MID-TERM EXAM

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SUBJECT	DIFFERENTIAL EQUATIONS
SEMESTER	4^{th}

QUESTION # 01

(i)

The order of matrix A is $m \times p$ and order of B is $p \times n$. Then order of matrix AB is ?

Sol :-

$[A]_{m \times p}$ m Rows, P Columns

$[B]_{p \times n}$ P Rows, n Columns

Now since the number of Rows in B is equal to number of columns so in A which is equal to p

Hence

$$[A]_{mp}\,[B]_{p \times n} = [AB]_{m \times n}$$

The order of Matrix

$\boxed{AB = m \times n}$ Ans

(ii)

The no of non-zero rows in an Echelon form ?

ANSWER :-

No. of non-zero rows are known as $\boxed{\text{RANK OF MATRIX}}$.

(iii)

If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is singular Matrix then $a = ?$

Sol

$$B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$$

$|B| = 0$ is called as Singular Matrix

$$|B| = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix} = 0$$

$$= 1 \times a - (4 \times 2) = 0$$

$$a - 8 = 0$$

$$\boxed{a = 8}$$ Ans

(iv)

If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Sol

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$= 2i \times (-i) - (i \times i)$$

$$= -2i^2 - i^2 \qquad \therefore i^2 = -1$$

$$= -2(-1) - (-1)$$

$$= 2 + 1$$

$$\boxed{|A| = 3}$$ Ans

(V)

The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Ans

The scalar matrix is basically a square matrix, whose all off diagonal elements are zero and all on-diagonal are equal.

Hence $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is a $\boxed{\text{SCALAR MATRIX}}$.

(Vi)

Solution of $\frac{dy}{dx} + 2xy = y$

Sol

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1-2x)$$

$$\frac{1}{y}\,dy = (1-2x)\,dx$$

Integration

$$\int \frac{1}{y}\,dy = \int (1-2x)\,dx$$

$$\ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + C$$

$$\ln y = x - x^2 + C$$

$$e^{\ln y} = e^{x - x^2 + C}$$

$$\boxed{y = e^{x - x^2 + C}}$$ Ans

(vii)

The order and degree of differential equation?

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \text{ is ?}$$

Ans

Order of DE = 1

Degree of DE = 6

(viii)

The order and degree of $\frac{d^2y}{dx^2} - 4xy = \sin\frac{d^2y}{dx^2}$?

Ans

Order of DE = 2

Degree of DE = undefined

(ix)

The differential equation $2\frac{dy}{dx} + x^2y = 2x+3,\ y(0)=5$ is ?

Sol

$$2\frac{dy}{dx} + x^2y = 2x+3$$

$$\Rightarrow 2\frac{dy}{dx} = 2x+3-x^2y$$

$$\Rightarrow 2dy = (2x+3-x^2y)\,dx$$

Taking integration of b/s

$$\Rightarrow \int 2dy = \int (2x+3-x^2y)\,dx$$

PTO

$$\Rightarrow 2y = \frac{2x^2}{2} + 3x - \frac{x^3 y}{3} + C$$

$$\Rightarrow 2y = x^2 + 3x - \frac{x^3 y}{3} + C$$

$$\Rightarrow 2y + \frac{x^3 y}{3} = x^2 + 3x + C$$

$$\Rightarrow y\left(2 + \frac{x^3}{3}\right) = x^2 + 3x + C$$

$$\Rightarrow y\left(\frac{6 + x^3}{3}\right) = x^2 + 3x + C$$

$$\Rightarrow y = (x^2 + 3x + C) \times \frac{3}{6 + x^3} \quad \text{—(i)}$$

Put $x = 0$ and $y = 5$

$$5 = ((0)^2 - 3(0) + C) \times \frac{\cancel{3}^{1}}{\cancel{6}^{2} + (0)^3}$$

$$5 = C \times \frac{1}{2}$$

$$C = 10$$

Hence

(it is Homogenous equation)

we will put the value of c in (i)

$$y = (x^2 + 3x + 10) \times \frac{3}{6 \times x^3}$$

$$y = \frac{3x^2 + 9x + 30}{6 + x^3}$$

Hence

$$\boxed{y = \frac{3x^2 + 9x + 30}{6 + x^3}}$$

(x)

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \text{ is ?}$$

Sol

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1-1 & a-b & a^2-b^2 \\ 1-1 & a-c & a^2-c^2 \end{vmatrix} \begin{matrix} \\ R_1 - R_2 \\ R_1 - R_3 \end{matrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & a-b & a^2-b^2 \\ 0 & a-c & a^2-c^2 \end{vmatrix}$$

Expand by Column 1

$$= 1 \begin{vmatrix} a-b & a^2-b^2 \\ a-c & a^2-c^2 \end{vmatrix} \Rightarrow 1 \begin{vmatrix} a-b & (a-b)(a+b) \\ a-c & (a-c)(a+c) \end{vmatrix}$$

Taking common $(a-b)$ from R_1 and $a-c$ from R_2

$$= (a-b)(a-c) \begin{vmatrix} 1 & (a+b) \\ 1 & (a+c) \end{vmatrix}$$

$$= (a-b)(a-c) [(a+c)-(a+b)]$$

$$= (a-b)(a-c) [\cancel{a}+c-\cancel{a}-b]$$

$$\boxed{= (a-b)(a-c)(c-b)} \quad \text{Ans}$$

QUESTION # 02

(i)

Express the determinant $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$ as the product of factors which are linear in a, b, c?

Sol

$$B = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$$|B| = a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$|B| = a(b^2c^3 - c^2b^3) - b(a^2c^3 - c^2a^3) + C(a^2b^3 - b^2a^3)$$

$$|B| = ab^2c^3 - ac^2b^3 - a^2bc^3 + a^3bc^2 + a^2b^3c - a^3b^2c$$

Taking abc common

$$|B| = abc(bc^2 - cb^2 - ac^2 + a^2c + ab^2 - a^2b)$$

Hence

$$\boxed{|B| = abc(bc^2 - cb^2 - ac^2 + a^2c + ab^2 - a^2b)}$$

ANS

Q-2 (ii)

Find the Eigen Value $\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$

Sol

$$A = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

$$[A - \lambda I] = 0$$

$$= \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix}$$

Expand by R_1

$$2-\lambda \begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix} - (-1) \begin{bmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix} + (-1) \begin{bmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix} + 0 = 0$$

$$\rightarrow \begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix}$$ Expand by R_1

$$3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + (-1) \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix} = 0$$

$$3-\lambda\left[(3-\lambda)(2-\lambda)-(-1)(-1)\right]+1\left[(-1)(2-\lambda)-(-1)(-1)\right]$$
$$-1\left[(-1)(-1)-(3-\lambda)(-1)\right]=0$$

$$(3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)-1(1+3-\lambda)=0$$

$$(3-\lambda)(5-5\lambda+\lambda^2)+(-3+\lambda)-(4-\lambda)=0$$

$$15-15\lambda+3\lambda^2-5\lambda+5\lambda^2-\lambda^3-3+\lambda-4+\lambda=0$$

$$\boxed{-\lambda^3+8\lambda^2-18\lambda+8=0 \quad -①}$$

$$\rightarrow +1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$-1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix}+1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix}$$

$$-1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$$-6+3\lambda+2\lambda-\lambda^2+1-2+\lambda-1$$

$$\boxed{-\lambda^2+6\lambda-8=0} \quad -②$$

$$\rightarrow -1\begin{bmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$$

Expand by C_1

$$-1\left[1(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right]=0$$

$$-1\left[2-\lambda+1+6-3\lambda-2\lambda+\lambda^2-1\right)=0$$

$$-2+\lambda-1-6+3\lambda+2\lambda-\lambda^2+1=0$$

$$\boxed{-\lambda^2+6\lambda-8=0} \quad -③$$

Putting equation ①, ② and ③ in A

$\Rightarrow (2-\lambda)(-\lambda^3+8\lambda^2-18\lambda+8) - \lambda^2+6\lambda-8 - \lambda^2+6\lambda-8 = 0$

$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2+6\lambda$
$-8-\lambda^2+6\lambda-8=0$

$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+18\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda$
$+6\lambda+6\lambda+16-8-8=0$

$\Rightarrow \quad \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$

By synthetic division

	1	-10	32	-32
2		2	-16	32
	1	-8	+16	0

$(\lambda-2)(\lambda^3-8\lambda^2+16\lambda)$

$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$

$\boxed{\lambda=0}$

$\lambda-2=0$

$\boxed{\lambda=2}$

$\lambda^2-8\lambda+16=0$

By factorization

$\lambda^2-4\lambda-4\lambda+16=0$

$\lambda(\lambda-4)-4(\lambda-4)=0$

$(\lambda-4)(\lambda-4)=0$

$\boxed{\lambda=4}, \boxed{\lambda=4}$

HENCE

$\boxed{\lambda=0}$ $\boxed{\lambda=2}$ $\boxed{\lambda=4}$ $\boxed{\lambda=4}$ ANS

QUESTION # 03

The rate of change in the form of differential equation is given by
$(x^2+3y^2)dx - 2xy\,dy = 0$.

Find the general solution at $x=2$ & $y=6$

Sol

$$(x^2+3y^2)\,dx - 2xy\,dx = 0$$

$$(x^2+3x^2)\,dx = 2xy\,dy$$

$$\frac{dy}{dx} = \frac{x^2+3x^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{x}{2y} + \frac{3y}{2x}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{x}{y} + \frac{3y}{x}\right] \longrightarrow (1)$$

Let us concider

$$y = VX$$

Diff: $dy = V\,dx + x\,dV$

Dividing by dx

$$\frac{dy}{dx} = V + x\frac{dV}{dx} \text{ —— } (2)$$

putting eq (2) in eq (1)

$$V + x \cdot \frac{dv}{dx} = \frac{1}{2}\left[\frac{x}{xv} + 3\frac{vx}{x}\right]$$

$$V + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

xing b.s by "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2x$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

Multiplying b.s by $\frac{dx}{dv}$ we get

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

Multiplying b.s by $\frac{v}{x(1+v^2)}$

we get

$$\frac{v}{1-v^2}\,dv = \frac{1}{x}\,dx$$

Taking Integration

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c.$$

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$$\ln|1+v^2| = \ln xC \quad \because \ln x + \ln y = \ln xy$$

$$\boxed{1+v^2 = xC} \longrightarrow (3)$$

Now reput $v = y/x$

$$1+\left(\frac{y}{x}\right)^2 = xC$$

$$1+\frac{y^2}{x^2} = xC$$

$$\frac{x^2+y^2}{x^2} = xC$$

$$\boxed{x^2+y^2 = x^3C} \longrightarrow (4)$$

Initial values $x=2$, $y=6$

$$2^2+6^2 = 2^3C$$

so $4+36 = 8C$

$$C = 5$$

Putting eq 4

$$x^2+y^2 = x^3 \times 5$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

Taking under root

$$\sqrt{y^2} = \sqrt{x^2(5x-1)}$$

$$\boxed{y = \pm y\sqrt{5x-1}}$$ Ans