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Subject	Advance Fluid Mechanics
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Question # 01 (A)

Drag

A body which is wholy immersed in a homogenious fluid may be subjected to two kind of forces arising from relatives motion between body & fluid. These force are termed as a drag and lift. Simply forces acting parallel to the motion then it is called drag. Drag forces on submerged body have two components.

Pressure Drag

It is equal to the integration of components in the direction of motion of all pressure force exerted on surface of the body.

$$F_p = C_p \rho \frac{V^2}{2} BL$$

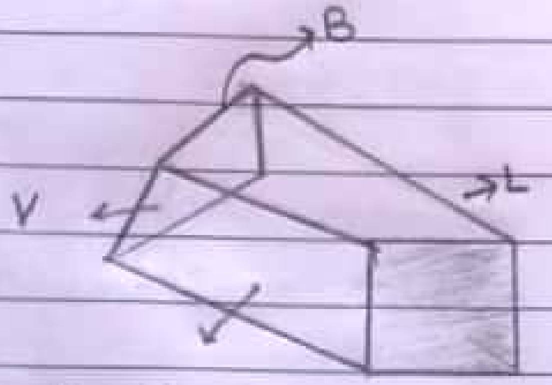
∴ C_p depends on shape

Fraction drag

It is equal to integration of components of shear stress along the surface of the body in direction of motion.

$$F_f = C_f \rho \frac{V^2}{2} BL$$

c_f = depends on viscosity
Area = $B \times L$

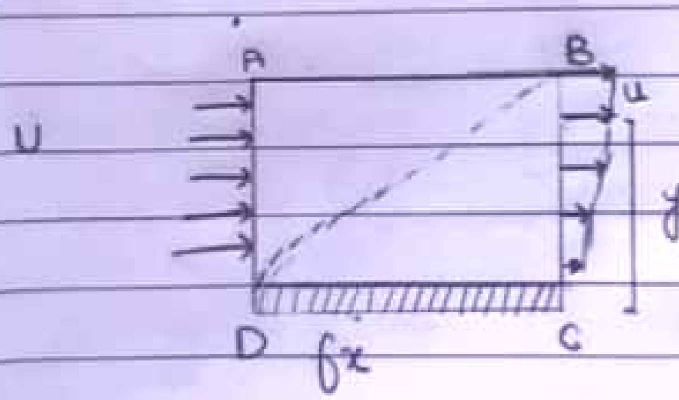


shear stress of 3rd Area

direction of motion

Friction Drag of Boundary Layer :-

figure shows growth of boundry layer along one side of smooth plate inside fluid.



When δ is thickness of boundry layer and U is undisturbed velocity.

Thus - F_x = drag = rate of momentum X direction.
leaving through BC + rate of momentum through AB is the rate of momentum entering through "DA"

$$\Delta P = P_{out} - P_{in}$$

Thus according to momentum

$$\Sigma F = \frac{d}{dt}(P) = \frac{dmv}{dt}$$

Where

$$\frac{dm}{dt} = \rho Q$$

$$\boxed{F = \rho Q v}$$

$$F = \rho A \cdot vv$$
$$F = \rho A v^2$$
$$DA \rightarrow \rho U(UB\delta)$$
$$BC \rightarrow \rho_B \int_0^{\delta} u^2 \cdot dy)$$

Thus putting values

$$F_x = \rho_B \int_0^{\delta}(U-u)\,dy$$

solving it $\Rightarrow$ $F_x = \int BU^2 \rho \alpha$

Where α is function of boundary layer.

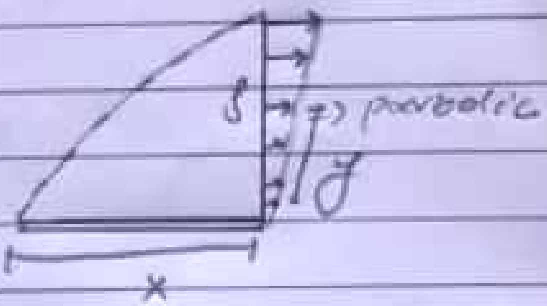
Now to find wall shear stress

$$\tau_0 = \frac{dF_x}{B\,dx}$$

$$F_x = \rho B U^2 \alpha$$

$$\tau_0 = \rho U^2 \alpha \frac{d\alpha}{dx}$$ = gets the general calculation of shear stress.

Laminar boundary layer :-



$$\frac{u}{v} = F\left(\frac{y}{\delta}\right)$$

Assume

$$\eta = \frac{y}{\delta} \quad \text{or} \quad y = \eta\delta$$

Thus

$$\frac{u}{v} = f(x) \quad \text{or} \quad u = v f(\eta)$$

In case of laminar flow

$$\tau_0 = \mu\left(\frac{du}{dy}\right) \Rightarrow \frac{\mu v}{\delta}\left[\frac{df}{d\eta}(\eta)\right]$$

Solving The equation

$$\tau_0 = \frac{\mu U \beta}{\delta} \longrightarrow A$$

The general equation is $\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$

Equating both equations.

$$\frac{\mu U \beta}{\delta} = \rho V^2 \alpha \frac{d\delta}{dx}$$

or

$$\delta \, d\delta = \frac{\mu \beta}{\rho U \alpha} dx$$

Integrating the equation.

$$\frac{\delta^2}{2} = \frac{\mu \beta}{\rho U \alpha} x + c$$

Now at $x = 0$ so thus $c = 0$

$$\frac{\delta^2}{2} = \frac{\mu \beta}{\rho U \alpha} x$$

or

$$\delta = \sqrt{\frac{2 \mu \beta}{\rho U \alpha} x} \text{ or } \sqrt{\frac{2\beta}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}}$$

xing and ÷ing by x,

$$\delta = \sqrt{\frac{2\beta}{\alpha}} \sqrt{\frac{\mu x}{\rho U}} \frac{x}{\sqrt{x}\sqrt{x}}$$

Where $\alpha = 0.135$

$\beta = 1.63$

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$$R_n = \frac{\rho U x}{\mu}$$

$$\delta = \frac{4.91}{\sqrt{R_n}} . x \quad \text{or} \quad \frac{\delta}{x} = \frac{4.91}{\sqrt{R_n}}$$

$$\tau_0 = \frac{\mu U B}{\delta}$$ Thus putting values

$$\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_n}$$

Where R_n is local reynold number

Now

$$F_y = B \int_0^a \underbrace{\tau_0 \, dx}_{\text{stress}}$$

Putting values

$$F_y = 0.664 B \sqrt{\rho \mu L U^3}$$

As general equation is

$$F_y = C_f \rho \frac{V^2}{2} BL$$ → equating both equation

$$\boxed{C_f = 1.32 B \sqrt{\frac{\mu}{\rho U L}} = \frac{1.328}{\sqrt{R}}}$$

Turbulent boundry Layer

resistone is less so curve become straight

Outer Transmission Turbulent

Figure show that velocity distribution in turbulent boundry layer shows a much steeper gradient near wall and flater throughout remaining layer.

The Shear stress is greater in turbulent than in laminar layer.

As we have.

$$\tau_0 = f\frac{\rho v^2}{8}$$ where v donates averge velocity of pipe.

Now we have obtained an aproximate relation between V and u by using pipe factor equation of

$$\frac{V}{U_{max}} = \frac{1}{1+1.33\sqrt{f}}$$ using friction factor of 0.028 from Chart which is middle critical Value so:

$$U = 1.235\,V$$

$$\tau_0 = f\frac{\rho v^2}{8}$$

As we know that:-

$$\boxed{f = \frac{0.316}{R^{0.25}}}$$

Thus $\tau_o = \frac{0.316}{\left(\frac{DU}{\nu}\right)^{1/4}} \frac{\rho V^2}{8}$, where $V = \frac{V}{1.235}$

Thus

$$\tau_o = \frac{0.316}{\left(\frac{DU}{\nu}\right)^{1/4}} \frac{\rho V^2}{8} \quad \text{where } V = \frac{V}{1.235}$$

$$\tau_o = \frac{0.316}{\left(\frac{D}{\nu}\left(\frac{V}{1.235}\right)\right)^{1/4}} \cdot \frac{\rho}{8}\left(\frac{V}{1.235}\right)^2$$

Thus $\tau_o = \frac{0.0235 \rho V^2}{\left(\frac{\delta v}{\nu}\right)^{1/4}}$

$$\tau_o = \rho V^2 \alpha \frac{d\delta}{dx}$$

Equating both and integrating for boundary condition of $x = 0, \delta = 0$

$$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\frac{\nu}{Ux}\right)^{1/5} x$$

For $\alpha = 0.0972$

$$\frac{\delta}{x} = \frac{0.377}{(R_x)^{1/5}}$$

putting values in equation.

$$\tau_0 = 0.0587 \rho \frac{v^2}{2} \left(\frac{\nu}{vx} \right)^{1/5}$$

Now $F_f = B \int_0^L \tau_0 \, dx$

$$F_f = 0.0735 \rho \frac{v^2}{2} \left(\frac{\nu}{UL} \right) BL$$

As

$$F_f = C_f \rho \frac{v^2}{2} BL$$

Equating Both

$$C_f = \frac{0.0735}{R^{1/5}}$$

R is less than 10^7 for $500000 < R < 10^7$

For $R > 10^7$

$$\boxed{C_f = \frac{0.455}{(\log R)^{2.58}}}$$

Part (B)

As specific energy $E = y + \frac{v^2}{2g}$

The flow Q per unit width "b" can be expressed as

$$q = \frac{Q}{b}$$

Now average velocity will be

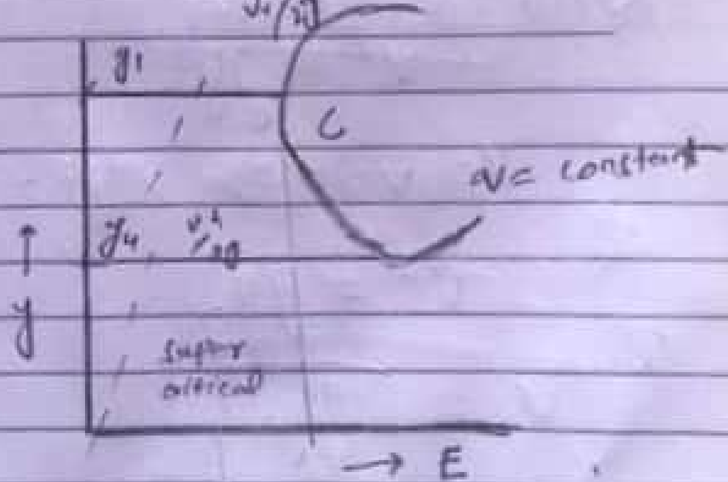
$$V = \frac{Q}{A} = \frac{qb}{by} = \frac{q}{y}$$

Thus

$$E = y + \frac{v^2}{2g} \Rightarrow y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

$$(E-y) = \frac{1}{2g}\left(\frac{q^2}{y^2}\right) \text{ or } (E-y)y^2 = \frac{q^2}{2g}$$

Thus plot of E vs y will be parabolic for particular q, there will be two kind of possible values of y for a given E.



The equation is cubic with three roots being negative point C represents dividing point between two region of flow. Thus for given q & value of E is minimum & flow of that point is critical flow. Depth of flow at that point is critical y_c and Velocity at that point is critical V_c.

Thus

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

For minimum specific energy $\frac{dE}{dy} = 0$

Thus

$$\frac{dE}{dy} = \left(1 - \frac{2}{2g}\left(\frac{q^2}{y^3}\right)\right) = 0$$

$$\frac{q^2}{gy^3} = 1 \Rightarrow q^2 = gy^3$$

$$\hookrightarrow \frac{q^2}{g} = y^3 \Rightarrow \left(\frac{q^2}{g}\right)^{1/3} = y_{cr}$$

Now

$$q^2 = gy^3$$

&

$$q = vy \Rightarrow v^2y^2 = gy^3$$

or

$$V^2 = gy_{cr}$$

or

$$\boxed{V_{cr} = \sqrt{gy_{cr}}}$$

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Question No # 2

Given Data

Water flows at rate, $Q = 3.5\ m^3/s$
Bed slope, $S_0 = 0.0008$
$n = 0.219$
Width of Bed in Student ID = 7728
= 7.728

Required

Depth of Rectangular Channel = ?
Critical depth y_c = ?
Critical velocity V_c = ?
Flow is critical or sub critical = ?

Solution

Manning equation

$$Q = \left(\frac{1}{n} R_n^{2/3} S_0^{1/2}\right) A \rightarrow (1)$$

Area $= 7.728 \times d$
Perameter $= d + 7.728 + d$
Hydralic Radius $R_n = \frac{\text{Area}}{\text{perameter}}$

$$R_n = \frac{7.728(d)}{2d + 7.728}$$

So we can put the values of "R_n" in eq ①

$$Q = \left(\frac{1}{n} R_n^{2/3} S_0^{1/2}\right) A$$

putting Values

$$3.5 = \left(\frac{1}{0.0219} \times \left(\frac{7.728d}{2d+7.728}\right)^{2/3} \times (0.0008)^{1/2} \times 7.728d\right)$$

$$\frac{3.5 \times 0.0219}{(0.0008)^{1/2}} = \left(\frac{7.728d}{2d+7.728}\right)^{2/3} \times 7.728$$

$$\left(\frac{3.5 \times 0.0219}{\sqrt{0.0008}}\right)^{3/2} = \frac{59.72d^2}{2d+7.728}$$

$$4.461(2d + 7.728) = 59.72d^2$$

$$8.922d + 34.47 = 59.72d^2$$

$$59.72d^2 - 8.922d - 34.47 = 0$$

$$\boxed{d = 0.671}$$

So the depth of Channel is

0.671

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Now As

q = discharge per unit width

$$q = \frac{Q}{b}$$

$$q = \frac{3.5}{7.728}$$

$$\boxed{q = 0.452\ m^2/s}$$

Critical depth y_{cr}

Using equation

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_{cr} = \left(\frac{(0.452)^2}{9.81}\right)^{1/3}$$

$$\boxed{y_{cr} = 0.275\ m}$$

Critical Velocity

Using equation

$$V_{cr} = \sqrt{g y_{cr}}$$

$$V_{cr} = \sqrt{(9.81)(0.274}$$

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$\boxed{V_{cr} = 1.63 \ m/s}$

$V = \frac{Q}{A} = \frac{3.5}{7.728 \times 0.671}$

$\boxed{V = 0.674}$

$y = 0.671 m$, $y_{cr} = 0.275 m$ $V_{cr} = 1.63 m/s$

As $y > y_{cr}$

and

$V < V_{cr}$

So the flow is subcritical flow.

Q.No3

Given Data

Width of smooth plate $B = 200mm = 0.2m$
length of smooth plate, $L = 200mm = 0.8m$
Oil with specific gravity, $S = 0.89$
Undisturbed velocity, $U = 5 \ m/s$
Kinematic velocity, $\nu = 0.93 \times 10^{-4} \ m^2/s$

Required Data

Friction drag one side of smooth plate, $F_f = ?$

Solution

Check the flow

As $\nu = 0.93 \times 10^{-4} \ m^2/s$

$$R = \frac{LU}{\nu} = \frac{(0.8)(5)}{0.93 \times 10^{-4}}$$

$$R = 43010.75 < 500{,}000$$

$\therefore$ Thus flow is laminar

Now

$$C_f = \frac{1.328}{\sqrt{R}}$$

$$= \frac{1.328}{\sqrt{43010.75}}$$

$$C_f = 6.403 \times 10^{-3}$$

$$\boxed{C_f = 0.0064}$$

$$F_f = C_f \, \rho \, \frac{v^2}{2} \, BL$$

$$= (0.0064)\,(S_{oil} \times \gamma_{water}) \times \left[\frac{(5)^2}{2} \times (0.2)(0.8)\right]$$

$$F_f = \left[(0.0064)(0.89 \times 1000) \times \frac{5^2}{2} \times (0.2)(0.8)\right]$$

$$\boxed{F_f = 11.392N}$$

So friction drag on one side of smooth plate is 11.392N