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Q: 1
Ans: 1

{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(1,7), (1,8)

(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
, (2,7), (2,8)

(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(3,7), (3,8)

(4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (4,7)
, (4,8)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (5,7)
(5,8)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6), (6,7), (6,8)
(7,1), (7,2), (7,3), (7,4), (7,5), (7,6), (7,7), (7,8)
(8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (8,7), (8,8)

Set

$A = \{$ The sum is 7 $\}$

B: $B = \{$ The sum is Even $\}$

$C = \{$ The sum is greater than 8 $\}$

$D = \{$ The two dischad the same out-come $\}$

$A = \{(1,6),(2,5),(3,4),(5,2),(6,1),(4,3)\}$

$B = \{(1,1),(1,3),(1,5),(1,7),(2,7),(2,4),(2,6)$
$(2,8),(3,1),(3,3),(3,5),(3,7),(4,2),(4,4)$
$,(4,6),(4,8),(5,1),(5,3),(5,5),(5,7),(6,2)$
$(6,4),(6,6),(6,8),(7,1),(7,3),(7,5),(7,7),(8,2)$
$,(8,4),(8,6),(8,8)\}$

$C = \{(1,8),(2,7),(2,8),(3,6),(3,7),(3,8),(4,5)$
$(4,6),(4,7),(4,8),(5,4),(5,5),(5,6),(5,7),(5,8)$
$(6,3),(6,4),(6,5),(6,6),(6,7),(6,8),(7,2),(7,3)$
$(7,4),(7,5),(7,6),(7,7),(7,8),(8,1),(8,2)$
$(8,3),(8,4),(8,5),(8,6),(8,7),(8,8)\}$

$D = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)$
$(7,7), (8,8)\}$

$A \cap B = \{\}$ OR ϕ

$A \cap C = \{\}$

$A \cap D = \{\}$

$P(A) = 6/64$, $P(B) = 32/64$

$P(C) = 36/64$, $P(D) = 8/64$

$P(A \cap B) = 0$, $P(A \cap C) = 0$, $P(A \cap D) = 0$

$$P(A/B) = \frac{P A \cap B}{P(B)} = 0 \times 32/6$$

$P(A/B) = 0$

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = 0 \times 36/64$$

$P(A/C) = 0$

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = 0 \times 8/64$$

$P(A/D) = 0$

Q:2

Ans: when we are rolling two dice there ar 36 different combination counting those up there are 15 possibilities less than 7

(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2) (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,1), (5,1)

The probability is of geting less than 7 is

$$\frac{15}{16} = \frac{5}{12}$$

There are 6 possible combinations of getting a 7: (1,6), (2,5), (3,4), (4,3) (5,2), (6,1) which gives a probability of $\frac{6}{36} = \frac{1}{6}$

This means that 21 possibility account for getting less than or equal to 7 so there are 15 remaining

Possibidities of getting more than 7 this is the same as the Probability of getting less than 7 so the Probability must be $\frac{5}{12}$ as well in

Calculating this must assume that each combination is equally likely to roll as any other and therefore the dice are fair or else the calculations don't work

Q:3

Ans:

$p = \frac{2}{3}$ $n = 8$

$q = 1 - p$

$= 1 - \frac{2}{3}$ $q = \frac{1}{3}$

1: $P(X=4)$

$= \binom{8}{4} (2/3)^4 (1/3)^4$

$= \frac{1120}{6561}$

$= 0.1707$

2: $P(X \geq 4)$

$1 - P(X<4)$

$= 1 - \sum^{3} \binom{8}{x} (2/3)^x (1/3)^{8-x}$

$= 1 - [(\frac{1}{3})^8 + 8(2/3)(1/3)^7 + 28(2/3)^2$

$(\frac{1}{3})^6 + 56(\frac{2}{3})^3(\frac{1}{3})^5]$

$= 1 - \frac{1}{6561} [1 + 16 + 112 + 448]$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561 - 577}{6561}$$

$$= \frac{5984}{6561}$$

$$= 0.9121$$

3: $P(3 \leq X \leq 6)$

$$\sum_{x=3}^{6} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3} \cdot \left(\frac{2}{3}\right)^{3} \left(\frac{1}{3}\right)^{5} + \binom{8}{4} \left(\frac{2}{3}\right)^{4} \left(\frac{1}{3}\right)^{4} + \binom{8}{5}$$

$$\left(\frac{2}{3}\right)^{5} \left(\frac{1}{3}\right) \binom{8}{6} \left(\frac{2}{3}\right)^{6} \left(\frac{1}{3}\right)^{2}$$

$$= \frac{8}{(3)^8} \left[56 + 140 + 224 + 224\right]$$

$$= \frac{8 \times 644}{6561} = \frac{5152}{6561} = 0.7852$$

Q:4

Ans:

Since the C_i's form a partition of the sample space we can apply the law of total probability for $A \cap B$

$$P(A \cap B) = \sum_{i=1}^{M} P(A \cap B / C_i) P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^{M} P(A / C_i) P(B / C_i) P(C_i)$$

A and B are conditionally independent)

$$P(A \cap B) = \sum_{i=1}^{M} P(A / C_i) P(B) P(C_i)$$

= (B is independent of all C_i's)

$$P(A \cap B) = P(B) \sum_{i=1}^{M} P(A / C_i) P(C_i)$$

$$P(A \cap B) = P(B) P(A)$$

Law of total probability)

Hence A and B are independent

Q:5

Ans:

The binomial distribution:

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

for $x = 0, 1, 2, \ldots, n$

$$\mu = np, \sigma^2 = np(1-p)$$

A binomial random variable can be thought of as the sum of n independent Bernoulli random variables each with mean p and variance p $p(1-p)$.

Let $U_1, \ldots U_n$ be independent Bernoulli random variables.

$$E(U_i) = p \text{ and } Var(U_i) = p(1-p)$$

$$X = U_1 + \ldots + U_n$$

$$E(x) = E(U_1 + \ldots + U_n)$$

$$E(x) = E(U_1) + \ldots + E(U_n$$

$X = U_1 + \cdots + U_n$

$var(X) = var(U_1 + \cdots + U_n)$

$var(X) = var(U_1) + \cdots + var(U_n)$

The binomial theorem:

$$(a+b)^m = \sum_{y=0}^{m} \binom{m}{y} a^y b^{m-y}$$

$$E(X) = \sum x p(x)$$

$$= \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!((n-1)-(x-1))!}$$

$x p^{x-1} (1-p)^{(n-1)-(x-1)}$

$$= np \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= np \sum \binom{m}{y} p^y (1-p)^{m-y}$$

Recall: $\sum_{y=0}^{m} \binom{m}{y} a^y b^{m-y} = (a+b)^m$

$$\text{var}(X) = E[(X-\mu)^2]$$

$$= \sum_{x} (x-\mu)^2 p(x)$$

$$E[(X-\mu)^2] = E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum_{x=0}^{n} x^2 \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$E[X(X-1)] = \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(x-x)!} p^x$$

$$(1-p)^{n-x}$$

$$E[X(X-1)] = \sum_{x=2}^{n} \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$E[X(X-1)] = n(n-1)p^2 \times$$

$$\sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!((n-2)-(x-2))!} p^{x-2} (1-p)^{(n-2)-(x-2)}$$

$$E[X(X-1)] =$$

$$n(n-1)p^2 \times \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$E[X(X-1)] = n(n-1)p^2$$

$$E[X^2 - X] = n(n-1)p^2$$

$$var(X) = E(X^2) - [E(X)]^2$$

$$= n(n-1)p^2 + np - (np)^2$$

$$= np[(n-1)p + 1 - np]$$

$$\boxed{= np(1-p)}$$

Q:6

Ans:

Bi-hominal Distribution:

A binominal distribution can be though of as simply probability of a success or Failure outcome in an experiment or survey that is repeat-ed multiple times

$$P(X=x)\,F(X) = {}^{n}C_{x}\, P^{x} q^{n-x}$$

Bi-nominal frequency distribution

if the bi-nomial probability distribution is multiplied by N the number of experiment or sets the resulting distrubution is known as the bi

$$N \binom{n}{x} p^{x}$$

Q:7
Ans:

Coefficient of variation

For Data set A

$$CV = \frac{\sigma}{\mu} \times 100$$

$$CV = \frac{3}{45} \times 100$$

$$\boxed{CV = 6.7}$$

For Data set B

$$CV = \frac{11}{60} \times 100$$

$$\boxed{CV = 18.3}$$

For Data set C

$$CV = \frac{5}{50} \times 100$$

$$\boxed{CV = 10}$$

For Dat set D

$$CV = \frac{15}{25} \times 100$$

$$\boxed{CV = 60}$$