Name = HASNAT KHAN

ID = 16631

SUBJECT = CALCULUS & ANALYTICAL GEOMETRY

TEACHER = Sir. HIMAYAT ULLAH

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SEMESTER = 2^{ND}

Q 1) part (a)

Estimate

$$\int \theta^4 \sqrt{1-\theta^2}\, d\theta$$

Sol:-

$$\int \theta^4 \sqrt{1-\theta^2}\, d\theta$$

Let $1-\theta^2 = u$

$$\frac{d}{d\theta}(1-\theta^2) = \frac{d}{d\theta}u$$

$$-2\theta = \frac{du}{d\theta}$$

$$\theta\, d\theta = \frac{-1}{2}\, du$$

Now

$$= \int (u)^{1/4} \cdot \left(-\frac{1}{2}\right) du$$

$$= \frac{-1}{2} \int u^{1/4}\, du \qquad \therefore \frac{1}{4}+1 = \frac{5}{4}$$

$$= \frac{-1}{1\cancel{2}} \cdot \frac{\cancel{4}^{2}}{5}\, u^{5/4} + C$$

$$= \frac{-2}{5}\, u^{5/4} + C$$

By back substitution

$$= -\frac{2}{5}(1-\theta^2)^{5/4} + C \text{ Ans.}$$

(Q 1) part (b):-

Estimate by using substitution method

$$\int_0^1 x^3 (1+x^4)^3\, dx$$

Sol:-

$$\int_0^1 x^3 (1+x^4)^3\, dx$$

Let $t = 1+x^4$

$$\frac{dt}{dx} = \frac{d}{dx}(1+x^4)$$

$$\frac{dt}{dx} = 4x^3$$

or

$$dt = 4x^3\, dx$$

$$\frac{dt}{4} = x^3\, dx$$

So

$$\frac{1}{4}\int_0^1 t^3\, dt$$

$$\frac{1}{4}\left(\frac{t^4}{4}\right)\Big|_0^1$$

solve through limit

$$\frac{1}{16}(1^4 - 0^4)$$

$$\frac{1}{16}(1)$$

$$= \frac{1}{16} \text{ Ans.}$$

Q 2 part (a)

Illustrate the centre & radius of sphere $x^2+y^2+z^2+3x-4z+1$

Sol:-

$$x^2+y^2+z^2+3x-4z+1=0$$

$$(x^2+3x)+y^2+z^2-4z+1=0$$

$$\left(x^2+3x+\left(\frac{3}{2}\right)^2\right)+(y-0)^2+\left(z^2-4z+\left(-\frac{b}{2}\right)^2\right)$$

$$=-1+\left(\frac{3}{2}\right)^2+\left(-\frac{b}{2}\right)^2$$

$$\left(x+\frac{3}{2}\right)^2+(y-0)^2+(z-2)^2=\frac{21}{4}$$

So,

$$(x_0, y_0, z_0) = \text{Centre}$$

$$=\left(-\frac{3}{2}, 0, 2\right)$$

&

Radius, $\boxed{a=\sqrt{\frac{21}{4}}}$ Ans.

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Q2 part (b):-

Find the curve

$y = \sqrt{x}$, $0 \leq x \leq 4$

Sol:-

Given that $y = \sqrt{x}$

$0 \leq x \leq 4 \Rightarrow a \leq x < b$

As

$$V = \int_a^b \pi y^2 \, dx$$

$$V = \int_0^4 \pi (\sqrt{x})^2 \, dx$$

$$V = \pi \int_0^4 x \, dx$$

$$V = \pi \frac{x^2}{2}$$

$$V = \frac{\pi}{2} \left[(4)^2 - 0 \right]$$

$$\boxed{V = 8\pi}$$

(Q3)

Illustrate the vector proje$_A$B.

Sol:-

$A = 2i - 4j + \sqrt{5}k$

$B = -2i + 4j - \sqrt{5}k$

Required:-

Projection $AB = ?$

Sol:-

By dot product

$B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$

$= ((-2)(2) + (4)(-4) + (-\sqrt{5})(\sqrt{5}))$

$= -4 - 16 + \sqrt{-5 \times 5}$

$= -4 - 16 - \sqrt{25}$

$= -4 - 16 - 5$

$\boxed{B \cdot A = -25}$

Now

$A \cdot A = (2i - 4j + \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$

$= ((2)(2) + (-4)(-4) + (\sqrt{5})(\sqrt{5}))$

$= 4 + 16 + \sqrt{5 \times 5}$

$= 4 + 16 + \sqrt{25}$

$= 4 + 16 + 5$

$\boxed{A \cdot A = 25}$

So,

$$\text{Proj}_A B = \left(\frac{B \cdot A}{A \cdot A} \right) A$$

Putting values

$$= \left(\frac{-\cancel{25}}{\cancel{25}} \right) (2i - 4j + \sqrt{5} k)$$

$$= (-1)(2i - 4j + \sqrt{5} k)$$

$$\boxed{\text{proj}_A B = -2i + 4j - \sqrt{5} k}$$ Ans.

(Q4) Find the area of region b/w graph & x-axis

$y = -x^2 + 5x - 4 \quad [0, 2]$

Sol:-

Given that

$$y = -x^2 + 5x - 4$$

&

$$[a, b] = [0, 2]$$

As $a = 0$

$b = 2$

So,

Area under graph will be

$$A = \int_a^b f(x)\,dx$$

Putting values

$$= \int_0^2 (-x^2 + 5x - 4)\,dx$$

By solving integration we will get

$$A = \left(-\frac{x^3}{3} + 5\frac{x^2}{2} - 4x\right)\Bigg|_0^2$$

$$A = \left(-\frac{1}{3}(2)^3 + \frac{5}{2}(2)^2 - 4(2)\right) - (0)^2$$

$$A = \left(-\frac{1}{3}(8) + \frac{5}{2}(4) - 8\right)$$

$$A = -\frac{8}{3} + \frac{20}{2} - 8$$

$$A = \frac{-8}{3} + \frac{20}{2} - \frac{8}{1}$$

$$A = \frac{2\times -8 + 3\times 20 - 6\times 8}{6}$$

$$A = \frac{-16 + 60 - 48}{6}$$

$$A = \frac{60 - 64}{6}$$

$$A = \frac{4}{6} = \frac{2}{3}$$

$$\boxed{A = 0.666}$$ Ans.

Q5 part (a)

Estimate the angle b/w A & B

$A = i - 2j - 2k$ & $B = 6i + 3j + 2k$

Sol:-

$A = i - 2j - 2k$

$|A| = \sqrt{(1)^2 + (-2)^2 + (-2)^2}$

$= \sqrt{1 + 4 + 4}$

$= \sqrt{9}$

$|A| = 3$

Now

$B = 6i + 3j + 2k$

$|B| = \sqrt{(6)^2 + (3)^2 + (2)^2}$

$= \sqrt{36 + 9 + 4}$

$= \sqrt{49}$

$|B| = 7$

So,

$$\theta = \cos^{-1}\left(\frac{A \cdot B}{|A|\,|B|}\right)$$

$$\theta = \cos^{-1}\left\{\frac{(i - 2j - 2k) \cdot (6i + 3j + 2k)}{3 \times 7}\right\}$$

$$\theta = \cos^{-1}\left\{\frac{(1)(6)+(-2)(3)+(-2)(2)}{21}\right\}$$

$$\theta = \cos^{-1}\left(\frac{6 - 6 - 4}{21}\right)$$

$$\theta = \cos^{-1}\left(\frac{-4}{21}\right)$$

$$\boxed{\theta = 100.97} \text{ Ans}$$

Q5 part (b)

Change into spherical coordinate equation

$$x^2+y^2+(z-1)^2=1$$

Sol:

$$x^2+y^2+(z-1)^2=1$$

$$(\rho\sin\phi\cos\theta)^2+(\rho\sin\theta\sin\theta)^2+(\rho\cos\phi-1)^2=1$$

$$\rho^2\sin^2\phi\cos^2\theta+\rho^2\sin^2\phi\sin^2\theta+\rho^2\cos\phi+1-2\rho\cos\phi=1$$

$$\rho^2\sin^2\phi(\cos^2\theta+\sin^2\theta)+\rho^2\cos^2\phi+1-2\rho\cos\phi=1$$

$$\rho^2(\sin^2\phi)+\rho^2\cos\phi^2+2\cdot\rho\cos\phi=1-1$$

$$\rho^2(\sin^2\phi+\cos^2\phi)-2\rho\cos\phi=0$$

$$\rho^2=2\rho\cos\phi$$

$$\boxed{\rho=2\cos\phi}$$ Ans.

Finish!

x———x———x———x