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SECTION : A

MODULE : 8TH SEMESTER

(Q1)

The function $g(t)$ is defined by $g(t) = 0$ $t < 0$

$t^2 =$, $0 \leq t \leq 3$

$2t + 3$ $3 < t \leq 4$

12 $t > 4$

a) State any point of discontinuity

b) Find, if they exist

i) $\lim_{t \to 3} g$

Sol:

To check possibility of discontinuity function is at $t = 0$ & 4

→ First at $t = 0$

$g(t) - t^2$

$g(0) = 0^2 = 0$

For R.H.L

$\lim g(1+H) = \lim_{h\to 0} (1+h)^2$

$= \lim_{h\to 0} 1 + h^2 + 2h$

Apply limits

$= 1 + 0^2 + 2(0)$

$= 1$

For L.H.L

$\lim_{h\to 0} g(1-h) = 2t+3$

$= \lim_{h\to 0} 2 - 2h + 3$

Apply limit

$= 2 - 2(0) + 3$

$= 5$

R.H.L $\neq$ L.H.L $= g(t) = 5$

Now at $t = 4$

$g(4) = 2(4) + 3$

$= 8+3$
$= 11$

For R.H.L

$\lim_{h\to 0} g(1+h) = \lim_{h\to 0} 2(1+h)+3$

$= \lim_{h\to 0} 2+2h+3$

Apply limits

$= 2+2(0)+3 \Rightarrow 5$

For L.H.L

$\lim_{h\to 0} g(1-h) = 12$

$g(4) = R.H.L \neq L.H.L$

Point ~~of~~ discontinuity is at $t=4$

(b) Find, if they exist

1. $\lim_{t \to 3} g$

For $g(t) = t^2$

R.H.L $\lim_{h \to 3} g(1+h) = \lim_{h \to 3} (1+h)^2$

$\Rightarrow \lim_{h \to 3} 1 + h^2 + 2h$

Apply limits

$= 1 + 3^2 + 2(3) \Rightarrow 16$

L.H.L

$\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t+3$

$= \lim_{h \to 3} 2(1-h)+3$

$= \lim_{h \to 3} 2-2h+3$

Apply limit

$= 2 - 2(3) + 3$

$= 2 - 6 + 3$

$\boxed{= -1}$ R.H.L $\neq$ L.H.L (doesn't exist, L.H.L is -ve)

QUESTION # 2

$y(x) = x^2 + \sin x$

Since we know that the madurin series is :

$y(x) = y(x) + y'(x)(x - x_0) + \frac{y''(x_0)(x - x_0^2)}{2!} + \cdots$

put $x_0 = 0$

$y(x) = y(0) + (x - 0) y'(0) + (x - 0^2) y''(0) + \cdots$

$y(x) = y(0) + x y'(0) + \frac{x^2 y''(0)}{2!} + \cdots$

Now find

$y(0) = ?$

$y(1) = x^2 + \sin x$

$y(0) = 0 + \sin 0$

$= 0 + 0$

0

$y(0) = 0$

$y(x) = x^2 + \sin x$

$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin x$

$y'(x) = 2x + \cos x$

$y'(0) = 2(0) + \cos(0)$

$y'(0) = 0 + 1$

$y'(0) = 1$

Since $y'(x) = 2x + \cos x$

$\frac{d}{dx} y'(x) = 2 \frac{d}{dx} x + \frac{d}{dx} \cos x$

$= 2 - \sin x$

$y''(x) = 2 - \sin x$

$y''(0) = 2 - \sin 0$

$= 2 - 0 = 2$

$y''(0) = 2$

Now

$y''(x) = 2 - \sin x$

$\frac{d}{dx} y''(x) = \frac{d}{dx} 2 - \frac{d}{dx} \sin x - x$

$= 0 - \cos x$

$y'''(x) = 0 - \cos x$

$y'''(0) = -\cos 0$

$\boxed{y'''(0) = -1}$ put in eq (1)

$y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \cdots$

$= x + \frac{\cancel{2}x^2}{\cancel{2}!} - \frac{x^3(-1)}{3!} + \cdots$

$= x + x^2 - \frac{x^3}{3!} + \cdots$

So,

$y(x) = x + x^2 - \frac{x^3}{3!} + \cdots$

QUESTION : 3

(1) $1 + xy = x^2 + y^2$

SOLUTION:

$$1 + xy = x^2 + y^2$$

$$\frac{d}{dx}(1 + xy) = \frac{d}{dx}(x^2 + y^2)$$

$$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$0 + x\frac{dy}{dx} + y\frac{dx}{dx} = 2x^{2-1} + 2y^{2-1}\frac{dy}{dx}$$

$$x\frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$\frac{dy}{dx}(x - 2y) = 2x - y$$

$$\frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

$$y' = \frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{2x - y}{x - 2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{(x - 2y)\frac{d}{dx}(2x - y) - (2x - y)\frac{d}{dx}(x - 2y)}{(x - 2y)^2}$$

②

$$y'' = \frac{(x-2y)(2-y') - (2x-y)(1-2y')}{(x-2y)^2}$$

$$y'' = \frac{2x - xy' - 4y + 2yy' - (2x - 4xy' - y + 2yy')}{(x-2y')^2}$$

$$y'' = \frac{\cancel{2x} - xy' - 4y + \cancel{2yy'} - \cancel{2x} + 4xy' + y - \cancel{2yy'}}{(x-2y)^2}$$

$$y'' = 4xy' - 3y - xy' / (x-2y)^2$$

$$y'' = 4x\left[\frac{2x-y}{x-2y}\right] - 3y - x\left[\frac{2x-y}{x-y}\right] \Big/ (x-2y)^2$$

$$y'' = \frac{4x(2x-y) - 3y(x-2y) - x[2x-y]}{(x-2y)(x-2y)^2}$$

$$y'' = \frac{8x^2 - 4xy - 3xy + 6y^2 - 2x^2 + xy}{(x-2y)^3}$$

$$y'' = \frac{6x^2 + 6y^2 - 6xy}{(x-2y)^3}$$

(Q3)

(ii)

Find y' by using logarithm diff

$y = x^3 (1+x)^9 e^{6x}$

Taking $\ln$ to b/s

$\ln y = \ln\left(x^3 (1+x)^9 . e^{6x}\right)$

$\ln y = \ln x^3 + \ln (1+x)^9 + \ln e^{6x}$

$\ln y = 3 \ln x + 9 \ln(1+x) + 6x \ln e$

Diff w.r.t x

$\frac{d}{dx} \ln y = \frac{d}{dx} 3 \ln x + \frac{d}{dx} 9 \ln(1+x) + \frac{d}{dx} 6x \ln e$

$\frac{1}{y} \frac{dy}{dx} = \frac{3}{x} + \frac{9}{1+x} + \frac{1}{e^{6x}} \cdot 6$

$\frac{dy}{dx} = y\left(\frac{3}{x} + \frac{9}{1+x} + \frac{6}{e^{6x}}\right)$

$\frac{dy}{dx} = x^3 (1+x)^9 e^{6x}\left(\frac{3}{x} + \frac{9}{1+x} + \frac{6}{e^{6x}}\right)$