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Deptt. BE (Electrical)

Semester 2nd

Subject Calculus

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Course Code:	MTH 102			Course Title:	Calculus and analytic geometry
Prerequisite:				Instructor:	HIMAYATULLAH
Module:	3	Program:	BEE	Total Marks:	50 :

Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1.	a	. **Estimate** $\int \theta \sqrt[4]{1-\theta^2}\, d\theta$	Marks 7 PLO2 C2
	b	**Estimate** $\int_c x^3(1+x^4)^3 dx$ using substitution method.	Marks 7 PLO2 C2
Q2	(a)	**Illustrate** the centre and radius of the sphere $x^2 + y^2 + z^2 + 3x - 4z + 1$.	Marks 5 PLO1 C3
	(b)	The region between the curve $y = \sqrt{x}$, $0 \le x \le 4$, and the x-axis is revolved about the x-axis to generate a solid. **Apply** the integration find the volume of solid.	Marks 4 PLO1 C3
Q3		If $A = 2i - 4j + \sqrt{5}k$, and $B = -2i + 4j - \sqrt{5}k$ then **Illustrate** the vector $proj_A B$	Marks 9 PLO1 C3
Q4		Find the area of the region between the graph and the x-axis Where $y = -x^2 + 5x - 4$, $[0,\ 2]$.	Marks 9 PLO1 C3
Q5	(a)	**Estimate** the angle between $A = i - 2j - 2k$ and $B = 6i + 3j + 2k$	Marks 5

			PLO1 C3
	(b)	**Change** into a spherical coordinate equation for the $x^2 + y^2 + (z-1)^2 = 1$	Marks 4 PLO1 C3
			PLO2 C2

Q (1)

Part (a)

Given that

$$\int \theta \cdot \sqrt[4]{1-\theta^2}\, d\theta$$

Solution:-

$$\int \theta \sqrt[4]{1-\theta^2}\, d\theta$$

$1-\theta^2$

$$\frac{d}{d\theta}(1-\theta^2) = \frac{d}{d\theta} u$$

$$-2\theta = \frac{du}{d\theta}$$

$$\theta d\theta = -\frac{1}{2} du$$

Now

$$\int (u)^{1/4} \cdot \left(-\frac{1}{2}\right) du$$

$$-\frac{1}{2} \int (u)^{1/4} du$$

$\because \frac{1}{4}+1 = \frac{5}{4}$

$$= -\frac{1}{2} \cdot \frac{4}{5} u^{5/4} + C$$

$$= -\frac{2}{5} u^{5/4} + C$$

now back substitution

$$-2/5 (1-\theta^2)^{5/4} + C$$

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Question 1 =>

part b =>

Given that:-

$$\int_0^1 x^3(\cancel{x}1+x^4)^3 \, dx$$

Solution:-

$$\int_0^1 x^3(1+x^4)^3 \, dx \sim (1)$$

let $t = 1 + x^4$

$$\frac{dt}{dx} = 4x^3$$

$$\frac{dt}{\cancel{dx}} = dx(4x^3)$$

$$\frac{dt}{4} = dx(x^3) \sim (2)$$

now put in above eq (1)

$$= \frac{1}{4}\int_0^1 t^3 \, dt$$

$$= \frac{1}{4}\left(\frac{t^4}{4}\right)\Big|_0^1$$ now solve the limit

$$= \frac{1}{16}(1^4 - 0^2)$$

$$= \frac{1}{16}(1)$$

$$= \boxed{\frac{1}{16}}$$ Ans

Question 2 :-
part a :-

Given that:

$$x^2+y^2+z^2+3x-4z+1=0$$

Solution:

$$x^2+y^2+z^2+3x-4z+1=0$$

$$(x^2+3x)+y^2+z^2-4z+1=0$$

$$\left(x^2+3x+\left(\frac{3}{2}\right)^2\right)+(y-0)^2+\left(z^2-4z\left(\frac{-b}{2}\right)^2\right)$$

$$=1+\left(\frac{3}{2}\right)^2+\left(\frac{-b}{2}\right)^2$$

$$\left(x+\frac{3}{2}\right)^2+\left(y-0\right)^2+\left(z-2\right)^2=\frac{21}{4}$$

now,

$$\left(x_0, y_0, z_0\right)=\text{center}$$

$$\therefore \; =\left(-\frac{3}{2}, 0, 2\right)$$

&

Radius $\boxed{a=\sqrt{\frac{21}{4}}}$

Question 2 :-

part b :-

Given that

$y = \sqrt{x}$, $0 \leq x \leq 4$

Solution:- $y = \sqrt{x}$

$0 \leq x \leq 4 \Rightarrow a \leq x < b.$

as $v = \int_a^b \pi y^2 \cdot dx$

$v = \int_0^4 \pi (\sqrt{x})^2 dx$

$v = \pi \int_0^4 x\, dx = \pi \frac{x^2}{2} \Big|_0^4$

$v = \frac{\pi}{2} \left((4)^2 - 0 \right)$

$\boxed{v = 8\pi}$

Question (3)

Solution :-

Given that

$A = 2i - 4j + \sqrt{5}k$

$B = -2i + 4j - \sqrt{5}k$

Projection AB = ?

By dot product

$B \cdot A = (-2i + 4j - \sqrt{5}k)(2i - 4j + \sqrt{5}k)$

$B \cdot A = ((-2)(2) + (4)(-4) + (-\sqrt{5})(\sqrt{5}))$

$B \cdot A = -4 - 16 + \sqrt{5 \times 5}$

$B \cdot A = -4 - 16 - \sqrt{25}$

$B \cdot A = -20 - 5$

$\boxed{B \cdot A = -25}$

$A \cdot A = (2i - 4j + \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$

$A \cdot A = ((2)(2) + (-4)(-4) + (\sqrt{5})(\sqrt{5}))$

$A \cdot A = 4 + 16 + \sqrt{5 \times 5}$

$A \cdot A = 20 + \sqrt{25}$

Question (4)

Given that:-

$y = -x^2 + 5x - 4 \qquad [0, 2]$

Solution:-

$y = -x^2 + 5x - 4$

$[a, b] = [0, 2]$

As

$a = 0$

$b = 2$

So

Area under graph will be

$A = \int_a^b f(x)\, dx$

putting values

$$A = \int_0^2 (-x^2+5x-4)\, dx$$

by solving integration we will get.

$$A = \left(\frac{-x^3}{3} + \frac{5x^2}{2} - 4x\right)\Big|_0^2$$

$$A = \left(-\frac{1}{3}(2)^3 + \frac{5}{2}(2)^2 - 4(2)\right) - (0)$$

$$A = \left(-\frac{1}{3}(8) + \frac{5}{2}(4) - 8\right)$$

$$A = \frac{-8}{3} + \frac{20}{2} - 8$$

$$A = \frac{-8}{3} + \frac{20}{3} - \frac{8}{1}$$

$$A = \frac{2\times-8 + 3\times20 - 6\times8}{6}$$

$$A = \frac{60-64}{6}$$

$$A = \frac{4}{6}$$

$$A = \frac{2}{3}$$

$$\boxed{A = 0.666}$$ Ans

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Question (5)
Part (a)

Given that:

$A = i - 2j - 2k$ and $B = 6i + 3j + 2k$

Solution:-

$$A = i - 2j - 2k$$

$$|A| = \sqrt{(1)^2 + (-2)^2 + (-2)^2}$$

$$|A| = \sqrt{1 + 4 + 4}$$

$$|A| = \sqrt{9}$$

$$|A| = 3$$

Now,

$$B = 6i + 3j + 2k$$

$$|B| = \sqrt{6^2 + 3^2 + 2^2}$$

$$|B| = \sqrt{36 + 9 + 4}$$

$$|B| = \sqrt{49}$$

$$|B| = 7$$

So, $\theta = \cos^{-1}\left(\frac{A \cdot B}{|A|\,|B|}\right)$

$$\theta = \cos^{-1}\left\{\frac{(i - 2j - 2k)\ (6i + 3j + 2k)}{3 \times 7}\right\}$$

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$$\theta = \cos^{-1}\left\{\frac{(1)(6)+(-2)(3)+(-2)(2)}{21}\right\}$$

$$\theta = \cos^{-1}\left(\frac{\cancel{6}-\cancel{6}-4}{21}\right)$$

$$\theta = \cos^{-1}\left(\frac{-4}{21}\right)$$

$$\theta = 100.97$$

Q5 :-
part (b) :-

Given that :-

$$x^2 + y^2 (z-1)^2 = 1$$

Solution :-

$$x^2 + y^2 (z-1)^2 = 1$$

$$(\rho \sin\phi \cos\theta)^2 + (\rho \sin\phi \sin\theta)^2 + (\rho \cos\theta - 1)^2 = 1$$

$$\rho^2 \sin^2\phi \cos^2\theta + \rho^2 \sin^2\phi \sin^2 \rho^2 \cos^2\theta + 1 - 2 \cos\theta = 1$$

$$\rho^2 (\sin^2 \theta) + \rho^2 (\cos^2\theta + \sin^2\theta) + \rho^2 \cos^2\theta + 1 - 2\rho \cos\theta = 1$$

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$$\rho^2(\sin^2\theta) + \rho^2\cos^2\theta - 2\rho\cos\theta = 1-1$$

$$\rho^2(\sin^2\theta + \cos^2\theta) - 2\rho\cos\theta = 0$$

$$\rho^2 = 2\rho\cos\theta$$

$$\rho = 2\cos\phi$$

$$\boxed{\rho = 2\cos\phi}$$ Ans