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Department : Civil Engineering

Subject : Intro. To Structural Dynamic & Earthquake

Assignment # 02

Iqra National University
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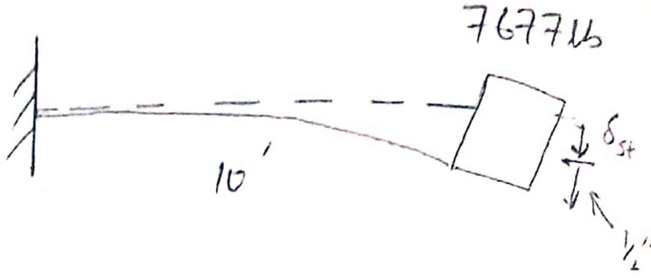
Q1 Given Data

$E = 29{,}000\ ksi$

$I = 150\ in^4$

$\delta_{st} = ?$

static load = 7677 lb



Sol

The general Eq of motion For SDOF System is

$$KU + C\dot{U} + m\ddot{U} = p(t)$$

in case of undamping (C=0) undergoing Free vibration p(t) = 0

Hence General EOM Become

$$KU + mU = 0 \quad \text{—— (1)}$$

$$K = 3EI/L^3$$

$$K = \frac{3 \times 29000 \times 150}{(10 \times 12)^3}$$

$$\boxed{K = 7.5\ K/in}$$ or

$$\boxed{K = 90625\ lb/ft}$$

$$m = \frac{7677}{32.2}$$

$$\boxed{m = 238.4\ slug}$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{90625}{238.4}}$$

$$\boxed{\omega_n = 19.49 \text{ rad/sec}}$$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{19.49}$$

$$\boxed{T_n = 0.322 \text{ sec}}$$

Substituting the corresponding value in eq (1)

$$90625u + 238.4\ddot{u} = 0$$

where "K" is in lb/ft & "m" is in lb sec/ft²

General Solution of the EOM For Undamped Free vibration is

$$u(t) = u(0)\cos(\omega_n t) + \frac{\ddot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$$u(0) = \frac{1}{2}\text{ inch} = \frac{1}{48}\text{ ft} \quad \text{and} \quad \ddot{u}(0) = 0$$

$$u(t) = \frac{1}{24} \times \cos(19.49) + \frac{0}{19.49} \times \sin(19.49) = 0$$

$$u(t) = \frac{1}{24} \times \cos(19.49)$$

Equivalent Static Force at any time "t"

$$f_s(t) = \frac{90625 \times \cos(19.49)}{24}$$

$$f_s(t) = \cos(19.49) \times 3776.04$$

Amplitude of dynamic displacement, u_0 for un-damped free vibration is

$$u_0 = \sqrt{\left((u(0))^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2 \right)}$$

$$= \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

$$u_0 = \frac{1}{24}$$

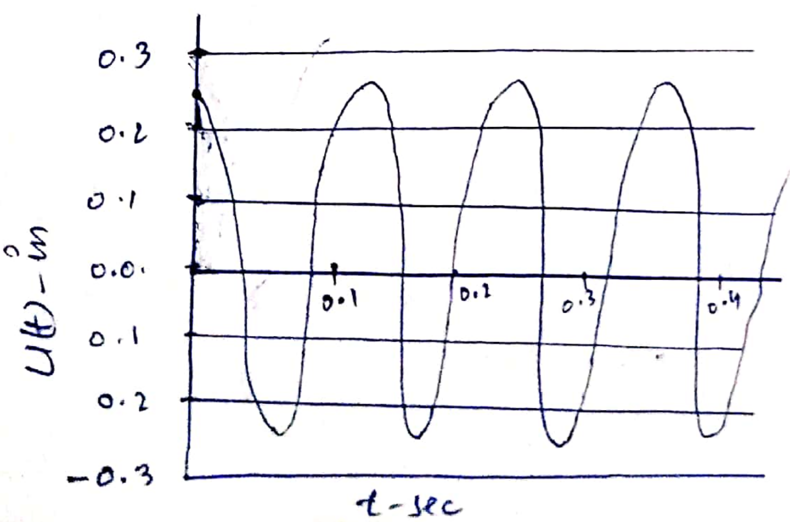
Amplitude of equivalent static force f_{s0}

$$K u_0 = 90625 \times \frac{1}{24}$$

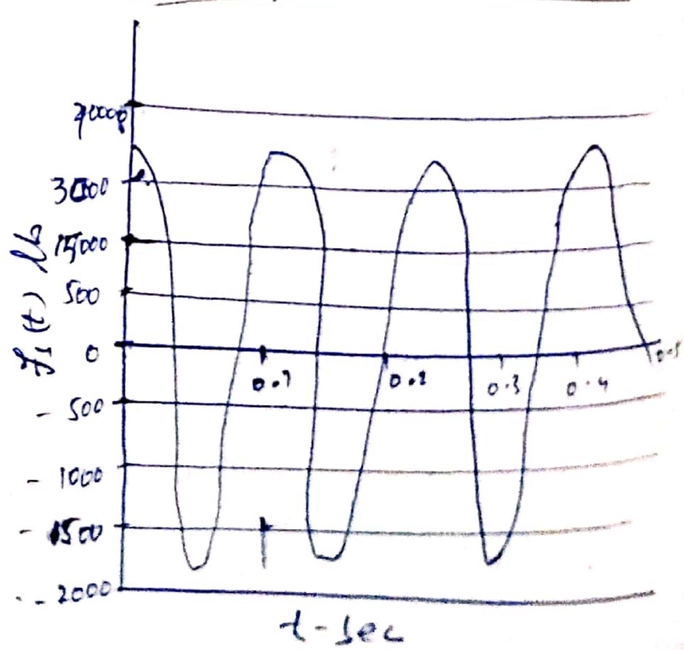
$K u_0 = 3776.04$ lb

[illegible]

Undamped free vibration



Undamped free vibration



Q2 Given Data

$E = 29{,}000\ ksi,\ I = 150\ in^4$

$\delta_{st} = 7677\ lb$

Damping ratio = 3

Reinforced Concrete with Considerate ranking 3-5%.

Sol

EOM For damped free vibration is

$ku + c\dot{u} + m\ddot{u} = 0$ —— ①

we know that from above eq ① Question ①

$K = 90625\ lb/ft$

$m = 238.4\ slug$

we know that

$c = \xi \times 2 m \omega_n$

Take damping ratio 3% = 0.03

Then

$c = 0.03 \times 238.4 \times 19.49 \times 2$

$$\boxed{c = 278.78\ lb \cdot sec^2/ft}$$

By substituting value of K, C and m in ① we get

$90625\bar{u} + 278.78\dot{u} + 238.4\, u = 0$

Solution to the E.O.M For damped free vibration is

$$u(t) = e^{-\xi \omega_n t}\left[u(0)\cos(\omega_d t) + \frac{1}{\omega_d}\left[\dot{u}(0) + u(0)\,\xi\,\omega_n\right) \times \sin \omega_d t\right]$$

$$u(t) = e^{-0.03 \times 19.49 t}\left[\frac{1}{24}\cos(19.49t) + \frac{1}{19.49}\left[\frac{1}{24} \times 0.03 \times 19.49 \times \sin(19.49)\right]\right]$$

$$u(t) = e^{-0.58}\left[0.41\cos(19.49) + (0.05(0.239\sin(19.49)))\right]$$

$$u(t) = e^{-0.58}\left[0.41\cos(19.49) + 0.0119\sin(19.49)\right]$$

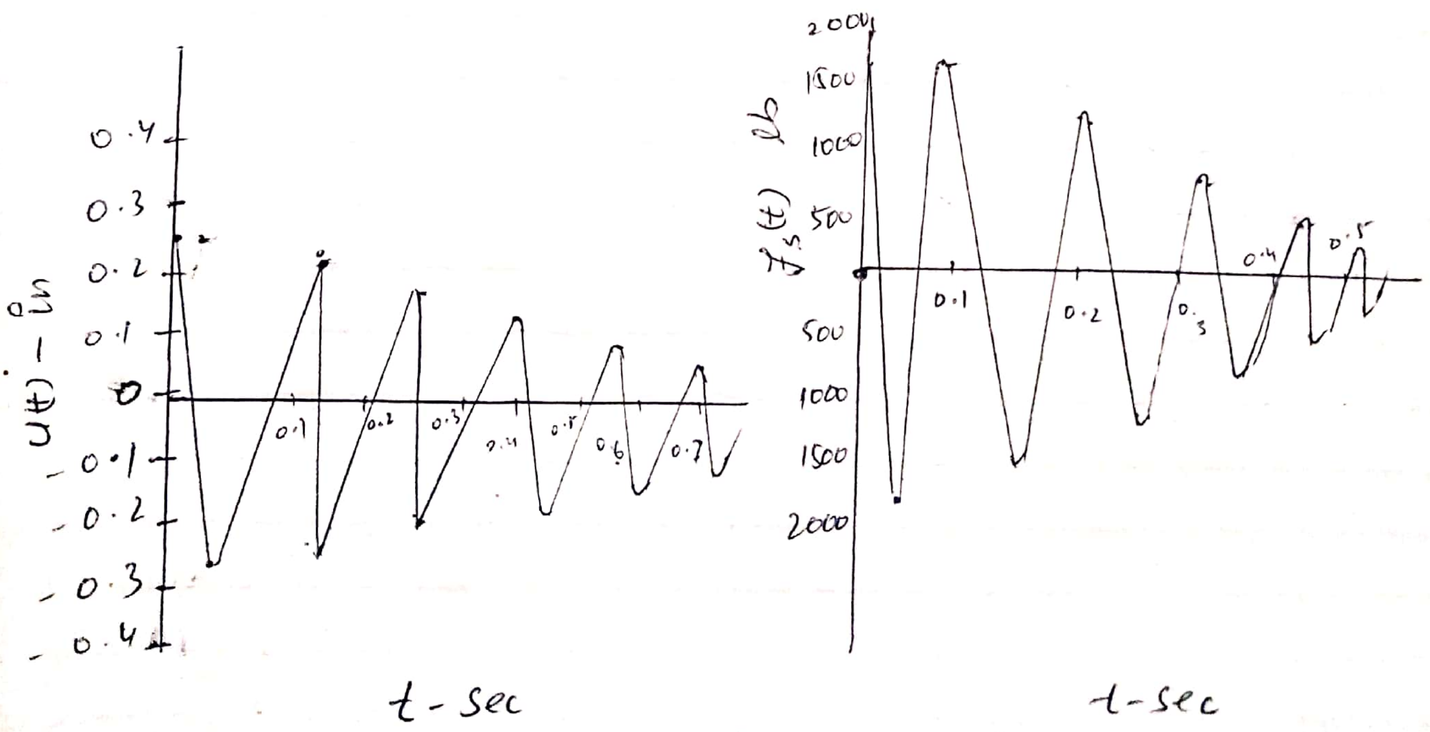
we know that

$$f_s(t) = K.u(t) = 90625 \times u(t)$$

$$f_s(t) = 90625 \times \left[e^{-0.58}\left[0.41 \times \cos(19.49) + 0.0119\sin(19.49)\right]\right]$$

$$\boxed{f_s(t) = e^{-0.58}\left[37156.2\cos(19.49) + 1078.4\sin(19.49)\right]}$$

<u>Damped Free vibration</u>

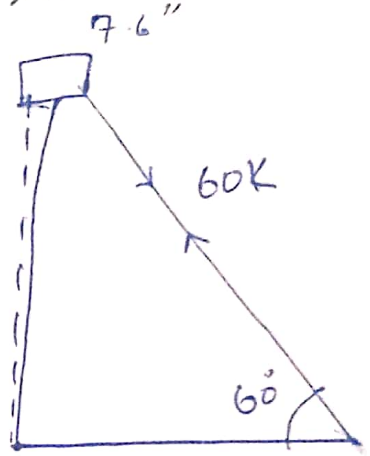


variation of static equivalent Force with Time(t)

Q3

Given Data

Force = 60 Kips
displace of tank = 7.6" horizontal
7 cycle in 3.57 sec complete
amplitude = 2.286 cm = 0.9 in



Find
(a) Damping Ratio
(b) Natural period of un-damped vib.
(c) stiffness of structure
(d) weight of tank
(e) Damping coefficient
(f) Number of cycle to reduce the displacement amplitude to 0.5"

sol

$u_1 = 7.6$ in.

After $j = 7$, $u_{j+1} = u_8 = 0.9''$

(a) ζ = Damping ratio

$$j = \frac{1}{2\pi\zeta} \ln \left[\frac{u_1}{u_{j+1}}\right]$$

$$\Rightarrow 7 = \frac{1}{2\pi\zeta} \ln \left(\frac{7.6}{0.9}\right)$$

$$\zeta = 0.0679 = 6.7\%$$

(b) $T_n = ?$

7 cycles of vibration are completed in 3.57 sec

Time require to complete one cycle = 3.57/7

$T_D = 0.51$ sec

Now

$$\omega_D = \omega_n \sqrt{(1-\xi^2)}$$

$$\frac{2\pi}{\omega_D} = 2\pi / \left(\omega_n \sqrt{(1-\xi^2)}\right)$$

$$\Rightarrow T_D = \frac{T_n}{\sqrt{1-\xi^2}}$$

$$T_n = T_D \times \sqrt{1-\xi^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.0679)^2}$$

$$T_n = 0.508 \simeq 0.51$$

$$\boxed{T_n = 0.51 \text{ sec}}$$

Scanned with CamScanner

(c) $K = ?$

$$K = \frac{60 \times \cos 60^\circ}{2} = 15 k/in = 18000 lb/ft$$

$$\boxed{K = 18000 lb/ft}$$

(d) weight of the tank, $W = ?$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{\left(\frac{W}{g}\right)}} = \sqrt{\frac{K \times g}{W}}$$

$$\omega_n^2 = K \times g / W$$

$$W = K \times g / \omega_n^2$$

Also $\omega_n = 2\pi / T_n$

$$W = Kg \Big/ \left(\frac{4\pi^2}{T_n^2}\right) = K \times g \times \frac{T_n^2}{4\pi^2}$$

$$W = \frac{18000 \times 32.2 \times (0.51)^2}{4\pi^2}$$

$$W = 3822.51 \, lb$$

$$\boxed{W = 38.2 K}$$

(e) $C = ?$

It is known that $\zeta = \frac{c}{2m\omega_n}$

$$c = \zeta \times 2m\omega_n$$

$$c = \zeta \times 2m(2\pi/T_n)$$

$$c = \frac{0.0679 \times 4 \times \pi \left(\frac{3822}{32.2}\right)}{0.51}$$

$$\boxed{c = 198.58 \text{ lb.sec/ft}}$$

(f) No. of cycles to reduce displacement amplitude From 7.6 to 0.5 $j = ?$

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{7.6}{0.5}\right]$$

$$\boxed{j = 6.37 \text{ or } 7 \text{ cycles}}$$