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paper - Linear Algebra

Department - BS(SE)-2

Sec - (A)

Q no 1:

Determine the following system is consistance or not?

$$x_1 - (3^{rd} - ID)x_2 + x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$5x_1 - 5x_3 = 10$$

ID = 15837
3^{rd} ID = 8

Solution:

$$\left[\begin{array}{ccc|c} 1 & -8 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -8 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 40 & -10 & 10 \end{array}\right] R_3 = R_3 - 5R_1$$

$$\left[\begin{array}{ccc|c} 1 & -8 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 4 & -1 & 1 \end{array}\right] R_2/4 \quad R_3/10$$

$$\left[\begin{array}{ccc|c} 1 & -8 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -15 & -15 \end{array}\right] R_3 - 4R_2$$

$$-15x_3 = -15$$
$$\boxed{x_3 = 1}$$

$$x_2 - 4x_2 = 4$$
$$x_2 = 4 + 4x_3$$
$$\boxed{x_2 = 8}$$

$$x_1 - 8x_2 + x_3 = 0$$
$$x_1 = 8x_2 - x_2$$
$$x_1 = 64 - 1$$
$$\boxed{x_1 = 63}$$

The solution is consistent.

consistent:

A system of linear system which has a solution is know as consistent.

Inconsistent,

A system of linear equation is call inconsistent if it has no solution.

So this system of linear equation is consistent because it has a solution.

p.t.o

Q No 2:

find the inverse of

$$\begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & \text{4th ID} \\ 5 & -2 & 7 \end{bmatrix}$$

by adjoint method. ID = 15837

Solution:

$$\begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 3 \\ 5 & -2 & 7 \end{bmatrix}$$

$|A| = +3(-7-(-6)) \cancel{+} 4(21-15) + 5(-5-(-6))$

$|A| = +3(-1) \cancel{+} -4(6) + 5(-1)$

$|A| = -3 - 24 - 5$

$\boxed{|A| = -32}$

$a_{11} = (-1)^{1+1}(-7+6) = (-1)^2(-1) = -1$

$a_{12} = (-1)^{1+2} \quad (21-15) = (-1)^3(6) = -6$

$a_{13} = (-1)^{1+3} \quad (-5+6) = (-1)^4(1) = 1$

$a_{21} = (-1)^{2+1} \quad (21+10) = (-1)^3(31) = -31$

$a_{22} = (-1)^{2+2} \quad (21-25) = (-1)^4(-4) = -4$

$a_{23} = (-1)^{2+3} \quad (-6-20) = (-1)^5(-26) = 26$

$a_{31} = (-1)^{3+1} \quad (2+15) = (-1)^4(17) = 17$

$a_{32} = (-1)^{3+2} \quad (9-15) = (-1)^5(-6) = 6$

$a_{33} = (-1)^{3+3} \quad (-3-12) = (-1)^6(-15) = -15$

$$A = \begin{bmatrix} -1 & -6 & 1 \\ -31 & -4 & 26 \\ 17 & 6 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\,\mathrm{adj}(A)$$

$$\mathrm{adj}(A) = \begin{pmatrix} -1 & -31 & 17 \\ -6 & -4 & 6 \\ 1 & 26 & -15 \end{pmatrix}$$

$$A^{-1} = \frac{1}{-32}\begin{pmatrix} 1 & -31 & 17 \\ -6 & -4 & 6 \\ 1 & 26 & -15 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -1/32 & 31/32 & 17/32 \\ +6/+32 & -4/-32 & 6/-32 \\ 1/-32 & 26/32 & -15/32 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -1/32 & 31/32 & 17/32 \\ \cancel{6/32} & 1/8 & 6/-32 \\ -1/32 & 26/32 & -15/32 \end{pmatrix}$$

p.t.o

Q No 3: Solve the following system of linear equation by Gauss Jordan method.

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

Solution:

$$\begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{bmatrix}$$

$R_1 = \frac{1}{2} R_1$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{bmatrix}$$

$R_2 = R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 3 & 2 & -3 & 14 \end{bmatrix}$$

$R_3 = R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -1 & -3 & 13 \end{bmatrix}$$

$R_2 = \frac{1}{2} R_2$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & -13 \end{bmatrix}$$

6

$R_3 = R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & -11 \end{bmatrix}$$

$R_1 = R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & 11 \end{bmatrix}$$

$R_3 = -1/3 \, R_3$

$$\begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/3 \end{bmatrix}$$

$R_1 = R_1 - 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 & -19/3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/3 \end{bmatrix}$$

$$\left.\begin{array}{l} \boxed{x = -19/3} \\ \boxed{y = 2} \\ \boxed{z = 11/3} \end{array}\right\} \text{Solution}$$

Q No 4 :

Show that this Matrix is Diagonsible.

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

Solution:

Matrix A is diagonsible if $A = CDC^{-1}$

let $(A - \lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{bmatrix}$$

$$|A - \lambda I| = (4-\lambda)[(3-\lambda)(1-\lambda)-8] - 2[-5(1-\lambda)+4] - 2[-20+2(3-\lambda)]$$

$$= (4-\lambda)(3-\lambda)(1-\lambda) - 8(4-\lambda) + 10(1-\lambda) - 8 + 40 - 4(3-\lambda)$$

$$= (12-4\lambda-3\lambda+\lambda^2)(1-\lambda) - 32 + 8\lambda + 10 - 10\lambda - 8 + 40 - 12 + 4\lambda$$

$$= (12-7\lambda+\lambda^2)(1-\lambda) - 2 + 2\lambda$$

$$= 12 - 12\lambda - 7\lambda + 7\lambda^2 + \lambda^2 - \lambda^3$$

$$= -\lambda^3 + 8\lambda^2 - 17\lambda + 10 = 0$$

$$= (\lambda-1)(-\lambda^2+7\lambda-10) = 0$$

$$\lambda - 1 = 0, \quad \boxed{\lambda = 1}$$

$$-\lambda^2 + 7\lambda - 10 = 0$$

p.t.o

8

$-\lambda^2 + 5\lambda + 2\lambda - 10 = 0$

$-\lambda(\lambda - 5) + 2(\lambda - 5) = 0$

$(-\lambda + 2), (\lambda - 5) = 0$

$\cancel{+}\lambda = \cancel{+}2$

$\boxed{\lambda = 2}$

$\lambda - 5 = 0$

$\boxed{\lambda = 5}$

$\boxed{\lambda = 1, 2, 5}$

There are three values of "λ" so it is diagonsable.

p.t.o

1) No 5
2) Determine the following homogenous has a non travial solution then describe the solution set.

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$-3x_1 - 25x_2 + 4x_3 = 0$$
$$6x_1 + x_2 - 8x_3 = 0$$

Solution:

$$A = \begin{vmatrix} 3 & 5 & -4 \\ -3 & -25 & 4 \\ 6 & 1 & 8 \end{vmatrix}$$

$$= 3\begin{vmatrix} -25 & 4 \\ 1 & -8 \end{vmatrix} - 5\begin{vmatrix} -2 & 4 \\ 6 & -8 \end{vmatrix} - 4\begin{vmatrix} -3 & -25 \\ 6 & 1 \end{vmatrix}$$

$$= 3(200-4) - 5(24-24) - 4(-3+150)$$
$$= 3(196) - 5(0) - 4(147)$$
$$= 3(196) - 4(147)$$
$$= 588 - 588 = 0$$

Here $|A| = 0$

The solution of the given system is travial so

i.e

solution set = $\{0, 0, 0\}$

P.T.O

Q NO 6:

Reduce the matrix to normal form and find its rank.

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Sol; By Row opreations

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$R_3 \to 2R_3 - R_2$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank = 2

Rank of Matrix by Normal form:

Now By column opreation

P.T.O

$C_2 \rightarrow C_2 - 3C_1$

$$\begin{vmatrix} 1 & 0 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

$C_3 \rightarrow C_3 - 4C_1$

$$\begin{vmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

$C_4 \rightarrow C_4 - 3C_1$

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

Swap $C_4 \rightleftarrows C_2$

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

Now Divide coloumn 2 by 6

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \rightarrow \text{normal form}$$