DISCRETE STRUCTURES FINAL PAPER

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QUESTION1 a)

BICONDITIONAL STATEMENT: Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is "p if and only if q". The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth value, and is false otherwise. The double headed arrow "↔" is called the biconditional operator. The words "if and only if" are sometimes abbreviated "iff".

There are some other common ways to express $p \leftrightarrow q$:

"p is necessary and sufficient for q "

"if p then q , and conversely"

Truth table for biconditional statements:

Truth table for Biconditional.

$p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

It is to be noted that $p \leftrightarrow q$ has exactly the same truth value as $(p \rightarrow q) \wedge (q \rightarrow p)$.

Truth table for $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

p	q	$p \leftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

QUESTION1 b) Let p, q, and r represent the following statements:

p: Sam had pizza last night.
q: Chris finished her homework.
r: Pat watched the news this morning

Give a formula (using appropriate symbols) for each of these statements.

i. Sam had pizza last night if and only if Chris finished her homework.

ii. Pat watched the news this morning iff Sam did not have pizza last night.

iii. Pat watched the news this morning if and only if Chris finished her homework and Sam did not have pizza last night.

iv. In order for Pat to watch the news this morning, it is necessary and sufficient that Sam had pizza last night and Chris finished her homework

ANSWER:

1) $p \leftrightarrow q$

2) $r \leftrightarrow \sim p$

3) $r \leftrightarrow (q \wedge \sim p)$

4) $r \leftrightarrow (p \wedge q)$

Q.2

a) Lets p, q, r represent the following statements:

p: it is hot today.

q: it is sunny

r: it is raining

Express in words the statements using Bicondtional statement represented by the following formulas:

i. $q \leftrightarrow p$

ii. $p \leftrightarrow (q \wedge r)$

iii. $p \leftrightarrow (q \vee r)$

iv. $r \leftrightarrow (p \vee q)$

ANSWER:

1) It is sunny if and only if it is hot today

2) it is hot today if and only if it is sunny and it is raining

3)it is hot today if and only if it is sunny or it is raining

4) it is raining if and only if it is hot today or it is sunny

Q.3

a) Explain Argument with proper examples. Differentiate Valid and Invalid argument through proper examples, also construct a truth table showing valid and invalid arguments.

ANSWER: An argument is a list of statements called premises (or assumptions or hypothesis) followed by a statement called the conclusion.

An argument is valid if the conclusion is true when all the premises are true.

An argument is invalid if the conclusion is false when all the premises are true

To test whether or not an argument is valid, we do the following:

(i) Identify the premises and the conclusion

(ii) Construct a truth table showing the truth values of the premises and the conclusion

(iii) Look for all the rows where the premises are all true - we call such rows critical rows. If the conclusion is false in a critical row, then the argument is invalid. Otherwise, the argument is valid (since the conclusion is always true when the premises are true).

Examples of valid argument:

Premise: “if it is cloudy outside, it will rain”
“it is cloudy outside”
Conclusion: “it will rain”

When both $p \rightarrow q$ and p are true, we know that q must also be true. We say this form of argument is valid because whenever all its premises (all statements in the argument other than the final one, the conclusion) are true, the conclusion must also be true.

Now suppose that both "If you have a current password, then you can log onto the network" and "You have a current password" are true statements. When we replace p by "You have a current password" and q by "You can log onto the network," it necessarily follows that the conclusion "You can log onto the network" is true. This argument is valid because its form is valid.

suppose p and q are statements forms, In this case there is only one critical row to consider, and its truth value is true. Hence this is a valid argument.

Example:-

Show that the following argument form is valid.

$p \to q$ premise

p premise

$\therefore q$ conclusion

(the symbol '$\therefore$' means therefore)

Truth table:-

p	q	$p \to q$ (Premise)	p (Premise)	q (conclusion)
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

Example of invalid argument:
Premise: “Your grandmother had cancer”
Premise: “your mother had cancer”
conclusion: “Therefore, you will get cancer”

Even if it is true that the person's mother and grandmother had cancer, this does not meant that it must be true that the person will also get cancer. This is an invalid argument.

This argument is invalid since the third row is a critical row with a false conclusion.

Show that the following argument form is invalid.

$p \to q$ premise

q premise

$\therefore p$ conclusion.

p	q	$p \to q$	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

a) Explain the concept of Union, also explain membership table for union by giving proper example of truth table.

ANSWER: Union of two given sets is the smallest set which contains all the elements of both the sets that are being compared.

To find the union of two given sets A and B is a set which consists of all the elements of A and all the elements of B such that no element is repeated.

The symbol for denoting union of sets is '∪'.

For example:

Let set A = {2, 4, 5, 6}

and set B = {4, 6, 7, 8}

Taking every element of both the sets A and B, without repeating any element, we get a new set = {2, 4, 5, 6, 7, 8}
This new set contains all the elements of set A and all the elements of set B with no repetition of elements and is named as union of set A and B.
The symbol used for the union of two sets is ‘u’.
Therefore, symbolically, we write union of the two sets A and B is $A \cup B$ which means A union B.
Therefore, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

MEMBERSHIP TABLES
We combine sets in much the same way that we combined propositions. Asking if an element xx is in the resulting set is like asking if a proposition is true. Note that
xx could be in any of the original sets.

- Analog to truth tables in propositional logic.
- b) Columns for different set expressions.
- c) Rows for all combinations of memberships in constituent sets.
- d) Use “1” to indicate membership in the derived set, “0” for non-membership.
- e) Prove equivalence with identical columns.

MEMBERSHIP TABLE for union:

Set Membership Table 1

A	B	C	A ∪ B	(A ∪ B) ∪ C
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Set Membership Table 2

A	B	C	B ∪ C	A ∪ (B ∪ C)
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

b) What is Intersection? Draw Membership table for intersection using different examples.

ANSWER:

Intersection of two given sets is the largest set which contains all the elements that are common to both the sets.

To find the intersection of two given sets A and B is a set which consists of all the elements which are common to both A and B.

The symbol for denoting intersection of sets is '∩'.

For example:

Let set A = {2, 3, 4, 5, 6}

and set B = {3, 5, 7, 9}

In this two sets, the elements 3 and 5 are common. The set containing these common elements i.e {3, 5} is the intersection of set A and B.

The symbol used for the intersection of two sets is '∩'.

Therefore, symbolically, we write intersection of the two sets A and B is A ∩ B which means A intersection B.

The intersection of two sets A and B is represented as

A ∩ B = {x : x ∈ A and x ∈ B}

Set Membership Table 1

A	B	C	A ∩ B	(A ∩ B) ∩ C
0	0	0	0	0

A	B	C	A ∩ B	(A ∩ B) ∩ C
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	1	0
1	1	1	1	1

Set Membership Table 2

A	B	C	B ∩ C	A ∩ (B ∩ C)
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Q5. a) Explain the concept of Venn diagram with examples.

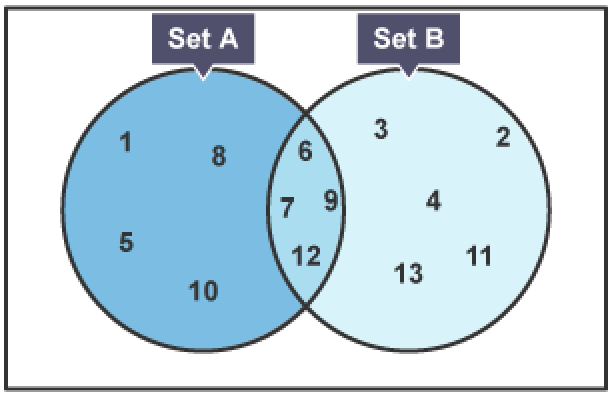
(a)

VENN DIAGRAM

A Venn diagram is an illustration of the relationships between and among sets, groups of objects that share something in common. Usually, Venn diagrams are used to depict set intersections (denoted by an upside-down letter U). This type of diagram is used in scientific and engineering presentations, in theoretical mathematics, in computer applications, and in statistics.

EXAMPLE:

In the diagram below, there are two sets, A = {1, 5, 6, 7, 8, 9, 10, 12} and B = {2, 3, 4, 6, 7, 9, 11, 12, 13}. The section where the two sets overlap has the numbers contained in both Set A and B, referred to as the intersection of A and B. The two sets put together, gives their union which comprises of all the objects in A, B which are {1 2 3 4 5 6 7 8 9 10 11 12 13}.



Example

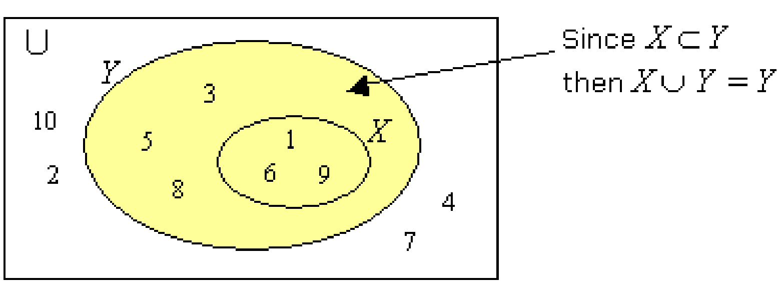
Given U = {1, 2, 3, 4, 5, 6, 7, 8, 10}

X = {1, 6, 9} and Y = {1, 3, 5, 6, 8, 9}

Find $X \cup Y$ and draw a Venn diagram to illustrate $X \cup Y$.

Solution

$X \cup Y$= {1, 3, 5, 6, 8, 9}



Q5.Given the set *P* is the set of even numbers between 15 and 25. Draw and label a Venn diagram to represent the set *P* and indicate all the elements of set *P* in the Venn diagram..

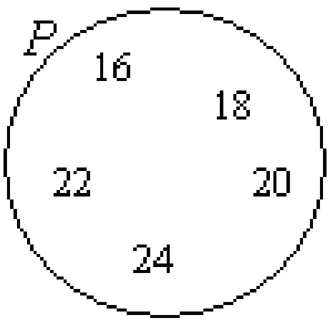
(b)

ANSWER

List out the elements of *F*.

P = {16, 18, 20, 22, 24} ← 'between' does not include 15 and 25

Draw a circle or oval. Label it *F*. Put the elements in *F*.



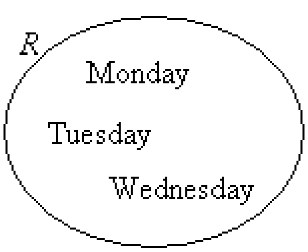
Q5.Draw and label a Venn diagram to represent the set

R = {Monday, Tuesday, Wednesday}.

(c)

ANSWER

Draw a circle or oval. Label it *R*. Put the elements in *R*.



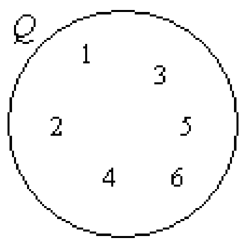
Q5.Given the set $Q = \{x : 2x - 3 < 11, x \text{ is a positive integer }\}$. Draw and label a Venn diagram to represent the set Q.

(d)

ANSWER

Since an equation is given, we need to first solve for x.

$2x - 3 < 11 \Rightarrow 2x < 14 \Rightarrow x < 7$



So, $Q = \{1, 2, 3, 4, 5, 6\}$

Draw a circle or oval. Label it Q.

Put the elements in Q.