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Q1 Sol:-

The sample Space S for this experiment is

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5)$
$(1,6), (2,1), (2,2), (2,3), (2,4),$
$(2,5), (2,6), (3,1), (3,2), (3,3)$
$(3,4), (3,5), (3,6), (4,1), (4,2)$
$(4,3), (4,4), (4,5), (4,6)$
$(5,1), (5,2), (5,3), (5,4), (5,5)$
$(5,6), (6,1), (6,2), (6,3), (6,4)$
$(6,5), (6,6)\}$

Let $A = \{$The sum is 7$\}$

$B = \{$The sum is even$\}$

$C = \{$The sum is greater than 6$\}$

$D = \{$Both lands the same outcome$\}$

Then

$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),$
$(6,1)\}$

$B = \{(1,1),(1,3),(1,5),(2,2),(2,4),$
$(2,6),(3,1),(3,3),(3,5),(4,2)$
$(4,4),(4,6),(5,1),(5,3),(5,5)$
$(6,2),(6,4),(6,6)\}$

$C = \{(3,6),(4,5),(4,6),(5,4),(5,5)$
$(5,6),(6,3),(6,4),(6,5),$
$(6,6)\}$

$D = \{(1,1),(2,2),(3,3),(4,4)$
$(5,5),(6,6)\}$

$A \cap B = \phi$

$A \cap C = \phi$

$A \cap D = \phi$

$P(A) = \frac{6}{36}$, $P(B) = \frac{18}{36}$

$P(C) = \frac{10}{36}$, $P(D) = \frac{6}{36}$

$P(A \cap B) = 0$, $P(A \cap C) = 0$

$P(A \cap D) = 0$

Hence

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{18/36} = 0$$

$P(A/C) = 0$

$P(A/D) = 0$

Ans

Question No 2

Sol:-

When we roll pair of die, there are 36 different combination There are 15 possible combinations of less than 7. That are (1,1), (1,2), (1,3), (1,4), (1,5) (2,1), (2,2), (2,3), (2,4), (3,1) (3,2), (3,3), (4,1), (4,2) (5,1). The posibility of getting less than 7 is

$$\Rightarrow \frac{15}{36} = \frac{5}{12}$$

There are 6 possible combinations of getting exact 7. That are (1,6), (2,5), (3,4), (4,3), (5,2) (6,1). which gives a probability of

$$\Rightarrow \frac{6}{36} = \frac{1}{6}$$

As the possibility of getting less than 7 is equall to posibility of getting greater than 7. So the ~~possible~~ probability for greater than 7 is also equal to $\frac{5}{12}$ as it is also obvious from the fact that 21 out of 36 possibility are taken by less then and equal to 7. ~~we~~ In calculating this, we must assume that each combination is equally likely to roll and therefore

the dice are fair,
or else the calculations
does'nt work.

Question No 3

Given that $p = \frac{2}{3}$, $n = 8$

$$q = 1 - p$$
$$= 1 - \frac{2}{3}$$

$$q = \frac{1}{3}$$

Let 'X' denotes the number of games won by A,
Then,

(1) $P(X = 4)$
$$= \binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4$$

$$= \frac{1120}{6561}$$

$= 0.1707$

(ii) $P(X \geq 4)$

$1 - P(X < 4)$

$= 1 - \sum_{x=0}^{3} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$

$= 1 - \left[\left(\frac{1}{3}\right)^{8} + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{7} + 28\left(\frac{2}{3}\right)^{2}\left(\frac{1}{3}\right)^{6} + 56\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5}\right]$

$= 1 - \frac{1}{6561}\left[1 + 16 + 112 + 448\right]$

$= 1 = \frac{577}{6561} \Rightarrow \frac{6561 - 577}{6561}$

$= \frac{5984}{6561}$

$= 0.9121$

(iii) $P(3 \le X \le 6)$

$$\sum_{x=3}^{6} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3}\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5} + \binom{8}{4}\left(\frac{2}{3}\right)^{4}\left(\frac{1}{3}\right)^{4}$$

$$+ \binom{8}{5}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{3} + \binom{8}{6}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{2}$$

$$= \frac{8}{(3)^{8}}\left[56 + 140 + 224 + 224\right]$$

$$= \frac{8 \times 644}{6561}$$

$$= \frac{5152}{6561}$$

$$= 0.7852$$

experiment or

Question No 4

Sol: Proof:

Since the C_i's from a partition of the sample space, we can apply the law of total probability for $A \cap B$.

$$P(A \cap B) = \sum_{i=1}^{M} P(A \cap B / C_i)\, P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^{M} P(A / C_i)\, P(B / C_i)\, P(C_i)$$

$\because$ (A and B are conditionally independent.)

$$P(A \cap B) = \sum_{i=1}^{M} P(A / C_i)\, P(B)\, P(C_i)$$

$\because$ (B is independent of all C_i's)

experiment ~~is~~ or

$$P(A \cap B) = P(B) \sum_{i=1}^{m} P(A|C_i)\, P(C_i)$$

$$P(A \cap B) = P(B)\, P(A)$$

$\because$ (law of total probability)

Hence A and B are independent.

Question No 5

Ans: Mean and Variance of a Bi-nomial distribution.

The probability function for a binomial random variable is

$$b(x, n, P) = \binom{n}{x} P^{x} (1-P)^{n-x}$$

This is the probability of having x successes

of experiment

In a series of n independent trails when the probability of success in any one of the trail is p. If X is a random variable with this probability distribution.

$$E(X) = \sum_{x=0}^{n} x \binom{n}{x} p^{x} (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \cdot \frac{n!}{x!(n-x)!} p^{x} (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^{x} (1-p)^{n-x}$$

Since the $x=0$ term vanishes. Let $y = x-1$ and $m = n-1$. Subbing $x = y+1$ and $n = m+1$ into the last sum.

$$E(X) = \sum_{y=0}^{m} \frac{(m+1)!}{y!(m-y)!} p^{y+1}(1-p)^{m-y}$$

$$= (m+1)p \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^{y}(1-p)^{m-y}$$

$$= np \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^{y}(1-p)^{m-y}$$

The binomial theorem says that

$$(a+b)^m = \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} a^{y} b^{m-y}$$

Setting $a = p$ and $b = 1-p$

$$\sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^{y}(1-p)^{m-y}$$

$$= \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} a^{y} b^{m-y} = (a+b)^m$$

$$= (p+1-p)^m = 1$$

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So that

$$E(x) = np$$

Similarly, but this time using $y = x-2$ and $m = n-2$

$$E(x(x-1)) = \sum_{x=0}^{n} x(x-1)\binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=2}^{n} \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

of experiment or

$= n(n-1)p^2(p+(1-p))^{m}$

$= n(n-1)p^2$

So the variance of X is

$E(X^2) - E(X^2) = E(X(X-1)) + E(X) - E(X)^2 = n(n-1)p^2 + np - (np)^2$

$= np(1-p).$

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Question No 6

Ans: Bi-nominal Distribution:

Many experiment consist of repeated independent trails, each trail do have possible two outcome e.g.

The two possible outcomes of a trail may head and tail, success and failure

$$P(X = x) \quad f(x) = {}^{n}C_{x}\, P^{x} q^{n-x}$$

Bi-nominal Frequency:

In Bi-nominal Frequency if the bi-nominal probability distribution is multiplied by N, The number of experiment ~~is~~ or sets, the result distribution is known as bi-nominal frequency distribution

$$N\binom{n}{x} P^x q^{n-x}$$

Question 7

Ans

Coefficient of Variation

For Data Set A:

$$CV = \frac{\sigma}{\mu} \times 100$$

$$CV = \frac{3}{45} \times 100$$

$$CV = 6.7.$$

For Data Set of B

$$CV = \frac{11}{60} \times 100$$

$$CV = 18.3$$

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For Data Set C:

$$CV = \frac{5}{50} \times 100$$

$$CV = 10$$