Department of Computer Science

Midterm Assignment –Spring 2020

Course Title: Differential Equations

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Q1:a) Define differential equation with example.

Ans:- Differential Equation:.

A differential equation is an equation that relates one or more function and their derivatives. In application, the function generally represent physical quantities, the derivation represent their rate of change, And differential equation defines a relationship b/w the two.

Example:- (1)

Rabbit Again!

our growth differential Equation

$$\frac{dN}{dt} = rN$$

well, that growth can't go on ~~fo~~ forever as they will soon run out of available food.

So, The maximum Population that the Food can Support K.

It called verhulst figure it all out and got Different equation. $\frac{dN}{dt} = rN(1-N/K)$.

Example:- (2)

i) $\frac{dy}{dx} + y^2 = 5x \rightarrow$ (First order)

ii) $\frac{d^2y}{dx^2} + xy = \sin(x) \rightarrow$ (second order)

iii) $\frac{d^3y}{dx^3} + x\frac{dy}{dx} + y = e^x \rightarrow$ (Third order).

b) Define a Separable Differential Equation?

Ans) Separable Differential Equation:-

A Separable differential equation is one that can be broken into a set of separate equations of lower dimensionality by a method of separation variable. This generally relies upon the Problem having some special form.

i) Solve the following IVP using Separable DE and find the interval of validity of the solution

i) $y' = e^{-y}(2x-4) \quad y(5)=0$

Soln

$= y' = e^{-y}(2x-4)$

$= \frac{dy}{dx} = e^{-y}(2x-4) \times dx$

$= \frac{dy}{e^{-y}} = \frac{\cancel{e^{-y}}}{\cancel{e^{-y}}}(2x-4)\,dx$

$= e^{y}dy = (2x-4)\,dx$

$= \int e^{y}dy = \int (2x-4)\,dx$

$= e^{y} = \int 2x\,dx - \int 4\cdot dx$

$= e^{y(x)} = x^2 - 4x + C \longrightarrow \star$

$= e^{y(5)} = 5^2 - 4(5) + C$

$= e^{0} = 25 - 20 + C$

$= 1 = 5 + C$

$1 - 5 = C$

$C = -4$

Put $C=-4$ in eq $(\star)$

$e^{y} = x^2 - 4x - 4$

$e^{y} = x(x-4) - 4 \longrightarrow$ Answer

ii) $y' = \frac{xy^3}{\sqrt{1+x^2}}$, $y(0) = -1$

Soln

$= \cancel{dx}\frac{dy}{\cancel{dx}} = \frac{xy^3}{\sqrt{1+x^2}} \times dx$

$= \frac{dy}{y^3} = \frac{x \cdot \cancel{y^3}}{\sqrt{1+x^2}\,(\cancel{y^3})} dx$

$= \frac{dy}{y^3} = \frac{x}{\sqrt{1+x^2}} dx$

$= \int y^{-3} dy = (1+x^2)^{-\frac12} x \cdot dx$

Taking Integrate

$= \int y^{-3} dy = \int (1+x^2)^{-\frac12} x \cdot dx$

$= \frac{y^{-3+1}}{-3+1} = \frac{2}{2} \int (1+x^2)^{-\frac12} x \cdot dx$

$= \frac{y^{-2}}{-2} = \frac12 \int (1+x^2)^{-\frac12} 2x\,dx$

$= -\frac12 y^{-2} = \frac12 \int (1+x^2)^{-\frac12} 2x\,dx$

$= -\frac12 y^{-2} = \frac12 \frac{(1+x^2)^{-\frac12+1}}{-\frac12+1} + C$

$= -\frac12 y^{-2} = \cancel{\frac12} \frac{(1+x^2)^{\frac12}}{-\frac12} + C$

$= -\frac12 y^{-2} = (1+x^2)^{\frac12} + C$

$= -\frac12 y^{-2} = \sqrt{1+x^2} + C$

$= -\frac12 y^{-2} = \sqrt{1+x^2} + C \longrightarrow \star$

Put $x = 0$

$-\frac12 y(0) = \sqrt{1+0^2}$

$-\frac12 y(0) = \sqrt{1} + C$

$+\frac12(+1) = 1 + C$

$\frac12 = 1 + C$

$\frac12 - 1 = C$

$\frac{1-2}{2} = C$

$C = -\frac12$

Put in ($\star$)

$-\frac12 y^{-2} = \sqrt{1+x^2} + (-\frac12)$ ~~Ans~~

$-\frac12 y^{-2} = \sqrt{1+x^2} - \frac12$ Ans

Q2:a) Solve the following IVP using Linear differential Equation?

i) Explain the Step for Solving Linear D.E?

Ans) Steps:-

- Substitue $y=uv$, $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ into

$$\frac{dy}{dx} + P(x)y = Q(x)$$

- Factor the Parts involving v
- Put the v term equal to zero.
- Solve using separable of variables to find u.
- Substitue u back into the equation we got at the step 2.
- Solve that to find v.
- Finally, Substitue u and v into $y=uv$ to get our solution.

ii) $x' + 2x = \sin t$

Sol^n

Given that

$= \frac{dx}{dt} + 2x = \sin t \rightarrow ①$

i) $\Rightarrow \int P dt = \int 2 dt = 2t$

ii) $\Rightarrow I \cdot F = e^{\int P dt} = 2t$

Multiply I·F with eq ①

$= e^{2t}\left(\frac{dx}{dt} + 2x\right) = e^{2t} \sin t$

$= \frac{d}{dt}(e^{2t}x) = e^{2t} \sin t$

$= d(e^{2t}x) = e^{2t} \sin dt$

Integrate b·s

$= \int d(e^{2t}x) = \underbrace{\int e^{2t} \sin t \, dt}$

$= e^{2t}x = I$

Now

$I = \int e^{2t} \sin t \, dt$

$= e^{2t}(-\cot) + \int \cos t \, dt \; e^{2t}(2)$

$= -e^{2t} \cos t + 2\int e^{2t} \cos dt$

$= -\cos t \, e^{2t} + 2[e^{2t} \sin t - \int \sin t \, dt \; e^{2t}(2)]$

$= -\cos t \, e^{2t} + 2e^{2t} \sin t t - 4I$

$= I + 4I = 2e^{2t} \sin t - e^{2t} \cos t$

$= 5I = e^{2t}(2 \sin t - \cos t)$

$= I = \frac{e^{2t}}{5}[2 \sin t - \cos t]$

Put ② in eq (i)

$= e^{2t} y = \frac{e^{2t}}{5}[2 \sin t - \cos t]$

$x = \frac{1}{5}(2 \sin t - \cos t)$

$5x = 2 \sin t - \cos t$. Ans

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iii) $\cos(x)y' + \sin(x)y = 2\cos^3(x)\sin(x) - 1$

Soln $\qquad y\left[\frac{\pi}{4}\right] = 3\sqrt{2}, \ 0 \le x \le \frac{\pi}{2}$

$= \cos(x)\frac{dy}{dx} + \sin(x)y = 2\cos^3(x)\cdot\sin(x) - 1$

Divide by $\boxed{\text{"}\cos x\text{"}}$

$= \frac{dy}{dx} + \frac{\sin x}{\cos x}y = \frac{2\cos^3 x \cdot \sin x}{\cos x} - \frac{1}{\cos x}$

$= \frac{dy}{dx} + \tan(x)y = 2\cos^2 x \sin x - \sec x$

$= 2\cos x \cdot \cos x \cdot \sin x - \sec x$

① $\frac{dy}{dx} + \tan(x)y = \cos x \cdot \sin 2x - \sec x$

$dy/dx + Py = Q$

2) $\int P dx = \int \tan x dx = \ln \sec x$

3) $I.F = e^{\int P dx} = e^{\ln \sec x} = \sec x$

4) $I.F \times 1$

$\sec x\left(\frac{dy}{dx} + \tan(x)y\right) = \overset{\sec x}{\cancel{\sec x}}(\cos x \cdot \sin 2x - \sec x)$

$\frac{d}{dx}(\sec(x)y) = \sin 2x - \sec^2 x$

$\cancel{\int d}\ \sec(x)\cdot y = \int \sin 2x dx - \int \sec^2 x dx$

Page(6)

$$\sec(x)\,y = \int \sin 2x\,dx - \int \sec^2 x\,dx$$

$$\sec(x)\,y = \cancel{\sin}\ \frac{-\cos 2x}{2} - \tan x + C \longrightarrow ☆(i)$$

Put $x = \pi/4$ and $y = 3\sqrt{2}$

$$\cancel{\sec(x)\,y = \frac{-\cos 2x}{2} - \tan x + C \longrightarrow ☆(ii)}$$

$$\sec(\pi/4) \times 3\sqrt{2} = -\frac{\cos 2x \times \pi/4}{2} - \tan(\pi/4) + C$$

$$1 \times 3\sqrt{2} = \frac{-\cos \pi/2}{2} - 1 + C$$

$$3\sqrt{2} = -\frac{1}{2} \times 0 - 1 + C$$

$$3\sqrt{2} = -1 + C$$

$$\boxed{3\sqrt{2} + 1 = C}$$

Put in ☆

$$\sec(x)\,y = \frac{-\cos 2x}{2} - \tan x + 3\sqrt{2} + 1 \quad \text{Ans}$$

Q3): Solve the following IVP for the exact equation and find the interval of validity for the solution.

i) $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0 \qquad y(0) = -3$

Soln

$= (2xy - 9x^2)dx + (2y + x^2 + 1)dy = 0$

where; $M = 2xy - 9x^2$ And $N = 2y + x^2 + 1$

$\Rightarrow \frac{\partial M}{\partial y} = 2x \quad , \quad \frac{\partial N}{\partial x} = 2x$

$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

So eq (i) is exact D.E

Now

$\underset{y\text{-constant}}{\int M dx} = \int (2xy - 9x^2)dx$

$= \int 2xy\,dx - \int 9x^2 dx$

$= 2y \int x\,dx - 9\int x^2 dx$

$= 2y \frac{x^2}{2} - 9\frac{x^3}{3}$

$= x^2 y - 3x^3$

$\underset{y\text{-Constant}}{\int M dx} = x^2 y - 3x^3 \longrightarrow (\star 1)$

$\int$ (term of N free from x) dy

$= \int (2y+1)\,dy$

$= \int 2y\,dy + \int dy$

$= \int \frac{\cancel{2}y^2}{\cancel{2}} + y$

$= y^2 + y \longrightarrow (\text{☆}2)$

So the required solution is

$= x^2y - 3x^3 + y^2 + y = c \longrightarrow (1)$

Now the initial value

$= y(0) = -3 \quad [x=0, y=-3]$

Putting these value in eq (1)

$= 9 + (-3) = c$

$\Rightarrow \boxed{c = 6}$

Put $c=6$ in eq (1)

$x^2y - 3x^3 + y^2 + y = 6$ Ans

ii) $\frac{2ty}{t^2+1} - 2t - (2 - \ln(t^2+1))y' = 0 \quad y(5) = 0$

Sol

$= \left(\frac{2ty}{t^2+1} - 2t\right)dx - (2 - \ln(t^2+1))dy = 0$

$Mdx + Ndy = 0$

where

$M = \frac{2ty}{t^2+1} - 2t \quad , \quad N = -2 + \ln(t^2+1)$

$\frac{\partial M}{\partial y} = \frac{2t}{t^2+1} \quad , \quad \frac{\partial N}{\partial t} = \frac{1}{t^2+1}(2t) \Rightarrow \frac{2t}{t^2+1}$

So, $\frac{\partial M}{\partial N} = \frac{\partial N}{\partial t}$

So eq(i) is E.D.E.

i) $\int M dt = \int \left(\frac{2t}{t^2+1} y - 2t\right) dt$

y-constant

$= \int \frac{2ty}{t^2+1} dt - \int 2t dt$

$= y \int \frac{2t}{t^2+1} dt - 2\int t dt$

$= y \ln(t^2+1) - t^2 \longrightarrow (i)$

page(10)

ii) $\int$ (term of N which is free from t) dy

$= \int -2dy$

$= -2y \rightarrow (☆2)$

Now

$☆1 + ☆2 = C$

$= y\ln(t^2+1) - t^2 - 2y = C \rightarrow (1)$

Since $y(5) = 0$

$\Rightarrow t = 5, \ y = 0$

Put in eq (1)

$= y\ln(26) - 25 = C$

Put in eq (1)

$= y\ln(t^2+1) - t^2 - 2y = 25$. Ans