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SEC : A

Subject : Applied Calculas

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Question # 1

The Function $g(t)$ is defined by

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \le t \le 3 \\ 2t+3 & 3 < t \le 4 \\ 12 & t > 4 \end{cases}$$

(a) state any point of discontinity

(b) Find if they exist

i) $\lim\limits_{t \to 3} g$

Solution:-

(a) To check possibidity of the discontinity of function is at $t=0$ & 4

→ First at $t=0$

$$g(t) = t^2$$

$$g(0) = 0^2 = 0$$

For R.H.L

$$\lim g(1+h) = \lim_{h \to 0} (1+h)^2$$

$$= \lim_{h \to 0} 1 + h^2 + 2h$$

Apply Limit

$$= 1 + 0^2 + (2)(0)$$

$$= 1$$

For L.H.L

$$\lim_{h\to 0} g(1-h) = 2t+3$$

$$= \lim_{h\to 0} 2(1-h)+3$$

$$= \lim_{h\to 0} 2-2h+3$$

Apply Limit

$$= 2-2(0)+3$$

$$= 5$$

R.H.L ≠ L.H.L = g(t) = 5

⇒ Now at t = 4

$$g(4) = 2(4)+3$$

$$= 8+3$$

$$= 11$$

For R.H.L

$$\lim_{h\to 0} g(1+h) = \lim_{h\to 0} 2(1+h)^2+3$$

$$= \lim_{h\to 0} 2+2h+3$$

Apply Limit

$$= 2+2(0)+3 \Rightarrow 5$$

For L.H.L

$$\lim_{h\to 0} g(1-h) = 12$$

$$\boxed{g(4) = R.H.L \neq L.H.L}$$

point of discontinuity is at t = 4

Ans

Page

(3) (b) Find if they exist

(1) $\lim_{t \to 3} g$

For $g(t) = t^2$

R.H.L

$$\lim_{h \to 3} g(1+h) = \lim_{h \to 3} (1+h)^2$$

$$= \lim_{h \to 3} + h^2 + 2h$$

Apply Limit

$$= 1 + 3^2 + 2(3) \Rightarrow 16$$

L.H.L

$$\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t + 3$$

$$= \lim_{h \to 3} 2(1-h) + 3$$

$$= \lim_{h \to 3} 2 - 2h + 3$$

Apply Limit

$$= 2 - 2(3) + 3$$

$$= 2 - 6 + 3$$

$$= -1$$

$\boxed{\text{R.H.L} \neq \text{L.H.L (do not exist since L.H.L is -ve)}}$

Ans

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Question No # 2

Maclaurin's series

$y(x) = x^2 + \sin x$

Solution :-

$Y(X) = x^2 + \sin x$

Since we know that maclaurin series is

$Y(x) = Y(0) + (x-0) y'(0) + (x-0)^2 Y''(0)$

$+ \cdots$

$Y(x) = y(0) + x y'(0) + \frac{x^2 y''(0)}{2!} + \cdots \quad (1)$

Now Find

$Y(0) = ?$

$Y(X) = x^2 + \sin X$

$Y(0) = 0 + \sin 0$

$= 0 + 0$

$= 0$

$\boxed{Y(0) = 0}$

$Y(X) = X^2 + \sin x$

$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin X$

$Y'(x) = 2x + \cos X$

P.T.O

$$y'(0) = 2(0) + \cos(0)$$
$$= 0 + 1$$
$$\boxed{y'(0) = 1}$$

Since $y'(x) = 2x + \cos x$

$$\frac{d}{dx} y'(x) = 2\frac{d}{dx}x + \frac{d}{dx}\cos x$$
$$= 2 - \sin x$$
$$y''(x) = 2 - \sin x$$
$$y''(0) = 2 - \sin 0$$
$$= 2 - 0 = 2$$
$$\boxed{y''(0) = 2}$$

Now

$$y''(x) = 2 - \sin x$$
$$\frac{d}{dx} y''(x) = \frac{d}{dx}2 - \frac{d}{dx}\sin x$$
$$= 0 - \cos x$$
$$y'''(n) = 0 - \cos x$$
$$y'''(0) = -\cos 0$$
$$\boxed{y'''(0) = -1}$$

Put in eq (1)

$$y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \cdots$$
$$= x + \frac{\cancel{2}x^2}{\cancel{2}!} - \frac{x^3}{3!} + \cdots$$
$$= x + x^2 - \frac{x^3}{3!} + \cdots$$

So

$$\boxed{y(x) = x + x^2 - \frac{x^3}{3!} + \cdots}$$ Ans

Duse (6)

Question No # 3

Part (ii)

$$Y = x^3(1+x)^9 e^{6x}$$

Solution:-

Find Y' by using Logrithium diff

$$Y = x^3(1+x)^9 e^{6x}$$

Takins $\ln$ to b/s

$$\ln y = \ln[x^3(1+x)^9 e^{6x}]$$

$$\Rightarrow \ln y = \ln x^3 + \ln(1+x)^9 + \ln e^{6x}$$

$$\Rightarrow \ln y = 3\ln x + 9\ln(1+x)^9 + 6x\ln e$$

$$\Rightarrow \ln y = 3\ln x + 9\ln(1+x) + 6$$

Differenting both sides

$$\Rightarrow \frac{1}{y} \cdot y' = 3 \cdot \frac{1}{x} + 9 \cdot \frac{1}{x} + 6$$

$$\Rightarrow \frac{Y'}{y} = \frac{3}{x} + \frac{9}{1+x} + 6$$

$$\Rightarrow y' = y\left(\frac{3}{x} + \frac{9}{1+x} + 6\right)$$

$$\boxed{\Rightarrow y' = x^3(1+x)^9 e^{6x}\left(\frac{3}{x} + \frac{9}{1+x} + 6\right)}$$

Ans

Question (3)

Part (1)

$$1 + xy = x^2 + y^2$$

Solution:-

$$1 + xy = x^2 + y^2$$

$$\frac{d}{dx}(1 + xy) = \frac{d}{dx}(x^2 + y^2)$$

$$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$0 + x\frac{dy}{dx} + y\frac{dx}{dx} = 2x^{2-1} + 2y^{2-1}\frac{dy}{dx}$$

$$x\frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$\frac{dy}{dx}(x - 2y) = 2x - y$$

$$\frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

$$Y' = \frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{2x - y}{x - 2y}\right)$$

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$$\frac{d^2y}{dx^2} = \frac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$y'' = \frac{(x-2y)(2-y') - (2x-y)(1-2y')}{(x-2y)^2}$$

$$y'' = \frac{2x - xy - 4y + 2yy' - (2x - 4xy' - y + 2yy')}{(x-2y)^2}$$

$$y'' = \frac{2\cancel{x} - xy - 4y + 2\cancel{y}y' - 2x + 4xy' + y - 2\cancel{y}y'}{(x-2y)^2}$$

$$y'' = \frac{4xy' - 3y - xy'}{(x-2y)^2}$$

$$y'' = \frac{4x\left[\frac{2x-y}{x-2y}\right] - 3y - x\left[\frac{2x-y}{x-2y}\right]}{(x-2y)^2}$$

$$y'' = \frac{4x(2x-y) - 3y(x-2y) - x[2x-y)}{(x-2y)(x-2y)^2}$$

$$y'' = \frac{8x^2 - 4xy - 3xy + 6y^2 - 2x^2 + xy}{(x-2y)^3}$$

$$\boxed{y'' = \frac{6x^2 + 6y^2 - 6xy}{(x-2y)^3}}$$

Ans