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Section:- B

Subject:- Applied Calculus

Department of Civil Engineering

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Question#1

The function $g(t)$ is defined by

$$g(t) = \begin{cases} 0 & t<0 \\ t^2 & 0 \leq t \leq 3 \\ 2t+3 & 3 < t \leq 4 \\ 12 & t>4 \end{cases}$$

a. State any point of discontinuity

b. Find, if they exist

i) $\lim\limits_{t \to 3} g$

Sol:- There is no discontinuity
(a)

(b) Now finding $\lim g(t)$ as $t \to 3$

R.H.L (Right hand limit)

$$\lim_{t \to 3} g(t) = \lim_{t \to 3} 2t+3$$

$= 2(3) + 3$

$= 6 + 3$

$= 9$

For L.H.L (Left hand limit)

$$\lim_{t \to 3} g(t) = \lim_{t \to 3} t^2$$

$$= (3)^2$$

$$= 9$$

L.H.L = R.H.L

Question # 2

i) Find the Maclaurin's series for

$$Y(x) = x^2 + \sin x$$

Sol:-

$y(x) = x^2 + \sin x$ | $y(0) = 0^2 + \sin(0) = 0$

$y'(x) = 2x + \cos x$ | $y'(0) = 2(0) + \cos(0) = 1$

$y''(x) = 2 - \sin x$ | $y''(0) = 2 - \sin(0) = 2$

$y'''(x) = -\cos x$ | $y'''(0) = -\cos(0) = -1$

Now the Maclaurin's series is

$$y(x) = y(0) + \frac{y'(0)\,x^1}{1!} + \frac{y''(0)\,x^2}{2!} + \frac{y'''(0)\,x^3}{3!} + \dots$$

$$x^2 + \sin x = 0 + \frac{1x}{1!} + \frac{\cancel{2}x^2}{\cancel{2}\times 1} + \frac{(-1)x^3}{3\times 2\times 1} + \dots$$

$$x^2 + \sin x = 0 + x + x^2 - \frac{x^3}{6} + \dots$$

Question # 3

i) Find y'' given

$$1 + xy = x^2 + y^2$$

ii) Find y' by using logarithmic differenciation

$$Y = x^3 (1+x)^9 e^{6x}$$

Sol :- $1 + xy = x^2 + y^2$

(i)

Differenciate w.r.t x on both sides

$$\frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2+y^2)$$

$$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$0 + x\frac{d}{dx}y + y\frac{d}{dx}x = 2x + 2y\frac{d}{dx}y$$

$$x\frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$\frac{dy}{dx}(x-2y) = 2x-y$$

$$\frac{dy}{dx} = \frac{(2x-y)}{(x-2y)}$$

Again differenciate w.r.t "x" on both sides

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\frac{(2x-y)}{(x-2y)}$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\left\{\frac{d}{dx}2x - \frac{d}{dx}y\right\} - (2x-y)\left\{\frac{d}{dx}x - \frac{d}{dx}2y\right\}}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\left(2-\frac{dy}{dx}\right) - (2x-y)\left(1-2\frac{dy}{dx}\right)}{(x-2y)^2}$$

Put $\frac{dy}{dx} = \frac{(2x-y)}{(x-2y)}$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\left(2-\left(\frac{2x-y}{x-2y}\right)\right) - (2x-y)\left(1-2\left(\frac{2x-y}{x-2y}\right)\right)}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\cancel{(x-2y)}\left(\frac{2(x-2y)-(2x-y)}{\cancel{x-2y}}\right) - (2x-y)\left(\frac{x-2y-2(2x-y)}{x-2y}\right)}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(\cancel{2x}-4y-\cancel{2x}+y) - (2x-y)\left(\frac{x-\cancel{2y}-4x+\cancel{2y}}{x-2y}\right)}{(x-2y)^2}$$

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$$\cancel{\frac{d^2y}{dx^2} = 2x - 4y - 2x + y}$$

$$\frac{d^2y}{dx^2} = \frac{(-3y) - (2x-y)\left(\frac{-3x}{x-2y}\right)}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(-3y) - \left(\frac{-6x^2+3xy}{x-2y}\right)}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{(-3y)(x-2y) - (-6x^2+3xy)}{x-2y}}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{-3xy+6y^2+6x^2-3xy}{(x-2y)}}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-6xy+6y^2+6x^2}{x-2y} \times \frac{1}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-6xy + 6y^2 + 6x^2}{(x-2y)^3}$$

ii) Sol:- $y = x^3 (1+x)^9 e^{6x}$

Applying ln on both sides

$$\ln y = \ln(x^3)(1+x)^9 e^{6x}$$

$$\ln y = \ln x^3 + \ln (1+x)^9 + \ln e^{6x}$$

$$\frac{1}{y}\frac{dy}{dx} = 3\ln x + 9 \ln (1+x) + \ln e^{6x}$$

$$\frac{1}{y}\frac{dy}{dx} = 3 \cdot \frac{1}{x} + \frac{9}{1+x}\frac{d}{dx}(1+x) + \frac{1}{e^{6x}}\frac{d}{dx}(e^{6x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{x} + \frac{9}{(1+x)}(1) + \frac{1}{\cancel{e^{6x}}} \cdot \cancel{e^{6x}} \cdot 6$$

$$\frac{dy}{dx} = y\left[\frac{3}{x} + \frac{9}{(1+x)} + 6\right]$$

Putting value of "y"

$$\frac{dy}{dx} = x^3(1+x)^9 e^{6x}\left(\frac{3}{x} + \frac{9}{1+x} + 6\right)$$