Department of Electrical Engineering
Sessional Assignment
Date: 01/06/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:		**Total Marks:**	20

Student Details

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Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$ To the input $x(n) = 4^n u(n)$.	Marks 6
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$$	
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	Marks 6
	(b)	Determine the partial fraction expansion of the following proper function $$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$	
Q 3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition $H(0) = 1$ and $\left\|H(\frac{\pi}{4})\right\|^2 = \frac{1}{2}$.	Marks 4

Q 4	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.	Marks 4
	(c)	A finite duration sequence of Length L is given as $$x(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & otherwise \end{cases}$$ Determine the N- point DFT of this sequence for $N \geq L$	
	(d)	Compute the DFT of the four-point sequence $$x(n) = (0\ 1\ 2\ 3)$$	

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Digital Singal proccessing Sessional Assigment :

Q'(a) Determine the response $y(n)$, $n \geq 0$, of the system described by the second-order difference equation.
$y(n) - 3y(n-1) - 4y(n-2) = x(x) + 2x(n-1)$ to the input $x(n) = 4^n u(n)$

Solucfion: consider the differance equaction :
$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ — ①
The homogenous equaction of the system is $y(n) - 3y(n-1) - 4y(n-2) = 0$
The characteristic equaction of the system is
$\lambda - 3\lambda^{-1} - 4\lambda^{-2} = 0$
$\lambda^2 - 3\lambda - 4 = 0$
Determine the root of the

Characteristic equaction:

$\lambda^{2} - 4\lambda + \lambda - 4 = 0$

$\lambda(\lambda-4) + 1(\lambda-4) = 0$

$(\lambda-4)(\lambda+1) = 0$

$\lambda = -1, 4$

The homogenous solucation is,

$y_h(n) = c_1(-1)^n u(n) + c_2(4)^n u(n)$

Since 4 is a characteristic root and the excitation is

$x(n) = 4^n u(n)$

We assume a particular solucation of the form $y_p(n) = Kn4^n u(n)$

Then:

$Kn4^n u(n) - 3K(n-1)4^{n-1}u(n-1) - 4K(n-2)4^{n-2}u(n-2) = 4^n u(n) + 2(4)^{n-1}u(n-1)$

Total solucation is:

$y(n) = y_p(n) + y_h(n)$

$= [6/5\, n4^n + c_1 4^n + c_2(-1)^n]\, u(n)$

The solve for c_1 and c_2, We assum that

$y(-1) = y(-2) = 0$ Then

$y(0) = 1$ and

$y(1) = 3y(0) + 4 + 2 = 9$

Hance

$c_1 + c_2 = 1$ and

$24/5 + 4c_1 - c_2 = 9$

$4c_1 - c_2 = 21/5$

Therefore,

$c_1 = 26/25$ and $c_2 = -1/25$

Total soluction is

$y(n) = \left[6/5\, n\, 4^n + 26/25\, 4n - 1/25 (-1)^n \right] u n.$

(b) Determine the impules response and unit step response of the system describe by the differance equaction

$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$

Sol:

consider the differance equaction:

$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$

$y(n) = 0.6y(n-1) + 0.08y(n-2) = x(n)$

To obtin the homogenous equaction set input.

$x(n) = 0$

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$y(n) - 0.6y(n-1) + 0.08y(n-2) = 0$
Determine the solution to the homogenous equaction.
$y_n(n) = \lambda^n$
Subsititute the solution abtained in the homogenous equaction.
$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$
$\lambda^{n-2}(\lambda^2 - 0.6\lambda + 0.08) = 0$
~~There fore~~
Thus, the general form of the solution to the homogenous equation is,
$y_n(n) = c_1(\lambda_1)^n + c_2(\lambda_2)^n$
$y_n = c_1(0.2)^n + c_2(0.4)^n$ ---- (1)
$\lambda = 0.2, \lambda = 0.4$ hence
$y_h(n) = c_1 1^n/5 + c_2 2^n/5$
With $x(n) = \delta(n)$, the initial condition are
$y(0) = 1,$
$y(1) - 0.6y(0) = 0$

$y(1) = 0.6$

Hence $c_1 + c_2 = 1$ and

$\frac{1}{5} c_1 + \frac{2}{5} = 0.6$

$\Rightarrow c_1 = -1, \quad c_2 = 3$

There fore $h(n) = \left[-(1/5)^n + 2(2/5)^n\right] u(n)$

The step responce is

$$(n) = \sum_{k=0}^{n} h(n-k), \; n > 0$$

$$= \sum_{k=0}^{n} \left[2(2/5)^{n-k} - (1/5)^{n-k}\right]$$

$$= \left\{\frac{1}{0.12}\left[\frac{2}{5}^{n+1} - 1\right] - \frac{1}{0.16}\left[(1/5)^{n-1} - 1\right]\right\} u(n)$$

Q2 (a) Determine the causal signal $x(n)$ having the Z - transform

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Hint: Take invers z-transform using Partial fraction method.

Soluation:

The z-transform is,

$$Y(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

The experssion is written as

$$X(z) = \frac{1}{(1-2/z)(1-1/z)^2}$$

$$= \frac{1}{\left(\frac{z-2}{z}\right)\left(\frac{z-1}{z}\right)^2}$$

$$\frac{1}{\frac{(z-2)(z-1)^2}{z_3}} \quad \text{——— (1)}$$

$X(z)$ has a simple pole at $p1 = 2$ and a double $p_2 = p_3 = 1$

In such a case the appropiate partial fraction is

$$X(z) = \frac{z^{2}}{(z-2)(z-1)} = \frac{A_1}{z-2} + \frac{A_2}{z-1} + \frac{A_3}{(z-1)^2}$$

The problem is to determine the coeffreient A_1, A_2 and A_3 we proceed as in the case of distinct pole to determine A_1 we multiply both side of by $(z-2)$ and evaluate the result $z-2$ -

$$(z-2)X(z)\ A_1 + \frac{z-2}{z-1} A_2 + \frac{z-2}{(z-1)^2} A_3$$

Which we evaluated at $z = 2$

$$A_1 = \frac{(z-2)\,X(z)}{2} \Bigg| \Rightarrow 2$$

$$A_1 = 4$$

$$A_2 = A_1 + \frac{z-2}{z-1}$$

$$A_2 = -3$$

$$A_3 = A_1 + \frac{z-2}{z-1} A_2$$

$$= -1$$

Hence

$$x(n) = [4(2)^n - 3 - n]\, u(n)$$

Q b 2 Determine the partial fraction expansion of the following proper function:

$$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

Sol:

First we eliminate the negative power by multiplying numerator and de-nominator by z^2 thus -

$$X(z) = \frac{z^2}{z^2 - 1.5z + 0.5}$$

The poles $X(z)$ are $p_1 = 1$

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$p_2 = 0.5$
consequently the expanion of the form.

$$X(z) = \frac{z^2}{z^2 - 1.5 + 0.5} = \frac{A_1}{z_1} + \frac{A_2}{z - 0.5}$$

~~The poles $X(z)$ are $p_1 = 1$ and $p_2 = 0.5$~~

A very simple method to determine A_1 and A_2 is to multiply the equation by the denominater term $(z-1)(z-0.5)$ thus we obtain

$z = (z - 0.5) A_1 + (z-1) A_2$ —— ①

Now if we set $z = p_2 = 1$ in eq ① we eliminate the term involving A_2, Hence $1 = (1 - 0.5) A_1$ Those we obtain the result $A_1 = 2$ Next we return eq ① and $z = p_2 = 0.5$ thus eleminating the term involving A_1 so we have

$$0.5 = (0.5-1)A_2.$$

and Hance $A_2 = -1$, Therefore the result of the partial fraction expansion is.

$$\frac{X(z)}{z} = \frac{2}{z-1} - \frac{1}{7.00} \quad \text{Ans}$$

Q NO 3(a) Solution:

A $W = 0$ we have

$$H(0) = \frac{b^0}{(1-P)^2} = 1$$

hance $b^0 = (1-P)^2$

At $We = \pi/4$

$$H(\pi/4) = \frac{1-P^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{(1-P\cos(\pi/4)+jP\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2}+jP\sqrt{2})^2}$$

Hance

$$\frac{(1-p)^4}{[(1-p)(\sqrt{2}^2 + p^2/2]} = 1/2$$

or equivalently

$$\sqrt{2}(1-p)^2 = 1+p^2-\sqrt{2}p$$

The value of $p = 0.32$ satiszies this equation consequently zilter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

Q3(b)

Sol :

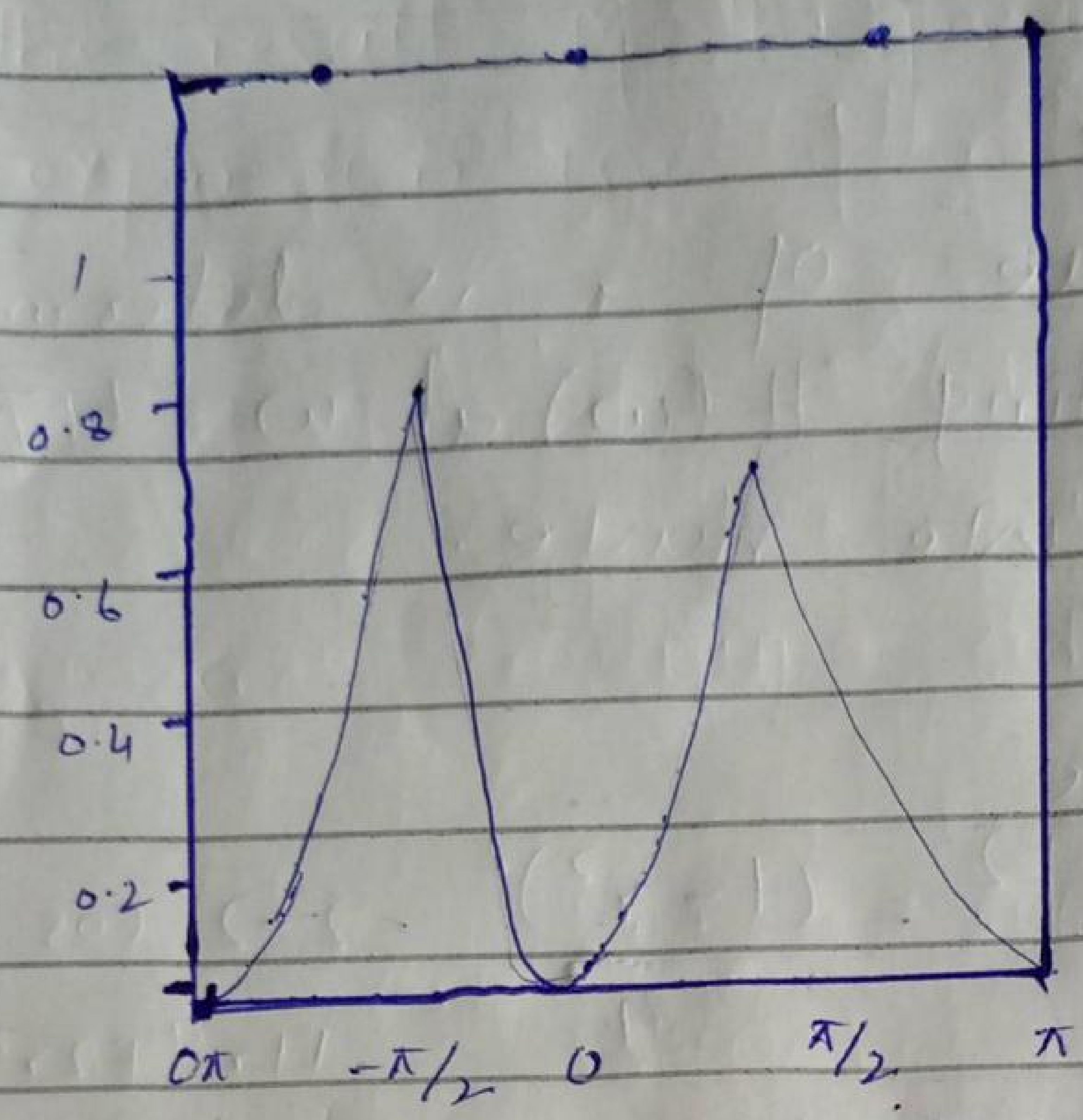
clearly the zilter must have poles at

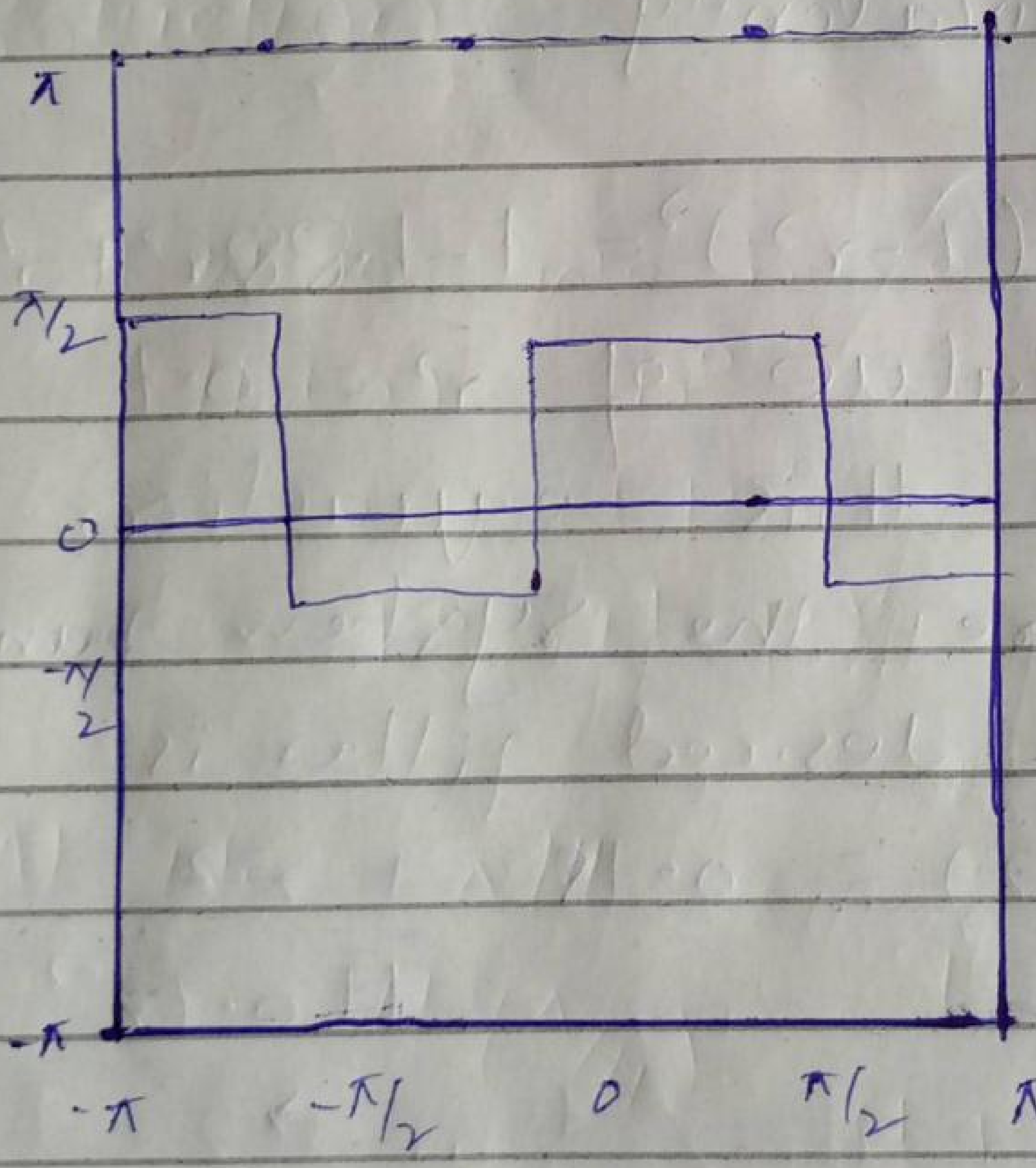
$p_{1.2} = re^{\mp j\pi/2}$

and zero at $z=1$ and $z=-1$ consequantly the system zunction is

$$(H)(z) = \frac{G(z-1)(z+1)}{(z+jr)(z-jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}.$$





The gain factor is deretmined by evaluating the frequancy responce $H(\omega)$ of the filter at $w = \left(\frac{\pi}{2}\right) = G\frac{2}{1-r^2} = 1$

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The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$.
Thus we have.

~~The value~~

$$|H(\frac{4\pi}{9})|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos r\pi/9}{1+r^4+2r^2\cos(r\pi/9)} = 1/2$$

or equivalently

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ satisfies this equaction
Therefore the system function for the desired filter is

$$H(z) = 0.15 \frac{1+z^{-2}}{1+0.7z^{-2}}$$

Question NO 4(a)

Sol :

The fourier transform of this sequance is:

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

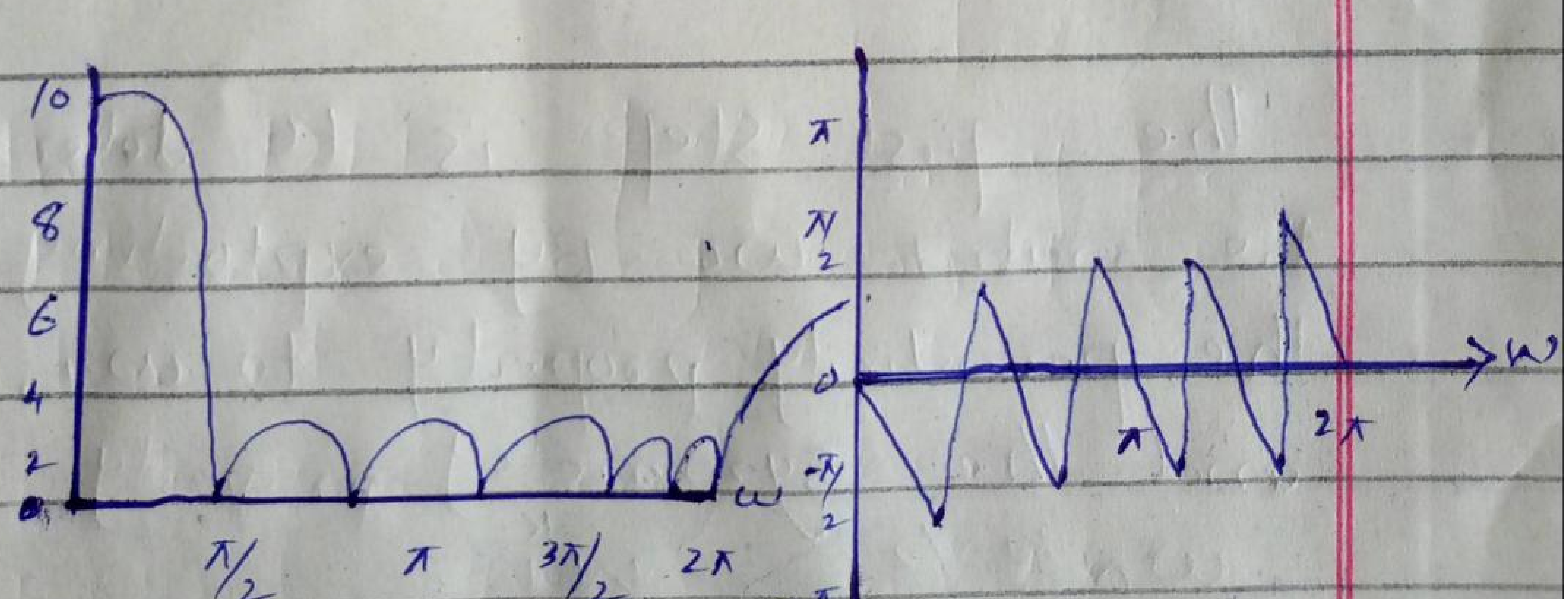
$$= \sum_{n=0}^{L-1} e^{-j\omega n} = 1 - e^{-j\omega L}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1/2)}.$$

The magnitude and phase of $x(\omega)$ are illertration for
The N-point DFT of $x(n)$ is simply $x(\omega)$ evaluated at the set of N equally speed frequancies $\omega_K = 2\pi K/N$ $K = 0, 1 \ldots N-1$

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1 - e^{-j2\pi K/N}} = K = 0, 1 \ldots, N-1$$

$$\frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$



If N is selected such that N = 2 then the DFT become

$$X(k) = \begin{cases} L, & k = 0 \\ 0, & k = 1, 2 \text{ ------- } 2-1 \end{cases}$$

Thus there is only one non zero value in DFT, This is apparent from observation of X (ω) since X(ω) = 0 at the frequancies $W_k = 2\pi k / L \quad k \neq 0$ the reador should verify that x(n) can be recoverd from X(k) by performing an L - point DFT

Question No 4 (b)

Sol :

The first step is to determine the matrix W_4. By exploiting the periodicity property to W_4, and the symmetory property

$$W_N^{k+N/2} = -W_N^{k}$$

the matrix W_4 may be

expressed as .

$$W4 \begin{bmatrix} W4^0 & W4^0 & W4^0 & W4^0 \\ W4^0 & W4^1 & W4^2 & W4^3 \\ W4^0 & W4^2 & W4^4 & W4^4 \\ W4^0 & W4^3 & W4^6 & W4^9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W4^1 & W4^2 & W4^3 \\ 1 & W4^2 & W4^0 & W4^2 \\ 1 & W4^3 & W4^2 & W4^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & +1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Then $y4 = W4\, Y4 \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$

The DFT of Y4 may be determined by conjugating element in W4 to obtain $W4^{+}$ and then appling the formula.