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Subject # Hydraulic Engineering

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Question # 1

A prototype gate valve which will control the flow

non dimensional groups. A 1/5 scale model is built to determine the pressure drop across the valve with water as the working fluid

- a) For a particular opening, when for dynamic similarity?
- b) What is the ratio of the quantities and model?
- c) Find the pressure drop in the prototype if it is 60 kPa in the model.

The density and viscosity of paraffin are 800 kg m^{-3} and $0.002 \text{ kg m}^{-1} \text{ s}^{-1}$ respectively. Take kinematic viscosity of water as $1.0 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$

The pressure drop ΔP is expected to depend upon the gate opening h , depth d , velocity V , density ρ and viscosity μ .

Sol:-

The pressure drop ΔP is expected to depend upon the gate opening h , the overall depth d , the velocity V , density ρ and Viscosity μ .

List the relevant variables:

$$\Delta P, h, d, V, \rho, \mu$$

Write down dimensions:

$$\Delta P \quad ML^{-1}T^{-2}$$

$$h \quad L$$

$$d \quad L$$

$$V \quad LT^{-1}$$

$$\rho \quad ML^{-3}$$

$$\mu \quad ML^{-1}T^{-1}$$

Number of variable - $n = 6$

Number of independent dimensions

$$m = 3 \text{ (M, L and T)}$$

Number of non dimensional groups

$$n - m = 3$$

Choose $m (=3)$ scaling variables:

geometric (d); kinematic / time-dependent (V)
dynamic / mass-dependent (ρ).

Form dimensionless groups by non-dimensionalising the remaining variables: $\Delta P, h$ and μ .

$$\Pi_1 = \Delta P d^a V^b \rho^c$$

$$M^0 L^0 T^0 = (ML^{-1}T^{-2})(L)^a (LT^{-1})^b (ML^{-3})^c$$

$$= M^{1+c} L^{-1+a+b-3c} T^{-2-b}$$

$$M: 0 = 1 + c \Rightarrow c = -1$$

$$T: 0 = -2 - b \Rightarrow b = -2$$

$$L: 0 = -1 + a + b - 3c \Rightarrow a = 1 + 3c - b = 0$$

$$\Rightarrow \Pi_1 = \Delta P V^{-2} \rho^{-1} = \frac{\Delta P}{\rho V^2}$$

$$\Pi_2 = \frac{h}{d} \text{ (by inspection, since } h \text{ is a length)}$$

$\Pi_3 = \mu d^a V^b \rho^c$ (Probably obvious by now, but here goes anyway)

$$M^0 L^0 T^0 = (ML^{-1}T^{-1})(L)^a (LT^{-1})^b (ML^{-3})^c$$

$$= M^{1+c} L^{-1+a+b-3c} T^{-1-b}$$

$$M: 0 = 1 + c \Rightarrow c = -1$$

$$T: 0 = -1 - b + 0 \Rightarrow b = -1$$

$$L: 0 = -1 + a + b - 3c \Rightarrow a = 1 + 3c - b = -1$$

$$\Rightarrow \Pi_3 = \mu d^{-1} V^{-1} \rho^{-1} = \frac{\mu}{\rho V d}$$

Recognition of the Reynolds number suggests that we replace Π_3 by

$$\Pi_3 = (\Pi_3)^{-1} = \frac{\rho V d}{\mu}$$

Hence dimensional analysis yields

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$$\Pi_1 = f(\Pi_2, \Pi_3')$$

i.e.

$$\frac{\Delta P}{\rho V^2} = f\left(\frac{h}{d}, \frac{\rho V d}{\mu}\right)$$

a) Dynamic similarity requires that all non-dimensional groups be the same in model and prototype i.e.

$$\Pi_1 = \left(\frac{\Delta P}{\rho V^2}\right)_p = \left(\frac{\Delta P}{\rho V^2}\right)_m$$

$$\Pi_2 = \left(\frac{h}{d}\right)_p = \left(\frac{h}{d}\right)_m$$

$$\Pi_3' = \left(\frac{\rho V d}{\mu}\right)_p = \left(\frac{\rho V d}{\mu}\right)_m$$

From the last we have velocity ratio

$$\frac{V_p}{V_m} = \frac{(\mu/\rho)_p d_m}{(\mu/\rho)_m d_p} = \frac{0.002/800 \times 1}{1.0 \times 10^6 \times 5} = 0.5$$

Hence

$$V_m = \frac{V_p}{0.5} = \frac{3.0}{0.5} = 6.0 \text{ m s}^{-1}$$

(b) The ratio of the quantities of flow is

$$\frac{Q_p}{Q_m} = \frac{(\text{Velocity} \times \text{Area})_p}{(\text{Velocity} \times \text{Area})_m} = \frac{V_p}{V_m} \left[\frac{d_p}{d_m}\right]^2$$

$$= 0.5 \times 5^2 = 12.5$$

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© Finally for the pressure drop.

$$\pi_1 = \left(\frac{\Delta P}{\rho V^2} \right)_p = \left(\frac{\Delta P}{\rho V^2} \right)_m \Rightarrow \frac{(\Delta P)_p}{(\Delta P)_m}$$

$$= \frac{\rho_p}{\rho_m} \left[\frac{V_p}{V_m} \right]^2 = \frac{800}{1000} \times 0.5^2 = 0.2$$

Hence,

$$\Delta P_p = 0.2 \times \Delta P_m = 0.2 \times 60$$

$$= 12.0 \text{ kPa}, \quad \text{Ans}$$

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Question # 2

Design a practical profile of gravity dam with the following data.

Sol:-

= Given data:-

- Maximum depth of water in the Reservoir is 75 m.
- Specific Gravity of Dam = $G = 2.42$
- Allowable compressive stress for the dam masonry = 751 T/m^2
- Height of wave = 1.21 m

$$u = 0.7$$

No uplift pressure. $C_u = 0$

Sol:-

$$\textcircled{1} H_{\text{limiting}} = \frac{G_{\text{all}}}{\gamma_w (G - C_u + 1)}$$

$$= \frac{751 \times 1000}{1000 (2.42 - 0 + 1)} = 219.60 \text{ m}$$

$$H_{\text{limiting}} = 219.60 \text{ m} > H_w = 75 \text{ m}$$

So it is low Gravity Dam.

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② Top width "d"

$$\text{Free board} = 1.5 \text{ h.w.} + 0.3$$

$$\text{Free board} = 1.5 \times 1.2$$

$$[\text{F.B} = 1.81 \text{ m}]$$

Height of Dam = H_0

$$H_0 = H_w + \text{F.B}$$

$$H_0 = 75 + 1.81 = [76.81 \text{ m}]$$

$$d = 14 \% \text{ of } H_0$$

$$d = 0.14 \times 76.81$$

$$[d = 10.753 \text{ m}]$$

③ Base width 'b' (without offset)

For no sliding Criteria

$$b' = H_w / 4G$$

$$= 75 / 0.7 \times 2.42$$

$$b' = 44.27$$

$$[b' \cong 45 \text{ m}]$$

For No tension:-

$$b' = \frac{Hw}{\sqrt{G}} = \frac{75}{\sqrt{2.42}}$$

$$[b' = 49\text{m}]$$

$$[\text{use } b' = 49\text{m}]$$

(ii) Depth of vertical portion on u/s side.

$$h' = 2a \sqrt{G - C_u}$$

$$h' = 2(10.753) \sqrt{2.42 - 0}$$

$$h' = 33.45\text{m} \cong 34\text{m}$$

(iii) Upstream offset.

$$a/16 = \frac{10.75}{16} = 0.67\text{m}$$

(iv) Depth of below the water level to end of inclined portion in u/s = $3.14 a \sqrt{G}$

$$= 3.14 \times 10.75 \sqrt{2.42}$$

$$= [52.51\text{m}]$$

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⑦ Total width of the Base of Dam

$$b = b' + \frac{a}{16}$$

$$b = 49 + 0.67$$

$$b = 49.67 \text{ m}$$

⑧ $\tan \theta = b' / H$

$$= 49 / 75$$

$$\theta = \tan^{-1} (49/75) = 33.15^\circ$$

⑨ Depth of vertical portion on D/S

$$\tan \theta = a / d' = \frac{10.75}{d'}$$

$$\frac{49}{75} d' = 10.75$$

$$d' = \frac{10.75 \times 75}{49} = [16.45 \text{ m}]$$

Depth of vertical portion

$$d = d' + F.B = 16.45 + 1.81$$

$$= 18.26 \text{ m}$$

Question #3

Using any hydraulic model. ?

Ans) Dimensional Analysis is a mathematical technique making use of study of dimensions.

It deals with the dimensions of physical quantities.

In dimensional analysis one first predicts the physical parameters that will influence the flow and then by grouping these parameters in dimensionless combination a better understanding of the flow phenomenon is made possible.

This mathematical technique is used in research work for design and for conducting model tests.

Types of Dimensions :-

- ① Fundamental Dimensions.
- ② Secondary Dimensions.

Fundamental Dimension:-

These are basic quantities. For example

- Time, T
- Distance, L
- Mass, M

- Time, T
- Distance, L
- Force, F

$$\text{Force} = \text{Mass} \times \text{Acceleration} = \text{MLT}^{-2}$$

Secondary Dimensions:-

These are those quantities which pass more than the fundamental dimensions.

For example:-

- Velocity is denoted by distance per unit time L/T .
- Acceleration is denoted by distance L/T^2 .
- Density is denoted by mass per unit volume M/L^3 .

Since velocity, density and acceleration involve more than one fundamental quantities so these are called derive quantities.

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Methodology of Dimensional Analysis :-

The basic principle is dimensional Homogeneity, which means the dimensions of each terms in an equation on both sides are equal.

So such an equation in which dimensions of each term in both sides of equation are same, is known as Dimensionally Homogeneous equation. Such equation are independent of system of units. For example:

Let Consider the equation $V = (2gH)^{1/2}$

- Dimensions of LHS = $V = L/T = LT^{-1}$
- Dimensions of RHS = $(2gH)^{1/2} = (L/T^2 \times L)^{1/2}$
- Dimension of L.H.S = Dimension of R.H.

So the equation $V = (2gH)^{1/2}$ is

dimensionally Homogeneous equation.

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Characteristics		Units SI	Dimension (MLT)	Dimension (FLT)
Geometry	Length	m	L	
	Area	m ²	L ²	
	Volume	m ³	L ³	
Kinematic	Time	s	L/T T	
	Velocity	m/s	L/T ² L/T	
	Acceleration	m/s ²	L/T ² L/T ²	
	Discharge	m ³ /s	L ³ /T ³	
Dynamic	Mass	kg	M	FL ⁻¹ T ²
	Force	N (kg-m/s ²)	MLT ⁻²	F
	Pressure	Pa (N/m ²)	ML ⁻¹ T ⁻²	FL ⁻²
	Energy	J (N-m)	ML ² T ⁻²	FL
	Power	Watt (N-m/s)	ML ² T ⁻³	FLT ⁻¹

Methods of Dimensional Analysis :-

If the number of variables involved in a physical phenomenon are known then the relation among the variables can be determined by the following two methods.

- ① Rayleigh's Method
- ② Buckingham's Π Theorem

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Rayleigh's Method :-

It is used to determining expression for a variable which depend upon the maximum three to four variables.

If the no of independent Variable are more than 4 then it is very difficult to obtain expression for dependent variables

Let X is dependent variable which depend upon $X_1, X_2,$ and X_3

$$X = f(X_1, X_2, X_3)$$

$$X = k X_1^a, X_2^b, X_3^c$$

Whv k is non dimensional constant.

The diameter of a sphere particle has same specific gravity and the terminal uniform settling velocity as the given particle in the same sedimentation.

Particle density:-

Particle density effect the settling fall velocity. As air density increases with decreasing altitude about 1% per 80 meter (260 ft) for every 160 meter of terminal speed decrease 1%.

Viscosity of water:-

Fluid Velocity through porous media is approximated as inversely proportional to the kinematic viscosity. A decrease in viscosity therefore increases velocity U of a compound through porous media.

Turbulence of water:-

Turbulence of water effect the fall velocity of water in reservoir. because the non-linearity ϵ zigzag path effect \cup the flow of water ϵ cause the variation in the flow.