Department of Electrical Engineering
Final – Term Assignment Spring 2020
Date: 24/06/2020

Course Details

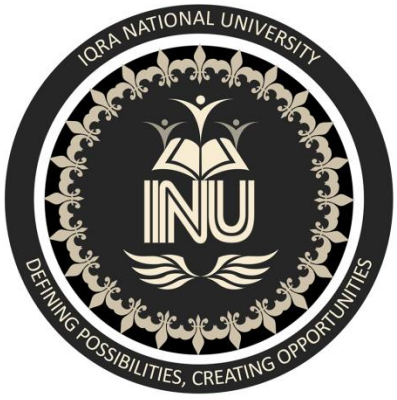
Course Title: Numerical Analysis **Module:** _____

Instructor: **Muhammad Waqas** **Total Marks:** 50

Student Details

Name: **Saad Bin Tariq** **Student ID:** **5534**

Q1.	(a)	Find the root of the equation given below by Bisection method, accuracy must be up to three decimal places $x^3 - x^2 + x - 7 = 0$	Marks 10 CLO 1
Q2.	(a)	Use Regula-Falsi method to compute the root of the following equation in the interval [0, 1] after third iteration. $f(x) = \cos x - xe^x$	Marks 07 CLO 1
	(b)	Use Regula-Falsi (method of false position) to solve the following equation, accuracy must be up to four decimal places. $x^3 - 4x - 9 = 0$	Marks 07 CLO 2
Q3.	(a)	Find the real root of the following equation using Newton-Raphson method in the interval [2,3] after third iteration. $x^3 - 3x - 5 = 0$	Marks 08 CLO 2
	(b)	Solve the following equation by using Muller's method, only perform three iterations. ($x_0 = 0.5$, $x_1 = 1$, $x_2 = 0$) $x^3 - 7x^2 + 14x - 6$	Marks 08 CLO 2
Q4.	(a)	Using Gaussian Elimination method, solve the following system of equations $2x - y + 2z = 2$ $x + 10y - 3z = 5$ $x - y - z = 3$	Marks 10 CLO 1



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Department: BE(E)

Subject: Numerical Analysis

Teacher : Sir Muhammad Waqas

Question No: 1

Find the root of the equation given below by bisection Method, accuracy must be upto three decimals places.

$$x^3 - x^2 + x - 7 = 0$$

Solution:

$$f(x) = x^3 - x^2 + x - 7 = 0$$

Step # 1:

Assume Limit

$$f(1) = (1)^3 - (1)^2 + (1) - 7 = 0$$

$$= 1 - 1 + 1 = 7$$

$$\boxed{= -6}$$

$$f(2) = (2)^3 - (2)^2 + (2) - 7$$

$$= 8 - 4 + 2 - 7$$

$$\boxed{= -1}$$

$$f(3) = (3)^3 - (3)^2 + 3 - 7$$

$$= 27 - 9 + 3 - 7$$

$$\boxed{= 14}$$

$$[2,3] = F(2) \times f(3)$$

$= (-1)(14)$

$= -14 < 0$

$c = \frac{2+3}{2}$

$= \frac{5}{2}$

$= 2.5$

Step # 2:

Mid Point:

$c = \frac{2+3}{2}$

$= \frac{5}{2}$

$= 2.5$

$f(2.5) = (2.5)^3 - (2.5)^2 + 2.5 - 7$

$= 4.875$

$f(2) \times f(2.5) = (-1) \times (4.875)$

$= -4.875 < 0$

STEP # 3:-

MID POINT:-

$$C = \frac{2 + 2.5}{2}$$

$$= \frac{4.5}{2}$$

$$= 2.25$$

$$F(2.25) = (2.25)^3 - (2.25)^2 + (2.25) - 7$$

$$= 1.5781$$

$$f(2) \times f(2.25) = (-1 \times 1.5781)$$

$$= (-1.5781) < 0$$

STEP # 4

MID POINT:-

$$C = \frac{2.25}{2}$$

$$= 2.125$$

$$f(2.125) = (2.125)^3 - (2.125)^2 + (2.0625) - 7$$

$$= -0.4177$$

~~f(2) x f(0~~

$$f(2) \times f(2.0625) = (1) \times (0.4177)$$

$$= +0.4177 > 0$$

Root of the equation lies in limit ~~t2~~ (2, 2.125) ie

0.20507

Question No :- 2

PART A:-

Use Regula-falsi method to compute the root of the following equation in the interval (0, 1) after third iteration.

$$f(x) = \cos x - xe^x$$

SOLUTION:-

$$f(x) = \cos x - xe^x$$

$[0, 1] =$

$f(0) = \cos(0) - (0)e^0$

$= 1 - 0$

$= 1$

$f(1) = \cos(1) - (1)e^1$

$= 0.999 - 2.7182$

$= -1.7192$

Here $a = 0$ $b = 1$

$f(a) = 1$ $f(b) = -1.7192$

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FORMULA:-

$$\frac{a f(a) - b f(b)}{f(b) - f(a)}$$

$$= \frac{0(1) - (1)(-1.71992)}{-1.7192 - 1}$$

$$= + \frac{1.7192}{-2.7192}$$

$$\boxed{= -0.6322}$$

$$f(-0.6322) = \cos(-0.6322) - e^{-(0.6322)}$$

$$= 0.9999 - (-0.6322)(0.5314)$$

$$= 0.9999 + 0.3359$$

$$\boxed{= 1.3358}$$

STEP 2:-

$a = -0.6322$ $\quad b = 1$

$f(a) = 1.3358$ $\quad f(b) = -1.7192$

FORMULA:-

$$= \frac{a f(a) - b f(b)}{f(b) - f(a)}$$

$$= \frac{-0.6322(1.3358) - 1(-1.7192)}{-1.7192 - 1.3358}$$

$$= \frac{-0.8444 + 1.7192}{-3.055}$$

$$= \frac{2.5636}{-3.055}$$

$$\boxed{= -0.839}$$

$f(-0.8391) = \cos(-0.8391) - (-0.8391)e^{-0.8391}$

$= 0.9998 - [-0.8391)(0.4320)$

$= 0.9998 + 0.3625$

$\boxed{= 1.3623}$

STEP:-

$a = -0.83912$ $b = 1$

$f(a) = 1.3623$ $f(b) = -1.7192$

$$= -\frac{0.8391(1.3623) - (1)(-1.7192)}{-1.7192 - 1.3623}$$

$$= \frac{1.1431 + 1.7192}{3.0815}$$

$$= \frac{2.862}{3.0815}$$

$\boxed{= 0.9287}$ Ans:

Question No:- 2

Part (B)

Use Regula-Falsi (method of false position) to solve the following equation, accuracy must be up to four decimal places

$$x^3 - 4x - 9 = 0$$

SOLUTION:-

$$f(x) = x^3 - 4x - 9$$

$$f(0) = (0)^3 - 4(0) - 9$$

$$= -9$$

$$f(1) = (1)^3 - 4(1) - 9$$

$$= 1 - 4 - 9$$

$$= -12$$

$$f(2) = (2)^3 - 4(2) - 9$$

$$= 8 - 8 - 9$$

$$= -9 \text{ (negative)}$$

$$f(3) = (3)^3 - 4(3) - 9$$

$$= 27 - 12 - 9$$

$= 6$ (Positive)

Root lies b/w $[2, 3]$

First Approx:-

$a = 2$, $b = 3$

Using formula:-

$$x_1 = \frac{a f(a) - b f(b)}{f(b) - f(a)}$$

$$= \frac{2 f(2) - 3 f(3)}{f(3) - f(2)}$$

$$= \frac{2(-9) - 3(6)}{6 - (-9)}$$

$$= \frac{-36}{-15}$$

$$= -2.4$$

$$f(-2.4) = (-2.4)^3 - 4(-2.4) - 9$$

$$= -5.76 - (-9.6) - 9$$

$$= -5.16$$

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Root lies b/w $f(-2.4)$ and $f(3)$

$a = (-2.4)$ $b = 3$

$f(-2.4) = -5.16$ $f(3) = 6$

$$\frac{a f(a) - b f(b)}{f(b) - f(a)}$$

$$= \frac{(-2.4)(-5.16) - (3)(6)}{6 - (-2.4)}$$

$$= \frac{12.384 - 18}{6 + 2.4}$$

$$= -\frac{5.016}{8.4}$$

$$= -0.5971$$

$f(-0.5971) = (-0.5971)^3 - 4(-0.5971) - 9$

$= -0.2128 - (-2.284) - 9$

$= -6.9288$

Roots lies b/w $(-0.5971, 3)$

$a = -0.5971$ $b = 3$

$f(a) = -6.9288$ $f(b) = 6$

$$= \frac{0.5971(-6.9288) - 3(6)}{6 - (-6.9288)}$$

$$= \frac{4.1371 - 18}{12.1288}$$

$$= 2.7448$$ Ans:-

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Q.NO:- Question No:- 3

PART (A)

Find the real root of the following equation using Newton-Raphson method in the interval [2, 3) after third iteration

$$x^3 - 3x - 5 = 0$$

Solution:-

$$f(x) = x^3 - 3x - 5$$

$$f'(x) = 3x^2 - 3$$

Since Roots Lies between [2, 3]

INITIAL POINT:-

$$x_0 = \frac{2+3}{2}$$

$$= \frac{5}{2}$$

$$\boxed{= 2.5}$$

NRM FORMULA:-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(x_n^3 - 3x_n - 5)}{3x_n^2 - 3}$$

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$$x_{n+1} = x_n \frac{(3x_n^2 - 3) - (x_n^3 + 3x_n - 5)}{3x_n^2 - 3}$$

$$x_{n+1} = \frac{3x_n^3 - 3x_n - x_n^3 - 3x_n - 5}{3x_n^2 - 3}$$

$$x_{n+1} = \frac{2x_n^3 - 6x_n - 5}{3x_n^2 - 3}$$

Iteration 1:-

$x_0 = 2.5$

$$x_0 + 1 = \frac{2(2.5)^3 - 6(2.5) - 5}{3(2.5)^2 - 3}$$

$$= \frac{31.25 - 15 - 5}{18.75 - 3}$$

$$= \frac{11.25}{15.75}$$

$$= 0.7142$$

Iteration 2:-

$$x_{n+1} = \frac{2(x_1)^3 - 6x_1 - 5}{3x_1^2 - 3}$$

$$x_2 = \frac{2(0.7142)^2 - 6(0.7142) - 5}{3(0.7142)^2 - 3}$$

$$x_2 = \frac{0.7286 - 4.2852 - 5}{1.5302 - 3}$$

$$x_2 = \frac{-8.5566}{-1.4697}$$

$$= 5.8220$$

Itteration 3:-

$$x_{2+1} = \frac{2(x_2)^3 - 6(x_2) - 5}{3x_2^2 - 3}$$

$$= \frac{2(5.8220)^3 - 6(5.8220) - 5}{3(5.8220)^2 - 3}$$

$$= \frac{394.68 - 34.93 - 5}{101.68 - 3}$$

$$= \frac{354.67}{98.687}$$

$$= 3.594 \quad \text{Ans.}$$

Question No:- 3
Part (B)
Solve the following eq by using Millers method only perform 2 iterations
($x_0 = 0.5$, $x_1 = 1$ $x_2 = 0$)

$$x^3 - 7x^2 + 14x - 6.$$

Solution::

$f(x) = x^3 - 7x^2 + 14x - 6$

$x_0 = 0.5 \quad x_1 = 1 \quad , x_2 = 0$

$f(x_0) = (0.5)^3 - 7(0.5)^2 + 14(0.5) - 6$

$f(0.5) = 0.125 - 1.75 + 7 - 6$

$f(0.5) = -0.625$

$f(x_1) = x_1^3 - 7x_1^2 + 14x_1 - 6$

$= (1)^3 - 7(1)^2 + 14(1) - 6$

$= 1 - 7 + 14 - 6$

$\boxed{f(1) = 2}$

$f(x_2) = x_2^2 - 7x_2^2 + 14x_2 - 6$

$= (0)^3 - 7(0)^2 + 14(0) - 6$

$\boxed{f(0) = -6}$

$h_1 = x_1 - x_0 = 1 - 0.5 = 0.5$

$h_2 = x_2 - x_1 = 0 - 1 = -1$

$\delta_1 = \dfrac{f(x_1) - f(x_0)}{h_1}$

$= \dfrac{2 - (-0.625)}{0.5}$

$= \frac{2.625}{0.5}$

$= 5.25$

$\delta_2 = \frac{f(x_2) - f(x_1)}{h_2}$

$= \frac{-6 - (-0.625)}{-1}$

$= 5.375$

$a = \frac{\delta_2 - \delta_1}{h_2 + h_1} = \frac{5.375 - 5.25}{-1 + 0.5}$

$= \frac{0.125}{-1.5}$

$= -0.0833$

$b = a \times h_2 + \delta_2 = -0.0833(-1) + 5.375$

$= 5.4583$

$C = f(x_2) = -6$

$x_3 = x_2 + \frac{-2C}{b \pm \sqrt{b^2 - 4ac}}$

$= \frac{0}{1} + \frac{-2(-6)}{5.4583 \pm \sqrt{(5.4583)^2 - 4(-0.083)(-6)}}$

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$$\frac{12}{5.468 \pm \sqrt{29.79 - (1.994)}}$$

$$= \frac{12}{5.4583 \pm 5.2717}$$

$$= \frac{12}{0.1866}$$

$$= 64.30$$

OR

$$\frac{12}{10.73}$$

$$= 1.11 \quad \text{Ans.}$$

RELATIVE ERROR:-

$$E_a = \left| \frac{x_3 - x_2}{x_3} \right| \times 100\% = \left| \frac{1.11 - 0}{1.11} \right| \times 100\%$$

$$= 100\% \text{ error}$$

Now

$x_0 = x_1 = 1$

$x_1 = x_2 - 0$

$x_2 = x_3 = 1.11$

2nd Iteration:-

$f(x_0) = (1)^3 - 7(1)^2 + 14(1) - 6$

$= 1 - 7 + 14 - 6 = 2$

$f(x_1) = (0)^3 - 7(0)^2 + 14(0) - 6 = -6$

$f(x_2) = (1.11)^3 - 7(1.11)^2 + 14(1.11) - 6$

$= 1.377 - 8.624 + 15.54 - 6$

$= 2.283$

$h_1 = x_1 - x_0 = -6 - 2$

$= -8$

$h_2 = x_2 - x_1 = 2.283 - 0$

$= 2.283$

$\delta_1 = \frac{f(x_1) - f(x_0)}{h_1} = \frac{-6-2}{-8} = 1$

$\delta_2 = \frac{f(x_2) - f(x_1)}{h_2} = \frac{2.283 - (-6)}{2.283}$

$= \frac{8.283}{2.283} = 3.628$

$a = \frac{\delta_2 - \delta_1}{h_2 + h_1} = \frac{3.628 - 1}{(2.283) + (-8)} = \frac{2.628}{-5.716}$

$= -0.459$

$b = a \times h_2 + \delta_2 = -(0.459)(2.283) + 3.628$

$= 2.580$

$C = f(x_2) = 2.283$

$$x_3 = x_2 + \frac{-2C}{b \pm \sqrt{b^2 - 4ac}}$$

$$= 0 + \frac{-2(2.283)}{2.580 \pm \sqrt{(2.580)^2 - 4(-0.459)(2.283)}}$$

$$= \frac{4.566}{2.580 \pm \sqrt{6.65 + 4.1925}}$$

$$= \frac{4.566}{2.580 \pm \sqrt{10.84}}$$

$$= \frac{4.566}{2.580 \pm 3.290}$$

$$= \frac{4.566}{5.873} = 0.7744$$

$$\varepsilon_a = \left| \frac{x_3 - x_2}{x_3} \right| \times 100\%$$

$$= \left| \frac{0.7744 - 0}{0.7744} \right| \times 100\% = 100\%$$

Now

$x_0 = x_1 = 0$

$x_1 = x_2 = 1.11$

$x_2 = x_3 = 0.7744$

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3rd Iteration:

$f(x_0) = (0)^3 - 7(0)^2 + 14(0) - 6 = \boxed{-6}$

$f(x_1) = (1.11)^3 - 7(1.11)^2 + 14(1.11) - 6$

$\boxed{= 2.283}$

$f(x_2) = (0.7744)^3 - 7(0.774)^2 + 14(0.7744) - 6$

$= 0.4118 - 3.874 + 10.416 - 6$

$\boxed{= 0.953}$

$h_1 = x_1 - x_0 = 1.11 - 0 = \boxed{1.11}$

$h_2 = x_2 - x_1 = 0.7744 - 1.11 = \boxed{-0.335}$

$\delta_1 = \frac{f(x_1) - f(x_0)}{h_1} = \frac{2.283 - (-6)}{1.11}$

$= \frac{8.28}{1.11}$ $\boxed{= 7.462}$

$\delta_2 = \frac{f(x_2) - f(x_1)}{h_2} = \frac{0.953 - 2.283}{0.335}$

$= 0$ $\boxed{= 3.970}$

$a = \frac{\delta_2 - \delta_1}{h_2 + h_1} = \frac{3.970 - 7.462}{-0.335 + 1.11}$

$\boxed{= -4.505}$

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$b = a \times h_2 + \delta_2 = (-4.505) \times (0.335) + 3.970$

$= 5.479$

$C = f(x_2) = 0.953$

$x_3 = x_2 + \frac{-2C}{b \pm \sqrt{b^2 - 4ac}}$

$= 1.11 + \frac{-2(0.953)}{5.479 \pm \sqrt{(5.479) - 4(-4.505)(0.953)}}$

$= +1.11 + \frac{(-1.906)}{5.479 \pm 3.584}$

$= 1.11 + \frac{(-1.906)}{9.333}$

$= \frac{9.333(1.11) - (1.906)}{9.333}$

$= \frac{10.3596 - 1.906}{9.333}$

$x = 8.45363$

$Ea = \left| \frac{x_3 - x_2}{x_3} \right| \times 100\%$

$$= \left| \frac{8.4536 - 1.11}{8.4536} \right| \times 100\%$$

$$= 0.868 \times 100\%$$

$$= 86.8\%$$

Question No 4:-

Using Gaussians Elimination method solve the following system of equations

$$2x - y + 2z = 2$$
$$x + 10y - 3z = 5$$
$$x - y - z = 3$$

SOLUTION:-

In Matrix form:-

$$\begin{bmatrix} 2 & -1 & 2 \\ 1 & 10 & -3 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$$

$R_2 \rightarrow 2R_2 - 1R_1$

$R_3 \rightarrow 2R_3 - R_1$

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In Augmented form:-

$$\begin{bmatrix} 2 & -1 & 2 & : & 2 \\ 1 & 10 & -3 & : & 5 \\ 1 & -1 & -1 & : & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 2 & : & 2 \\ 0 & 19 & 4 & : & 8 \\ 0 & -1 & -4 & : & 4 \end{bmatrix}$$

Applying Operations:-

$R_3 \rightarrow 19R_3 + R_2$

$$\begin{bmatrix} 2 & -1 & 2 & : & 2 \\ 0 & 19 & 4 & : & 8 \\ 0 & 0 & -70 & : & 84 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 2 \\ 0 & 19 & 4 \\ 0 & 0 & -70 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 2 \\ 8 \\ 84 \end{bmatrix}$$

Writing in equation form:-

$2x - y + 2z = 2 \Rightarrow (i)$

$19y + 4z = 8 \Rightarrow (ii)$

$= 70z = 84$

$z = \frac{84}{-70} = \boxed{-1.2}$

Putting in eq (ii)

$19y + 4(-1.2) = 8$

$19y + (-4.8) = 8$

$19y = 8 + 4.8 = \boxed{12.8}$

$y = \frac{12.8}{19} = \boxed{0.673}$

Putting values of y, z in eq (i)

$2x - (0.673) + 2(-1.2) = 2$

$2x - 0.673 + (-2.4) = 2$

$2x - 3.073 = 2$

$2x = 5.073$

$x = \frac{5.073}{2}$

$\boxed{x = 2.5365}$

$\boxed{x = 2.536 \quad y = 0.673 \quad z = -1.2}$