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Subject.

probability method in Engineering

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Q1 (a) Given:

$a_x = 30m$

$\sigma_x = 5m$

$f_x = 26m$

Req:

(i) What is the probability of being disqualified in the qualifying round?

(ii) In the main event what is the probability the record will be broken.

Sol:

As we know that.

(i)

$$P\{x \leq 26m\} = F_x(26)$$

$$= F\left(\frac{26-30}{5}\right)$$

$$= F = (-0.8)$$

$$= 1 - F(0.8)$$

$$= 1 - 0.7881$$

$$= 0.2113.$$

(ii)

$$P\{x > 42\} = 1 - F_x(42)$$

$$= 1 - F\left(\frac{42-30}{5}\right)$$

$$= 1 - F\left(\frac{42-30}{5}\right)$$

$$= 1 - F(2.4)$$

$$= 1 - 0.9918$$

$$= 0.0082.$$

(0.82%)

Q1 (b) Given

$\sigma x = 1800m$

$6r = 80m.$

Req: Target located distance = 1980m.

(i) Find the probability that the shell will fall within $\pm$ 68m of the target.

(ii) Find the probability that the shells will fall at distance of 2050m or more

Sol.

As we know that.

(i) $P\{1980-68 < x \leq 1980+68\}$

$F_X(2048) - F_X(1912)$

$F\left[\frac{2048-1800}{80}\right) - F\left(\frac{1912-1800}{80}\right)$

$F(3.1) - F(1.4)$

$F(0.9990) - (1-0.9112)$

$= 0.080\,7192.$

(ii)

$P\{x > 2050) = 1 - P\{x < 2050\}$

$1 - F_X(2050) = 1 - F\left(\frac{2050-1800}{80}\right)$

$F(3.125)$

$= 1 - 0.9992.$

$= 0.0008.$

Q1 (c) Given:

$$F_x(x) = \begin{cases} \frac{e^{5x}}{8} & 0 \le x \le b \\ 0 & \text{elsewhere} \end{cases}$$

Req.

valid probability density?

Sol.

As we know that

$$F_x(x) = \int_0^b \frac{e^{5x}}{8}\, dx = 1$$

$$= \frac{1}{8}\left[\int_0^b e^{5x}\, dx\right] = 1$$

$$= \frac{1}{8}\left[\frac{e^{5x}}{5}\Big|_0^b\right] = 1$$

$$= \frac{1}{8}\left[\frac{e^{5b}}{5} - \frac{e^{5(0)}}{5}\right] = 1$$

$$= \frac{1}{8}\left[\frac{e^{5b}}{5} - \frac{1}{5}\right] = 1$$

$$= \frac{e^{5b}-1}{5} = 4$$

$$e^{5b} - 1 = 20$$

$$e^{5b} = 20+1 = e^{5b} = 21$$

$$\left\{ b = \frac{1}{5}\ln(21) \right\}$$ Ans

Q2 (a) Given:

average 3 murder per week.

Req.

(i) What is the probability that there will be 5 or more murder in a given week?

(ii) on average how many weeks a year can this city expect to have no murders

(iii) How many weeks/year (average) can the city expect the number of murders per week equal to or exceed the average number per week.

Sol.

As we know that.

(i)

$P(5 \text{ or more})$

$= 1 - P(0) - P(1) - P(2) - P(3) - P(4)$

$= 1 - e^{-3}\left[1 + 3 + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!}\right]$

$= 1 - \frac{131}{8} e^{-3}$

$= 0.1847.$

(ii) $P(0) = e^{-3} = 0.0498$ average number of weeks per year with no murders $= 52(e^{-3})$

$= 2.5864$ weeks.

(iii)

3 or more murders exceeds the average no

$p(3 \text{ or more}) = 1 - p(0) - p(1) - p(2)$

$= 1 - e^{-3}\left[1 + 3 + \frac{3^2}{2}\right]$

$= 1 - \frac{17}{2} e^{-3}$

$= 0.5768$ (2.5889)

average number of weeks per year that number of murders exceeds the average $= 52\left[1 - \frac{17}{2} e^{-3}\right]$

$= 29.9941$ weeks.

Q2 (b) Given:

$$F_X(x) = \sum_{n=1}^{N} \frac{n^3}{650} u(x-n)$$

Req:

(i) $P\{-\infty < x \leq 6.5\}$

(ii) $P\{x > 4\}$

(iii) $P\{6 < x \leq 9\}$

Sol:

As we know that

Distribution function:

$F_X(x_2) - F_X(x_1)$

(i)

$$F_X(6.5) = \sum_{n=1}^{6} \frac{n^3}{650} u(x-n)$$

$$= \frac{1^3}{650} + \frac{2^3}{650} + \frac{3^3}{650} + \frac{4^3}{650} + \frac{5^3}{650} + \frac{6^3}{650}$$

$$\frac{1}{650} + \frac{8}{650} + \frac{27}{650} + \frac{64}{650} + \frac{125}{650} + \frac{216}{650}$$

$$= \frac{441}{650} = 0.6784$$

(ii) $P(X>4) = 1 - P(X \le 4)$

$$= 1 - F_X(4)$$

$$= 1 - \sum_{n=1}^{4} \frac{n^3}{650} u(x-n)$$

$$= 1 - \left(\frac{1^3}{650} + \frac{2^3}{650} + \frac{3^3}{650} + \frac{4^3}{650}\right)$$

$$= 1 - \left(\frac{1}{650} + \frac{8}{650} + \frac{27}{650} + \frac{64}{650}\right)$$

$$1 - \left(\frac{100}{650}\right)$$

$$\frac{650-100}{650} = \frac{550}{650} = 0.846$$

(iii) $P(6 < x \leq 9)$

According to property v

$F_x(9) - F_x(6)$

$$\sum_{n=1}^{9} \frac{n^3}{650} - \sum_{n=1}^{9} \frac{n^3}{650}$$

$$\frac{7^3}{650} + \frac{8^3}{650} + \frac{9^3}{650}.$$

$$\frac{343}{650} + \frac{512}{650} + \frac{729}{650}$$

$$\frac{1584}{650} = 2.436$$

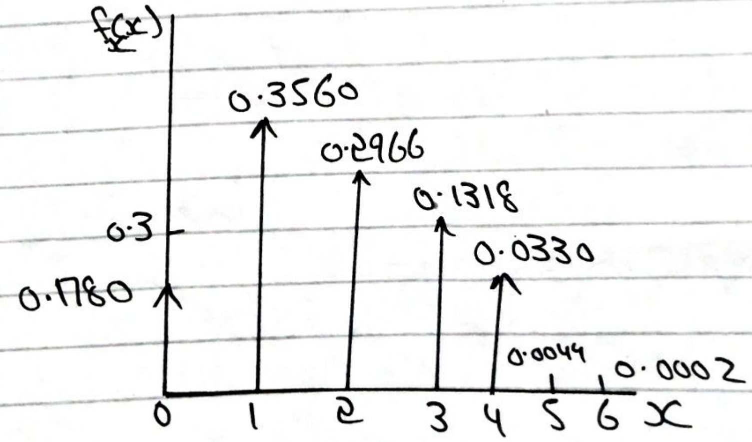
Q2 (c) Given.

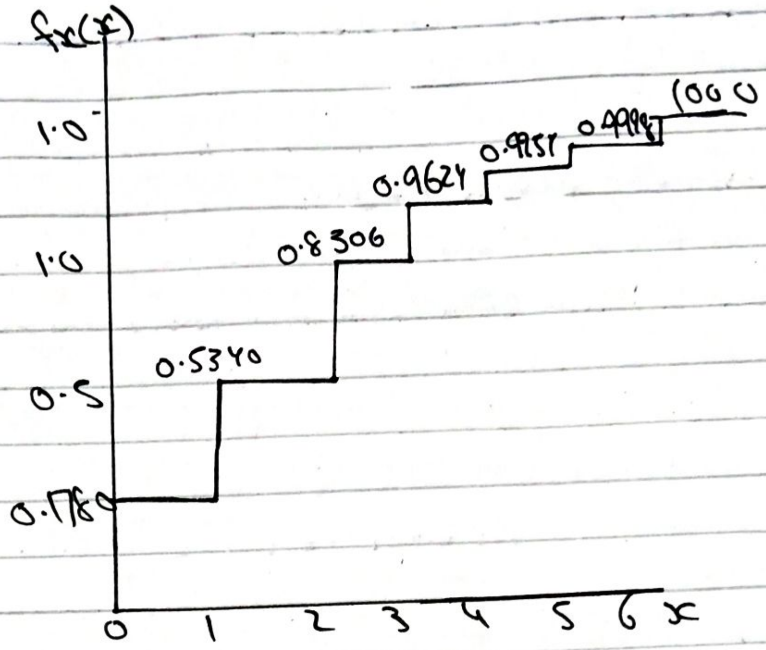
$N = 5$
$P = 0.25.$

Req.
Find the binomial Density and distribution function,

Sol.
As we know that.

(a)





(b)

Binomial density (a) and distribution (b) function for the case $N=6$ $p=0.25$.

Q3 (a) Given.

$$f(x) = \left(\frac{1}{2}\right) u(x) e^{-x/2}$$

$$g(x) = x^3$$

Req.

Find the expected value.

Sol.

As we know that.

$$E[g(x)] = E[x^3]$$

$$= \int_0^\infty \frac{1}{2} x^3 e^{-x/2} \, dx.$$

use C-48

$$= \frac{1}{2}\left(\frac{6}{(1/2)^4}\right)$$

$$= 0.5\left[\frac{6}{0.0625}\right]$$

$$= 48 \text{ Ans.}$$

Q3 (b) Given

person A B, C

Req.

(i) What is probability you will adance to step 2 after the first toss?

(ii) what is the probability that person A will be out after the first toss?

Sol.

As we know that.

i:

you ar advancing ~~was~~ immediately if event has a result and you are on wining side

$$\frac{6}{8} \times \frac{2}{3} = \frac{1}{2} \text{ or } 50\%.$$

(ii) Out mean you are facing $\frac{1}{3}$ chance of $\frac{6}{8}$ fatal events

$= 25\%$.