Q1 Apply both Euler's method and modified method to the following solution of

$\frac{dy}{dx} = 2x$, $y(0) = 1$.

for $0 \le x \le 0.5$ using $h = 0.1$ compare your answer with the analytic solution. work throughout to three decimal places.

Ans:- By Euler's method:-

Given data

$y(0) = 1$
$\downarrow \quad y_0$

$x_0 = 1$
$\quad y_0$

$x_0 = 0$

$y_0 = 1$

$h = 0.1$

By using formula

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(2x_n)$$

1st Iteration :-

$n=0$.

$$y_1 = y_0 + h(2x_0)$$

$$y_1 = 1 + 0.1(2(0)).$$

$$y_1 = 1 + 0.1(0)$$

$$y_1 = 1 + 0.1$$

$$\boxed{y_1 = 1.1.}$$

$$x_{n+1} = x_n + h.$$

$$x_1 = x_0 + h$$

$$x_1 = 0 + 0.1.$$

$$\boxed{x_1 = 0.1.}$$

2^{nd} Iteration:

$n = 1.$

$y_{n+1} = y_n + h(2x_n)$

$y_2 = y_1 + h(2x_1).$

$y_2 = 1.1 + 0.1[2(0.1)).$

$y_2 = 1.1 + 0.1(0.2)$

$y_2 = 1.1 + 0.02$

$$\boxed{y_2 = 1.12}$$

$x_{n+1} = x_n + h$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$$\boxed{x_2 = 0.2}$$

3rd Iteration:

$n = 2.$

$y_{n+1} = y_n + h(2x_n).$

$y_3 = y_2 + h(2x_2)$

$y_3 = 1.12 + 0.1(2(0.2))$

$\boxed{y_3 = 1.16.}$

$x_{n+1} = x_n + h.$

$x_3 = x_2 + 0.1.$

$x_3 = 0.2 + 0.1.$

$\boxed{x_3 = 0.3.}$

Now By Modified:-

$$\frac{dy}{dx} = 2x.$$

Given

$y_0 = 1,\ x_0 = 0,\ h = 0.1.$

FORMULA:

$$y^{*}_{n+1} = y_n + h\left[f(x_n)\right].$$

$$y^{p}_{n+1} = y_n + h(2x_n) \rightarrow (1)$$

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y^{*}_{n+1})\right]$$

$$= y_n + \frac{h}{2}\left[2x_n + 2x_n\right]$$

$$= y_n + \frac{h}{2}\left[4x_n\right].$$

NOW.

put $n=0$

$$x_1 = x_0 + h.$$

$$x_1 = 0 + 0.1$$

$$\boxed{x_1 = 0.1}$$

1st iteration:-

$y_1 = y_0 + \frac{h}{2}(4x_0)$

$y_1 = 1 + \frac{0.1}{2}(4(0))$

$y_1 = 1.$

2nd iteration

$n = 1.$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$x_2 = 0.2.$

$y_2 = y_1 + \frac{h}{2}(4x_1).$

$y_2 = 1 + \frac{0.1}{2}(4(0.1)).$

$y_2 = 1.02$

3rd Iteration :-

If $n = 2$.

$$x_3 = x_2 + h$$

$$x_3 = 0.2 + 0.1$$

$$x_3 = 0.3$$

$$y_3 = y_2 + \frac{h}{2}(4x_2)$$

$$= 1.02 + \frac{0.1}{2}(4(0.2))$$

$$= 1.06$$

①

Q.No 11

use the fourth order of Runge Kutta to obtain a solution of $\frac{dy}{dx} = x^2 + x - y$

subject to $y = 0$, when $x = 0$, for $0 \le x \le 0.6$ with $h = 0.2$. work throughout to 4 decimal places.

Given Data:-

$x_0 = 0, \quad y_0 = 0, \quad h = 0.2.$

$$y_{n+1} = y_n + K \rightarrow (i)$$

$$x_{n+1} = x_n + h$$

~~x~~ put

$n = 0$

$$x_{0+1} = x_0 + h$$

$$x_1 = 0 + 0.2$$

$$x_1 = 0.2$$

NOW

$$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) \rightarrow (ii)$$

NOW

$$K_1 = hf(x_0, y_0)$$

$$K_1 = h(x_0 - y_0)$$

$$K_1 = 0.2(0^2 - 0 - 0)$$

$$\boxed{K_1 = 0}$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$$

$$K_2 = 0.2f\left(0 + \frac{0.2}{2}, 0 + \frac{0}{2}\right)$$

$$K_2 = 0.2f(0 + 0.1, 0 + 0$$

$$K_2 = 0.2(0.1^2 - 0 + 0)$$

$$K_2 = 0.2(0.01)$$

$$\boxed{K_2 = 0.002}$$

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$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_3 = 0.2 f\left(0 + \frac{0.2}{2}, 0 + \frac{0.002}{2}\right)$$

$$k_3 = 0.2 f(0.1, 0.001)$$

$$k_3 = 0.2(0.1^2 + 0.1 - 0.001)$$

$$\boxed{k_3 = 0.0218}$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

$$k_4 = 0.2 f(0 + 0.2, 0 + 0.0218)$$

$$k_4 = 0.2(0.2^2 + 0.2 - 0.0218)$$

$$\boxed{k_4 = 0.04364}$$

Now

step 2.

$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \rightarrow (*)$$

Put the value of k_1, k_2, k_3 & k_4 in Eq (*).

$$K = \frac{1}{6}(0 + 2(0.002) + 2(0.0218) + 0.0436)$$

$$\boxed{K = 0.0146}$$

NOW

$$y_{n+1} = y_n + 0.0146$$

$$y_1 = 0 + 0.0146$$

$$\boxed{y_1 = 0.0146}$$

3rd Given data:-

$a = 0,\ b = 10\ \&\ n = 10$

$h = \frac{b-a}{n} = \frac{10-0}{10}$

$h = 10/10 = 1.$

Sol:-

x	0	1	2	3	4	5	6	7	8	9	10
$f(x_0)$	10.1	17.2	24.4	29.2	34.6	41.6	50.9	57.8	60.3	61.2	62.1

using formula.

$f(x)dx = h/2\ [f(x_0) + 2f(x_1) + f(x_2) + f(x_3) \ldots f(x_9) + f(x_{10})].$

$= \frac{1}{2}[10.1 + 2(17.2 + 24.4 + 29.2 + 34.6 + 41.2 + 50.9 + 57.8) + 62.1 \Rightarrow \boxed{412.9}$ Ans

Q4 $\int_{2}^{3} \ln(x^3+1)\,dx$

use 10 strips.

Sol:

$n = 10$

$h = \frac{3-2}{10} = 0.1$.

x	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
$f(x)$	0.693	0.846	1.003	1.162	1.320	1.476	1.628	1.777	1.922	2.062

By using formula.

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4(f(x_1) + f(x_3) \ldots\ldots) + 2f(x_2 \ldots) + \right] f(x_n).$$

$$= \frac{0.1}{3}[0.693 + 4(0.846 + 1.162 + 1.476 + 1.777) + 2(1.008 + 1.320 + 1.628 + 1.922) + 2.062].$$

$$= 1.184 \quad \text{ANS.}$$