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Subject ≠ Digital Signal processing.

Submitted To ≠ Engr Pir Meher Ali Shah Sir.

Question: 1 (i)-(A)

Answer: $x(t) = 3\cos 100\pi t + 4\sin 200\pi t$

(i)- minimum sampling rate?

As we have two frequency components. So, we have to find maximum One.

$3\cos 100\pi t \Rightarrow w = 200\pi$

$$2\pi f_1 = 100\pi$$

$$f_1 = \frac{100\pi}{2\pi} = 50\,Hz$$

$4\sin 200\pi f \Rightarrow$

$$w = 200\pi$$

$$2\pi f_2 = 200\pi$$

$$f_2 = \frac{200\pi}{2\pi} = 100\,Hz$$

As, $f_2 = 100\,Hz$ So, $f_{max} = 100\,Hz$

Now,
manimum rampling rate to avoid aliasing

$$f_b = 2f_{man}$$

$$f_b = 2 \times 100$$

$$f_b = 200$$

(ii)-

We have

$$F_s = 100 \text{ Hz}$$

So, $$\frac{f_1}{f_s} = \frac{50}{100} = 0.5 \text{ Hz}$$

$\Rightarrow$ F_2 becomes,

$$f_2' = \frac{F_2}{F_s} = \frac{100}{100}$$

$$f_2' = 1 \text{ Hz}$$

So,

$$w_1' = 2\pi f_1'$$

$$w_1' = 2\pi \times 0.5$$

$$w_1' = \pi$$

&

$$w_2' = 2\pi f_2'$$

$$w_2' = 2\pi \times 1$$

$$w_2' = 2\pi$$

$$x[n] = 3\cos 100\pi t + 4\sin 200\pi t$$

$$x[n] = 3\cos \pi t + 4\sin 2\pi t$$

(iii)-

We can construct the orignal signal & also frequency components. Since as 50 Hz and 100 Hz are present in the sampaled signal.

The signal we can recover is ;

$y_a(t) = 3\cos\pi t + 4\sin 2\pi t.$

$x_a(t)$, orignal signal is different from it. due to low sampling rate. Distortion the orignal analog signal was caused by the aliasing effect.

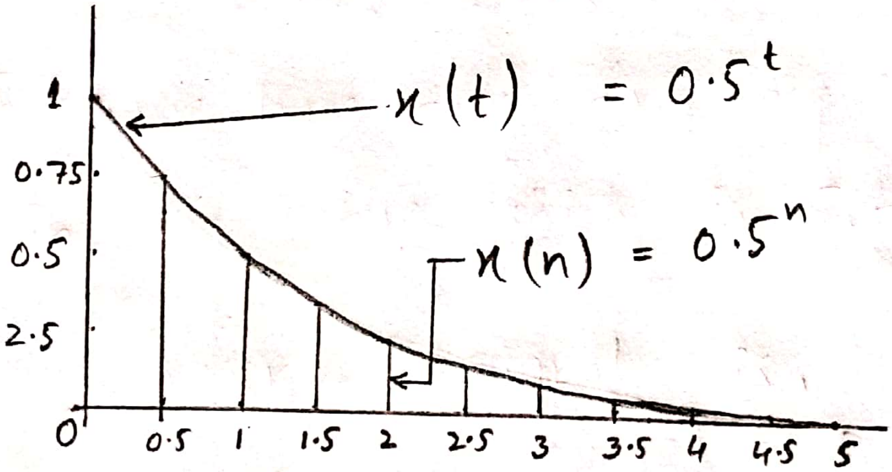
Question :1 (B)

(i)-

$$x(n) = \begin{cases} 0.5^n & ; \ n \geq 0 \\ 0 & ; \ n < 0 \end{cases}$$

$$F_s = 2 \text{ Hz}$$

$$T_s = 1/2 = 0.5 \text{ sec}$$



x_n	0.5^n
0	1
0.5	0.75
1	0.5
1.5	0.25

(ii):

here $n = 3$ bits per sample.

$L = 2^n$ $\therefore$ L is quantization level.

$L = 2^3$

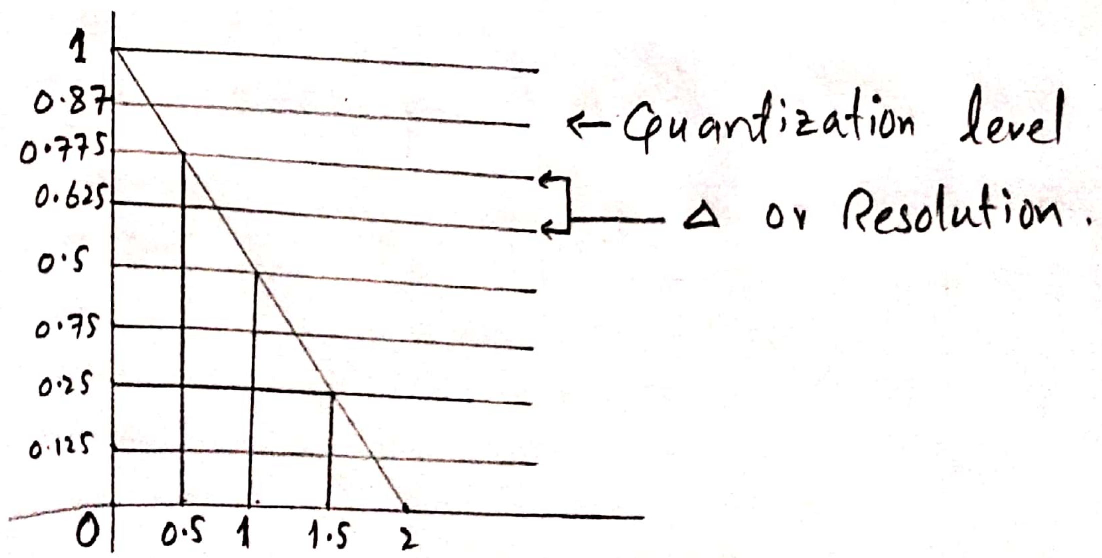
$\boxed{L = 8 \text{ levels}}$

Quantization resulation / Steps Size Δ is ;

$$\Delta = \frac{x_{max} - x_{min}}{L - 1}$$

$$\Delta = \frac{1 - 0}{8 - 1} = 1/7$$

$$\Delta = 0.142$$



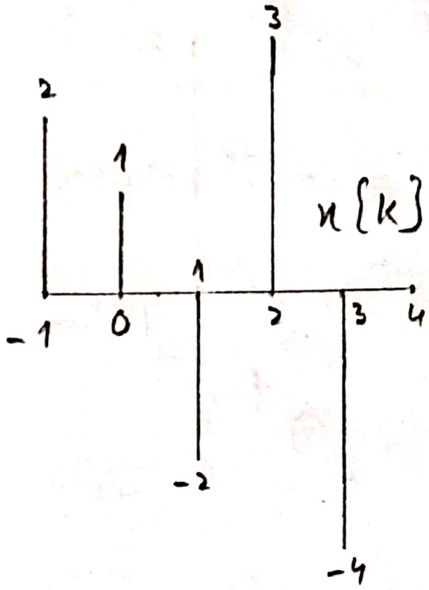
(iii)- 79 Truncation.

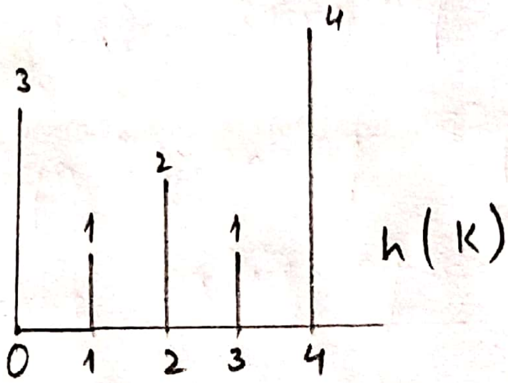
n	x(n)	$x_q(n)$	Roundoff	Error $e_q = x_q(n) - x(n)$
0	1	1.0	1.0	0.0
1	0.5	0.5	0.5	0.0
2	0.25	0.2	0.3	+0.05
3	0.015625	0.0	0.0	-0.015625
4	0.0625	0.0	0.1	+0.0375
5	0.0.3185	0.0	0.0	-0.03125
6	0.015625	0.0	0.0	-0.015625
7	0.0078125	0.0	0.0	-0.0078125
8	0.00390625	0.0	0.0	-0.001953125
9	0.001953125	0.0	0.0	-0.0009765625
10	0.0009765625	0.0	0.0	-0.007
0.5	0.707	0.7	0.7	0.0465
1.5	0.3535	0.3	0.4	0.02323
2.5	0.17677	0.1	0.2	0.01162
3.5	0.08838	0.0	0.1	0.00128
4.5	0.04419	0.0	0.0	-0.04419

Question : 2 (A)-

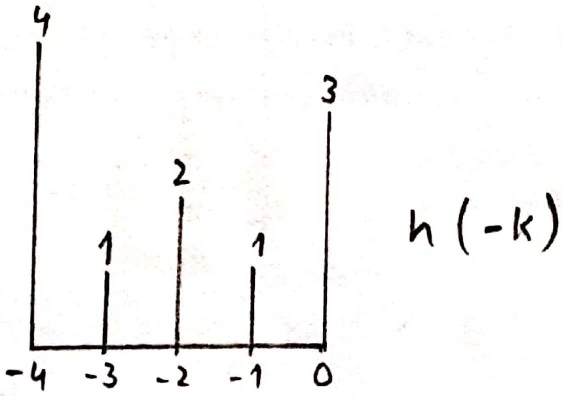
$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$$

$$x[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$





Convert $h[k]$ into $h[-k]$



As,

$$y[n] = \sum_{k=\infty}^{\infty} x[k]\, h[n-k]$$

For $n=0$;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$y[0] = \sum_{k=-1}^{0} x[k]\, h[-k]$$

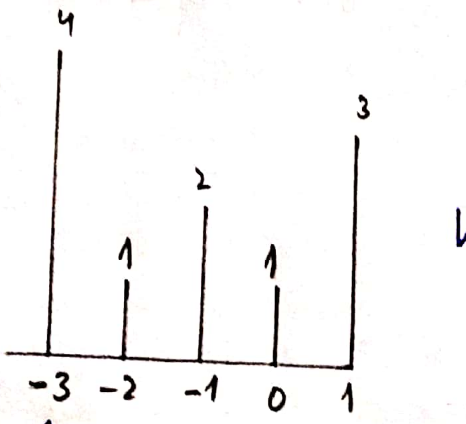
$$x[-1]\, h(-1) + x(0)\, h(0)$$

$$(2)(1) + (1)(3)$$

$$2 + 3$$

$$y[0] = 5$$

For $n=1$;



$h[1-k]$

$$y[1] = \sum_{k=-1}^{1} x[k]\, h[1-k]$$

$$= x(-1)\,h(-1) + x(0)\,h(0) + (1)\,h(1)$$

$$= (2 \times 2) + (1 \times 1) + (-2 \times 3)$$

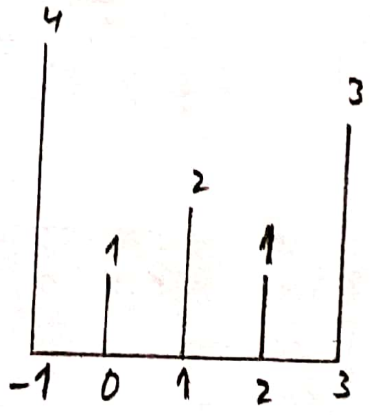
$$= 4 + 1 - 6$$

$$= 5 - 6$$

$$y[1] = -1$$

For $n = 2$;

$h[3-k]$



$$y[3] = \sum_{x=-1}^{3} x[n]\; h[3-k]$$

$$= x(-1)\,h(-1) + x(0)\,h(0) + x(1)\,h(1) + x(2)\,h(2) + x(3)\,h(3).$$

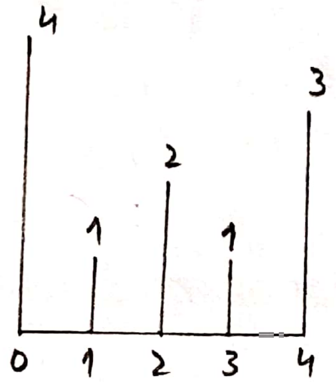
$$y[3] = (2)(4) + (1)(1) + (-2)(2) + 3(1) + (-4)(3)$$

$$y[3] = 8 + 1 - 4 - 12 + 3$$

$$y[3] = 1 - 4 - 4 + 3$$

$$y[3] = -7 + 3 \quad = -4$$

$h[4-k]$



$$y[4] = \sum_{k=0}^{3} x[k]\, h[4-k]$$

$$y[4] = x(0)\,h(0) + x(1)\,h(1) + x(2)\,h(2) + x(3)\,h(3)$$

$$y[4] = (1)(4) + (-2)(1) + (3)(2) + (-4)(1)$$

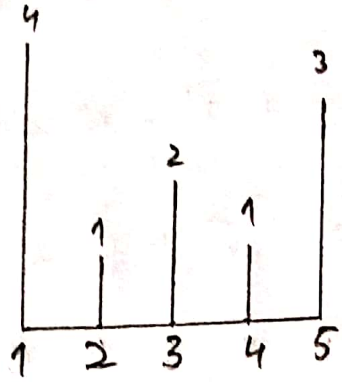
$$= 4 - 2 + 6 - 4$$

$$= -2 + 6$$

$$y[4] = 4$$

$\Rightarrow$ $n = 5$

$h[5-k]$



$$y[5] = \sum_{k=1}^{3} x[k]\, h[5-k]$$

$$= x(1)h(1) + x(2)h(2) + x(3)h(3)$$

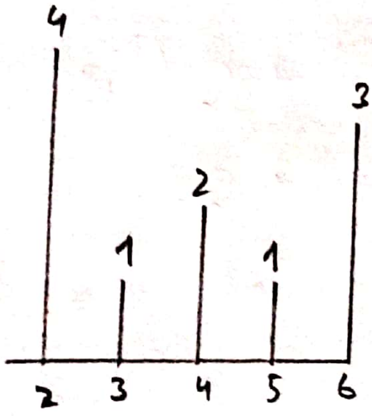
$$= (-2)(4) + (3)(1) + (-4)(2)$$

$$= -8 + 3 - 8$$

$$y[5] = -13$$

$\Rightarrow$ $n = 6$;

$h[6-k]$



$$y[6] = \sum_{k=2}^{3} x(k)\, h[6-k]$$

$$y[6] = x(2)\,h(2) + x(3)\,h(3)$$

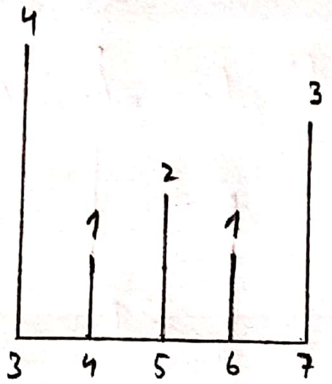
$$= (3)(4) + (-4)(1)$$

$$= 12 - 4$$

$$y[6] = 8$$

$\Rightarrow$ $n = 7$;

$h[7-k]$



$$y[7] = \sum_{k=3}^{3} x(k)\, h(7-k)$$

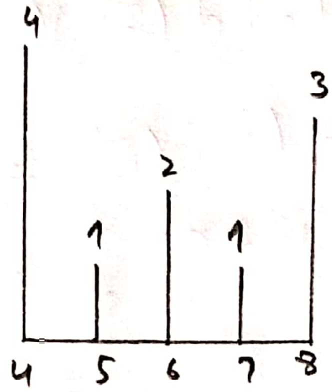
$$= x(3)\,h(3)$$

$$= (-4)(4)$$

$$y[7] = -16$$

$\Rightarrow$ $n = 8$;

$y[8] = x(k)\, h(8-k)$



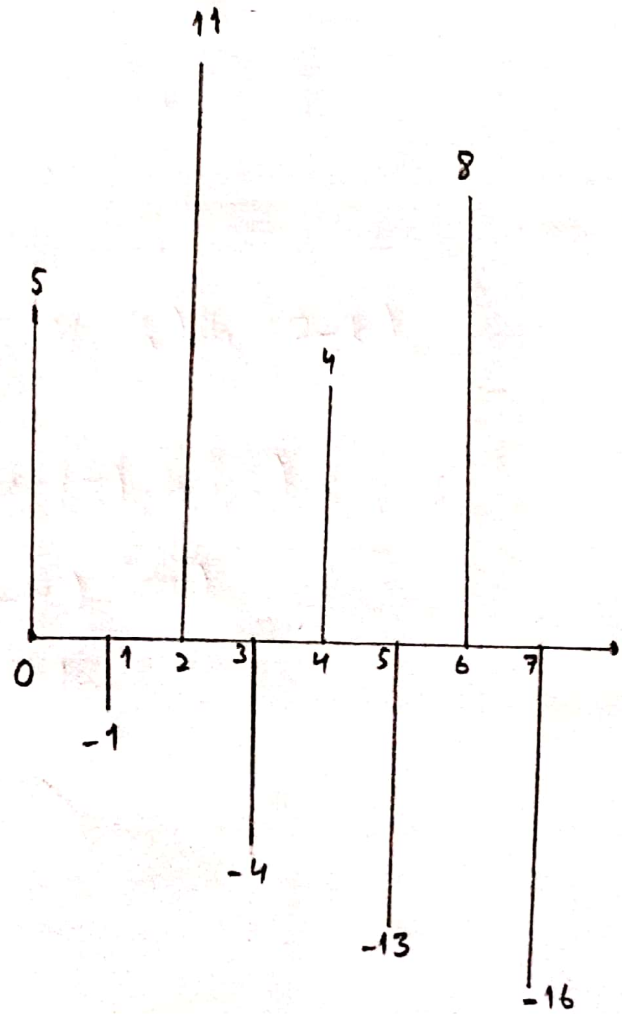
$h[8-k]$

$y[8] = 0$

There is no over-lapping on $n=8$.

So,

$y[n]$;

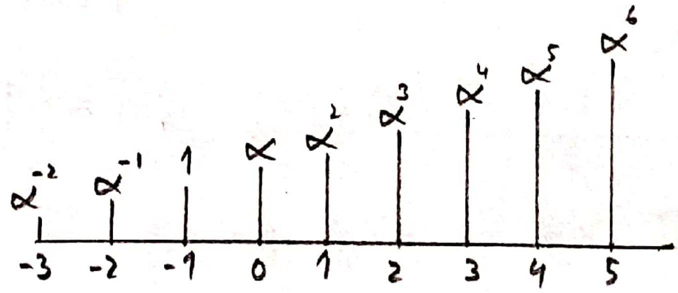


Question: 2 (B)

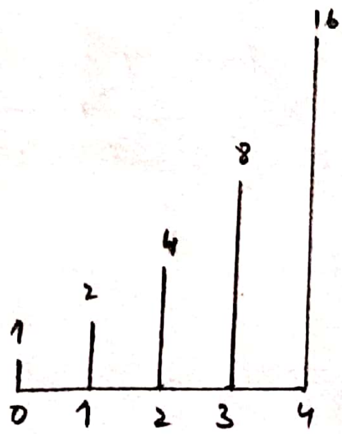
$$x(n) = \begin{cases} \alpha^{n+1} & , -3 \le n \le 5 \\ 0 & , \text{else} \end{cases}$$

$$h(n) = \begin{cases} 2^{n} & , 0 \le n \le 4 \\ 0 & , \text{else} \end{cases}$$

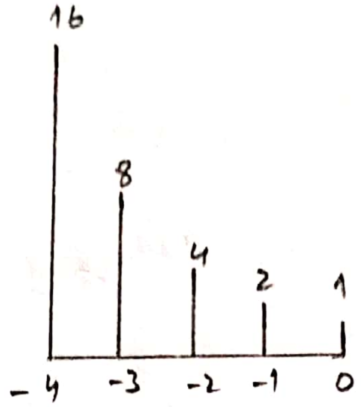
$$x(n) = \{\alpha^{-2}, \alpha^{-1}, 1, \alpha, \alpha^{2}, \alpha^{3}, \alpha^{4}, \alpha^{5}, \alpha^{6}\}$$



$$h(x) = \{1, 2, 4, 8, 16\}$$



$h[-h]$



$$y[-3] = (\alpha^{-2})(1)$$

$$y[-3] = \alpha^{-2}$$

$$y[-2] = (\alpha^{-1})(1) + (\alpha^{-2})(2)$$

$$y[-2] = \alpha^{-1} + \alpha^{-2}$$

$$y[-2] = \alpha^{-3}$$

$$y[-1] = (\alpha^{1})(1) + (2)(4) + (\alpha^{-1})(4) + (\alpha^{-2})(8)$$

$$y[-1] = (1 \times 10) + 2\alpha^{-1} + 4\alpha^{-2}$$

$$y[-1] = 16\alpha^{-3} + 1$$

$$y[0] = (8)(\alpha^{-2}) + (\alpha^{-1})(4) + (1)(2) + (\alpha)(1)$$

$$y[0] = 8\alpha^{-2} + 4\alpha^{-1} + 2 + \alpha$$

$$y[0] = 12\alpha^{-3} + 2 + \alpha$$

$$y[1] = (\alpha^{-1})(8) + (1)(4) + (\alpha)(2) + (\alpha^2)(1)$$

$$= 8\alpha^{-1} + 4 + 2\alpha + \alpha^2$$

$$y[2] = (\alpha^{-1})(16) + (1)(8) + (\alpha)(4) + (\alpha^2)(2) + (\alpha^3)(1)$$

$$y[2] = 16\alpha^{-1} + 8 + 4\alpha + 2\alpha^2 + \alpha^3$$

$$y[3] = (1)(16) + (\alpha)(8) + (\alpha^2)(4) + (\alpha^3)(2) + (\alpha^4)(1)$$

$$= 16 + 8\alpha + 4\alpha^2 + 2\alpha^3 + \alpha^4$$

$$y[4] = (\alpha)(16) + (\alpha^2)(8) + (\alpha^3)(4) + (\alpha^4)(2) + (\alpha^5)(1)$$

$$y[4] = 16\alpha + 8\alpha^2 + 4\alpha^3 + 2\alpha^4 + \alpha^5$$

$$y[5] = (\alpha^2)(16) + (\alpha^3)(8) + (\alpha^4)(4) + (\alpha^5)(2) + (\alpha^6)(1)$$

$$y[5] = 16\alpha^2 + 8\alpha^3 + 4\alpha^4 + 2\alpha^5 + \alpha^6$$

$$y[6] = (\alpha^3)(16) + (\alpha^4)(8) + (\alpha^5)(4) + (\alpha^6)(2)$$

$$y[6] = 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$$

$$y[7] = (16)(\alpha^4) + (\alpha^5)(8) + (\alpha^6)(4)$$

$$y[7] = 16\alpha^4 + 8\alpha^5 + 4\alpha^6$$

$$y[8] = (\alpha^5)(16) + (\alpha^6)(8)$$

$$y[8] = 16\alpha^5 + 8\alpha^6$$

$$y[9] = 16\alpha^6$$

$$y[10] = 0$$

There is no envelop in $y(10)$.

Question: 3

(i)-

$$x(n) = \begin{cases} (1/4)^n & , n \geq 0 \\ (1/3)^{-n} & , n < 0 \end{cases}$$

The Z-Transform is;

$$x(n) = a^n u(n)$$

$$X(z) = \frac{1}{1 - \alpha z^{-1}} \quad \text{ROC} |z|$$

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} - 1$$

Using Geometric series;

$$\Rightarrow \frac{1}{1 - \frac{1}{4} z^{-1}} + \frac{1}{1 - \frac{1}{3} z} - 1$$

$$\Rightarrow \frac{1 - \frac{1}{3} z + 1 - \frac{1}{4} z^{-1} - \left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}$$

Scanned with CamScanner

$$\Rightarrow \frac{1-\frac{1}{3}z+1-\frac{1}{4}z^{-1}-\left(1-\frac{1}{3}z-\frac{1}{4}z^{-1}+\frac{1}{12}z^{-1}z\right)}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$\Rightarrow \frac{1-\frac{1}{3}z+1-\frac{1}{4}z^{-1}-1+\frac{1}{3}z+\frac{1}{4}z^{-1}+\frac{1}{12}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$\Rightarrow \frac{1+\frac{1}{12}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$\Rightarrow \frac{13/12}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$\Rightarrow$ So, ROC $1/4 < |z| < 3$.

(ii)- $x(n) = \begin{cases} (1/2)^n - 3^n, & n \geq 0 \\ 0, & \text{else} \end{cases}$

So, the Z-Transform is;

$$x(n) = \sum_{n=0}^{\infty} (1/2)^n z^{-n} - \sum_{n=0}^{\infty} 3^n z^{-n}$$

$$\Rightarrow \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-3z^{-1}}$$

$$\Rightarrow \frac{1-3z^{-1} - 1 - 1/2\, z^{-1}}{(1-1/2\, z^{-1})(1-3z^{-1})}$$

$$\Rightarrow \frac{-5/2\, z^{-1}}{(1-1/2\, z^{-1})(1-3z^{-1})}$$

$\therefore \frac{-6z^{-1} - z^{-1}}{2}$

$\therefore -3z^{-1} - \frac{1}{2}z^{-1}$

$\therefore \frac{-5z^{-1}}{2}$