

Name ≠ Asad shoaib khalid.

ID ≠ 13095

Paper ≠ Probability.

Teacher ≠ Sir Daud.

Date ≠ 23 - 6 - 20

Question # 1

Answer

Solution:-

Let A be the event that the sum is 7. Let B the sum is odd.
So.

$$A = (1,6) (2,5) (3,4) (4,3) (5,2) (6,1).$$

$$B = (1,2) (1,4) (1,6) (2,1) (2,3) (2,5), \\ \dots (6,1) (6,3) (6,5) \text{ and.}$$

$$A \cap B = (1,6) (2,5) (3,4) (4,3) (5,2) (6,1)$$

Now there are 36 outcomes in all tossing 2 Fair dice, Hence

$$P(A) = 6/36 = 1/6$$

$$P(B) = 18/36 = 1/2$$

and

$$P(A \cap B) = 6/36 = 1/6 \text{ so } P(A|B)$$

$$P(A \cap B) / P(B) = 1/3 \quad \text{Answer}$$

Question # 2. !

Answer:-

Solution:-

Sum of 2 has 1 way 1,1

Sum of 3 has 2 way 1,2 & 2,1

Sum of 4 has 3 way 1,3; 2,2; 3,1

5 has 4 ways

6 has 5 ways

8 has 5 ways

9 has 4 ways

10 has 3 ways

11 has 2 ways

12 has 1 way

There are 15/36 for each side with a sum of 30/36. That leaves a $6/36 = 1/6$ probability for a sum of 7.

Question # 3.

Answer:-

Solution:-

$$\text{Given that } p = \frac{2}{3} \quad n = 8$$

$$q = 1 - p$$

$$q = 1 - \frac{2}{3} \Rightarrow \frac{1}{3}$$

(i) $P(x=4)$

$$= \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4$$

$$= \frac{1120}{6561} \Rightarrow \boxed{0.1707} \text{ Ans.}$$

(ii) $P(x \geq 4)$

$$1 - P(x < 4)$$

$$= 1 - \sum_{x=0}^3 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$$

$$= 1 - \left[\left(\frac{1}{3}\right)^8 + 8 \left(\frac{2}{3}\right) \left(\frac{1}{3}\right)^7 + 28 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^6 + \right.$$

$$\left. 56 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 \right]$$

$$= 1 - \frac{1}{6561} [1 + 16 + 112 + 448]$$

$$= 1 - \frac{577}{6561} \Rightarrow \frac{6561 - 577}{6561} \Rightarrow \boxed{0.9121} \text{ Ans}$$

(iii) $P(3 \leq x \leq 6)$

$$\sum_{x=3}^6 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 + \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4 + \binom{8}{5} \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^3 + \binom{8}{6} \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^2$$

$$= \frac{8}{(3)^3} [56 + 140 + 224 + 224]$$

$$= \frac{8 \times 644}{6561} = \frac{5152}{6561} = \boxed{0.7852} \text{ Ans}$$

Question # 4.

Answer:-

Proof:-

Since the C_i 's form a partition of the sample space, we can apply the law of total probability for $A \cap B$

$$P(A \cap B) = \sum_{i=1}^M P(A \cap B | C_i) P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^M P(A | C_i) P(B | C_i) P(C_i)$$

\therefore (A & B are conditionally independent)

$$P(A \cap B) = \sum_{i=1}^M P(A | C_i) P(B) P(C_i)$$

\therefore (B is independent of all C_i 's)

$$P(A \cap B) = P(B) \sum_{i=1}^M P(A | C_i) P(C_i)$$

$$P(A \cap B) = P(B) P(A)$$

\therefore law of total probability.

Hence A & B are independent.

Question #5.

Answer:-

* Mean & Variance of Binomial Random Variables:-

The probability ftn for the binomial random variable is

$$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

This is the probability of having x successes in a series of n independent trials when the probability of success in any one of the trials is p . If X is a random variable with this probability distribution.

$$E(X) = \sum_{x=0}^n x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^n x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^n \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

Since the $x=0$ term vanishes, let $y = x-1$ and $m = n-1$ into the last sum.

$$E(X) = \sum_{y=0}^m \frac{(m+1)!}{y!(m-y)!} p^{y+1} (1-p)^{m-y}$$

$$= (m+1)p \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= \cancel{np} np \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

The binomial theorem says that.

$$(a+b)^m = \sum_{y=0}^m \frac{m!}{y!(m-y)!} a^y b^{m-y}$$

Setting $a = p$ & $b = 1 - p$

$$\sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} = \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} = (p+1-p)^m$$

$$= (p+1-p)^m = 1.$$

So that.

$$E(x) = np$$

Similarly, but this time using $y = x-2$ & $m = n-2$

$$E(x(x-1)) = \sum_{x=0}^n x(x-1) \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^n x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=2}^n \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^n \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$= n(n-1)p^2 (p + (1-p))^n$$

$$= n(n-1)p^2$$

So the variance of x is

$$E(x^2) - E(x)^2 = E(x(x-1)) + E(x) - E(x)^2$$

$$= n(n-1)p^2 + np - (np)^2$$

Question #6.

Answer:-

* Bi-nomial frequency Distribution:-

If the bi-nomial probability distribution is multiplied by N , the no. of experiments or sets, the resulting distribution is known as bi-nomial frequency Distribution.

$$N \binom{n}{x} p^x q^{n-x}.$$

* Bi-nomial Distribution:-

A binomial Distribution can be thought of as simply the probability of a success or failure outcome in an experiment or survey that is repeated multiple times

$$P(x) f(x) = {}^n C_x p^x q^{n-x}.$$

Question #7.

Answer:-

Coefficient of Variation:-

For Data Set A:-

$$CV = \frac{\sigma}{\mu} \times 100$$

$$CV = 3/45 \times 100 \Rightarrow \boxed{CV = 6.7}$$

For Data Set B:-

$$CV = 11/60 \times 100$$

$$\boxed{CV = 18.3}$$

For Data set C:-

$$CV = 5/50 \times 100$$

$$\boxed{CV = 10}$$

For Data set D:-

$$CV = 15/25 \times 100$$

$$\boxed{CV = 60}$$