Name : M. Daud

I'D : 7769

Sec : A

Subject : Applied Calculas

Submitted to : Engr Shomaila Mazhar

Date : 21/8/020

Scanned with CamScanner

Qno 1

The Function $g(t)$ is define by

$$g t = 0 \quad t < 0$$

$$t^2 \quad 0 \leq t \leq 3$$

$$2t + 3 \quad 3 < t \leq 4$$

$$12 \quad t > 4$$

(a) State any point of discountinity

Sol :.

To check possibility of the discontinity of the function is at $t = 0$ & 4

→ First at $t = 0$

$$g(t) = t^2$$

$$g(0) = 0^2 = 0$$

→ For R.H.L

→ $$\lim g(1+h) = \lim_{h \to 0} (1+h)^2$$

→ $$= \lim_{h \to 0} 1 + h^2 + 2h$$

Apply Limit $1 + 0^2 + 2(0)$

→ $$= 1$$

For L.H.L

$$\lim_{h\to 0} g(1-h) = 2t+3$$

$$= \lim_{h\to 0} 2(1-h)+3$$

Apply limit

$$= 2-2(0)+3$$

$$= 5$$

$$R.H.L \neq L.H.L = g(t) = 5$$

Now at $t = 4$

$$g(4) = 2(4)+3$$

$$= 8+3$$

$$= 11$$

For R.H.L

$$\lim_{h\to 0} g(1+h) = \lim_{h\to 0} 2(1+h)+3$$

$$= \lim_{h\to 0} 2+2h+3$$

Apply limit

$$= 2+2(0)+3$$

$$= 5$$

Scanned with CamScanner

For L. H.L

$$\lim_{h \to 0} g(1-h) = 12$$

$g(4) =$ R. H. L + L. H. L

point of discontinuity is at $t = 4$

(b) Find, if they exist

(i) $\lim_{t \to 3} g$

For $g(t) = t^2$

R. H. L $\lim_{h \to 3} g(1+h) = \lim_{h \to 3} (1+h)^2$

$$= \lim_{h \to 3} 1 + h^2 + 2h$$

Apply limit

$$= 1 + 3^2 + 2(3) \Rightarrow 16$$

Scanned with CamScanner

L.H.L

$$\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t + 3$$

$$= \lim_{h \to 3} 2(1-h)+3$$

$$= \lim_{h \to 3} 2 - 2h+3$$

Apply limit

$$= 2-2(3)+3$$

$$= 2-6+3$$

$$= -1$$

R.H.L $\neq$ L.H.L

(do not exist since L.H.L is negative)

Scanned with CamScanner

Q No 2. Maclaurin's Series

$$Y(x) = x^2 + \sin x$$

Sol:

$$Y(x) = x^2 + \sin x$$

Since we know that the M. Series is

$$Y(x) = Y(x_0) + Y'(x_0)(x - x_0) + \frac{Y''(x_0)(x - x_0)^2}{2!} + \cdots$$

put $x_0 = 0$

$$Y(x) = Y(0) + (x - 0) Y'(0) + \frac{(x - 0)^2 Y''(0)}{2!} + \cdots$$

$$Y(x) = Y(0) + x Y'(0) + \frac{x^2 Y''(0)}{2!} + \cdots \quad (1)$$

Now Find

$Y(0) = ?$

$$Y(x) = x^2 + \sin x$$

$$Y(0) = 0 + \sin 0$$

$$= 0 + 0$$

$$= 0$$

Scanned with CamScanner

$$\boxed{Y(0) = 0}$$

$$Y(x) = x^2 + \sin x$$

$$\frac{d}{dx} Y(x) = d/dx\ x^2 + d/dx\ \sin x$$

$$Y'(x) = 2x + \cos x$$

$$Y'(0) = 2(0) + \cos 0$$

$$= 0 + 1$$

$$\boxed{Y'(0) = 1}$$

Since $Y'(x) = 2x + \cos x$

$$d/dx\ Y'(x) = 2\ d/dx\ x + \frac{d}{dx} \cos x$$

$$= 2 - \sin x$$

$$Y''(x) = 2 - \sin x$$

$$Y''(0) = 2 - \sin 0$$

$$= 2 - 0 = 2$$

Scanned with CamScanner

$$\boxed{y''(0) = 2}$$

Now

$$y''(x) = 2 - \sin x$$

$$\frac{d}{dx} y''(x) = \frac{d}{dx} 2 - \frac{d}{dx} \sin x$$

$$= 0 - \cos x$$

$$y'''(x) = 0 - \cos x$$

$$y'''(x) = -\cos 0$$

$$\boxed{y'''(0) = -1}$$

Put in eq ①

$$y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \cdots$$

$$= x + \frac{\cancel{2}x^2}{\cancel{2}!} - \frac{x^3}{3!} + \cdots$$

$$= x + x^2 - \frac{x^3}{3!} + \cdots$$

Scanned with CamScanner

So

$$Y(x) = x + x^2 - \frac{x^3}{3!} + \cdots\cdots$$

Scanned with CamScanner

Q no 3 part 1

Find y'' given

$1 + xy = x^2 + y^2$

Sol:

$$1 + xy = x^2 + y^2$$

$$\frac{d}{dx}(1 + xy) = \frac{d}{dx}(x^2 + y^2)$$

$$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx} x^2 + \frac{d}{dx} y^2$$

$$0 + x\frac{dy}{dx} + y\frac{dx}{dx} = 2x^{2-1} + 2y^{2-1}\frac{dy}{dx}$$

$$x\frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$\frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

Scanned with CamScanner

$$y' = \frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{2x - y}{x - 2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\, d/dx\,(x-2y)}{(x-2y)^2}$$

$$y'' = \frac{(x-2y)(2-y') - (2x-y)(1-2y')}{(x-2y)^2}$$

$$y'' = \frac{2x - xy' - 4y + 2yy' - (2x - 4xy' - y + 2yy')}{(x-2y)^2}$$

$$y'' = \frac{\cancel{2x} - xy' - 4y + \cancel{2yy'} - \cancel{2x} + 4xy' + y - \cancel{2yy'}}{(x-2y)^2}$$

$$y'' = 4xy' - 3y - xy' / (x-2y)^2$$

Scanned with CamScanner

(2)

$$y'' = 4x\left[\frac{2x-y}{x-2y}\right] - 3y - x\left[\frac{2x-y}{x-2y}\right] \Big/ (x-2y)^2$$

$$y'' = \frac{4x(2x-y) - 3y(x-2y) - x[2x-y)}{(x-2y)(x-2y)^2}$$

$$y'' = \frac{8x^2 - 4xy - 3xy + 6y^2 - 2x^2 + xy}{(x-2y)^3}$$

$$y'' = \frac{6x^2 + 6y^2 - 6xy}{(x-2y)^3}$$

Ans

Q.No 3 part (ii)

Find y by using logarithmic differentitation

$$Y = x^3 (1+x)^9 e^{6x}$$

Sol.:

$$y = x^3 (1+x)^9 e^{6x}$$

Taking ln to b/s

$$\ln y = \ln (x^3 (1+x)^9 \cdot e^{6x})$$

$$\ln y = \ln x^3 + \ln (1+x)^9 + \ln e^{6x}$$

diff w.r.t x

$$\frac{d}{dx} \ln y = \frac{d}{dx} 3 \ln x + d/dx\, 9 \ln (1+x) + d/dx\, 6x \ln e$$

$$1/y \frac{dy}{dx} = 3/x + \frac{9}{1+x} + \frac{1}{e^{6x}} \cdot 6$$

$$\frac{dy}{dx} = y \left(3/x + \frac{9}{1+x} + \frac{6}{e^{6x}} \right)$$

Scanned with CamScanner

$$\frac{dy}{dx} = x^3 (1+x)^9 e^{6x} \left(\frac{3}{x} + \frac{9}{1+x} + \frac{6}{e^{6x}} \right)$$

Ans.

Scanned with CamScanner