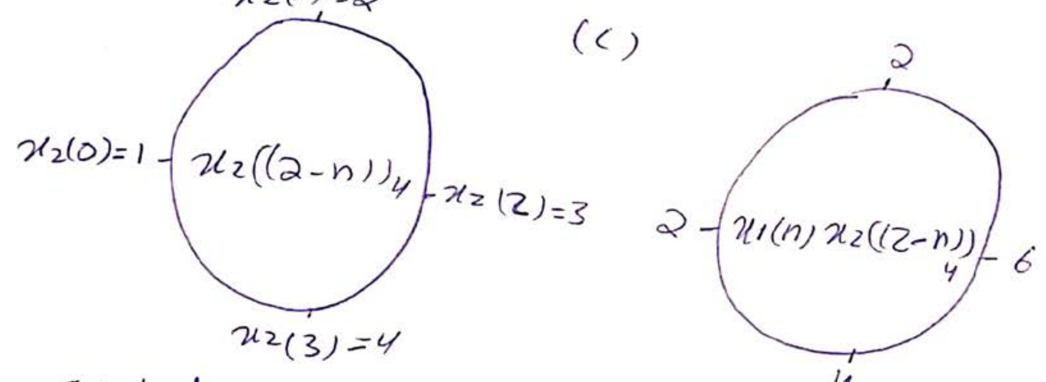
Product sequence

(b)



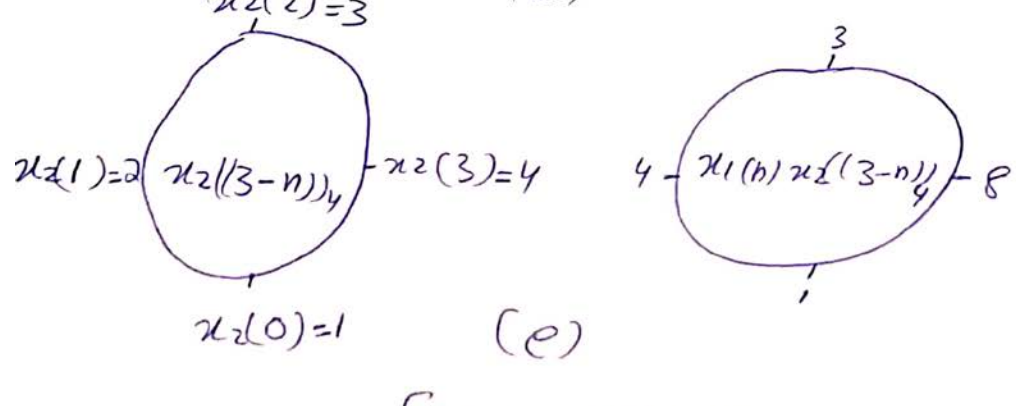
Folded sequence by one unit time Product seq

(c)



Folded seq by one unit in time, product seq

(d)



(e)

Figure Circular convolution of two sequence along with the product sequence $x_1(n)\, x_2((2-n))_4$. By summing the four terms in the product seq we obtain

$$x_3(2) = 14$$

For $m=3$ we have

$$x_3(3) = \sum_{n=0}^{3} x_1(n)\, x_2((3-n))_4$$

The sum of the values in the product sequence is

$$x_3(3) = 16$$

we observe that if the computa-tion above is continued beyond $m=3$, we simply repeat the sequence of four values obtained above. Therefore, the circular convolution of the two sequences $x_1(n)$ and $x_2(n)$ yields the sequence

$$x_3 = \{\underset{\uparrow}{14}, 16, 14, 16\}$$

The two sequences may be folded and rotated without changing the result of the circular convolution. Thus

$$x_3(m) = \sum_{n=0}^{n-1} x_2(n)\, x_1((m-n))_N \quad m = 0, 1, \ldots, N-1$$