-: COURSE DETAILS:-

Course Title :- (S and S)

Instructor Name :- Engr. Mujtaba Ihsan.

Module :- 4th

-: STUDENT DETAILS:-

Name :- Naveed Alam

Student ID :- 14965

Student Sign. :-

Q No. 1

ⓐ Show with a help of an equation that the differentiation of a function in time domain result in the multiplication by jw in frequency domain-

Fourier transform of Differentiation Integration of continous- time -

Let $x(t)$ be a continous - time Signal with a fourier transform of $x(j\omega)$.

i.e :-

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

Differentiation both side with respect to "(t)"

$$\frac{dx}{dt}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(j\omega) \frac{d}{dt} \left\{ e^{j\omega t} \right\} d\omega$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(j\omega) \left\{ e^{j\omega t} \cdot j\omega \right\} \cdot d\omega$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left\{ j\omega\, x(j\omega) \right\} e^{j\omega t}\, d\omega$$

$$\Rightarrow \mathcal{F} \left\{ \frac{d}{dt} x(t) \right\} = j\omega\, x(j\omega)$$

↳ Result:-

We concluded that if a function is differentiated in time domain. It is multiplied by $j\omega$ in frequency domain.

Q NO. 1 part (B)

(B) If $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$
$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$
produce $Y(z)$ and $Y[n]$-

Sol:- $x[z] = 2 - 4z^{-2} + 2z^{-3}$

$H(z) = 3 + z^{-1} + 2z^{-2}$

Now:- $Y(z) = H(z) * X(z)$

$Y(z) = (3 + z^{-1} + 2z^{-1}) * (2 - 4z^{-2} + 2z^{-3})$

$Y(z) = 6 - 12z^{-2} + 6z^{-3} + 2z^{-1} - 4z^{-3} + 2z^{-4} + 4z^{-2}$
$- 8z^{-4} + 4z^{-5}$

$Y(z) = 6 + 2z^{-1} - 8z^{-2} + 2z^{-1} - 8z^{-2} + 2z^{-3} - 6z^{-4} + 4z^{-5}$

To find $Y[n]$ use the delay property

$Y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2] + 2\delta[n-3]$
$- 6\delta[n-4] + 4\delta[n-5]$.

-:X ———— X:-

Q NO. 2 :-

$$f(x) = \begin{cases} -\pi/2 & -\pi \leq x \leq 0 \\ \pi/2 & 0 \leq x \leq x \end{cases}$$

Retrieve the ~~[illegible]~~ fouriers series for the given function -

Solution:-

We know that fouriers series:-

$$\Rightarrow a_0 + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

so we find first (a_0)

$$a_0 = \frac{1}{2\pi} \left[\int_{-\pi}^{0} f(x)\, dx \right.$$

$$a_0 = \frac{1}{2\pi} \left[\int_{-\pi}^{0} \frac{-\pi}{2}\, dx + \int_{0}^{\pi} \frac{\pi}{2}\, dx \right]$$

$$a_0 = \frac{1}{2\pi} \left[\frac{-\pi}{2} \Big|_{-1}^{0} + \frac{\pi}{2} x \Big|_{0}^{x} \right]$$

$$a_0 = \frac{1}{2\pi} \left[+\frac{\pi}{2} (-\pi) + \frac{\pi^2}{2} \right]$$

so

DC componint

$$a_0 = \frac{1}{2\pi} \left[\frac{-\pi^2}{2} + \frac{\pi^2}{2} \right] = 0$$

so

$\boxed{a=0} \rightarrow (1)$

the find

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2}\cos nx\,dx + \int_{0}^{\pi} \frac{\pi}{2}\cos nx\,dx\right.$$

$$a_n = \frac{1}{\pi}\left[\frac{-\pi}{n2}\sin nx\Big|_{-x}^{0} + \frac{\pi}{n2}\sin nx\Big|_{0}^{\pi}\right.$$

$$a_n = \frac{1}{\pi}\left[\frac{-\pi}{2n}(0-0)+0\right] = 0$$

then find bn.

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2}\sin nx\,dx + \int_{0}^{\pi}\frac{\pi}{2}\sin nx\,dx\right]$$

$$b_n = \frac{1}{\pi}\left[\frac{\pi}{n2}\cos nx\Big|_{-x}^{0} - \frac{\pi}{2}\cos nx\Big|_{0}^{\pi}\right]$$

$$b_n = \frac{1}{\pi}\left[\frac{\pi}{n2}(1+1) - \frac{\pi}{2n}[-1-1]\right]$$

Scanned with CamScanner

$$b_n = \frac{1}{\pi}\left[\frac{2\pi}{2n} + \frac{2\pi}{2n}\right]$$

$$b_n = \frac{1}{n} + \frac{1}{n} = \frac{2}{n} \rightarrow (3)$$

so put in main function

$$\text{fourier series} = 0 + \sum_{n}^{\infty}\left(0\cos nx + \frac{2}{n}\sin nx\right)$$

$b_1 = \frac{2}{1} = \boxed{2}$, $b_2 = \frac{2}{2} = \boxed{1}$

$b_3 = \frac{2}{3} = \boxed{0.6}$ and upto so. on..

Scanned with CamScanner

Q NO. 3:-

If $x(z) = 2z^2+2z/(z^2+2z-3)$

Retrieve $x[n]$ using inverse z-transform method.

Sol:-

$$x(z) = \frac{2z^2+2z}{z^2+2z-3}$$

$$x(z) = \frac{2z(z+1)}{z^2+2z-3}$$

$$x(z) = \frac{2z(z+1)}{z(z+3)-1(z+3)}$$

$$\frac{x(z)}{z} = \frac{2(z+1)}{(z+3)(z-1)}$$

or $$\frac{2(z+1)}{(z+3)(z-1)} = \frac{A}{(z+3)} + \frac{B}{(z-1)}$$

Or $\Rightarrow$ put.

$$z(z+1) = A(z-1) + B(z+3) \rightarrow 1$$

put $z = 1$ in eq 1

$$z(1+1) = B(1+3)$$

$$4 = 4B$$

$$\frac{4}{4} = B \quad \boxed{B = 1}$$

Now put $z = -3$ in eq 1

$$z(-3+1) = A(-3-1)$$

$$-4 = -4A$$

$$\boxed{A = 1}$$

Now put A, and B in eq 1.

$$\frac{z(z+1)}{(z+3)(z-1)} = \frac{1}{(z+3)} + \frac{1}{(z-1)}$$

$$x(z) = \frac{z}{z+3} + \frac{z}{z-1}$$

inverse z - transform

$$x[n] = u[3] + 1[-1]^{k}.$$

x ——— x.

Q NO. 4 :-

Express the transfer function using the given data:-

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \end{bmatrix}$$

Solution:-

We know that:-

$$\frac{Y(s)}{X(s)} = H(s)$$

$$H[s] = C(sI - A)^{-1} B + D$$

putting the value

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \left[s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ +1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1}$$

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & +1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Adj = (s+2)s + 1 = s^2 + 2s + 2$$

$$\Rightarrow \quad s^2 + 2s + 2$$

Scanned with CamScanner

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \times \frac{1}{s^2+2s+2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

So.

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \times \frac{1}{s^2+2s+2} \begin{bmatrix} s & 0 \\ 1 & \end{bmatrix}$$

$$H[s] = \frac{\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}}{s^2+2s+2}$$

$$H[s] = \frac{s+2}{s^2+2s+2}$$

Scanned with CamScanner

Q5 No.

Apply Fourier transform on the signal, $x(t) = e^{-a|t|}\, u(t)$ where $u(t)$ is a unit step function:-

Given data:-

$$x(t) = e^{-a|t|}$$

$$a > 0$$

Find :- $x(j\omega) = ?$

Solution:-

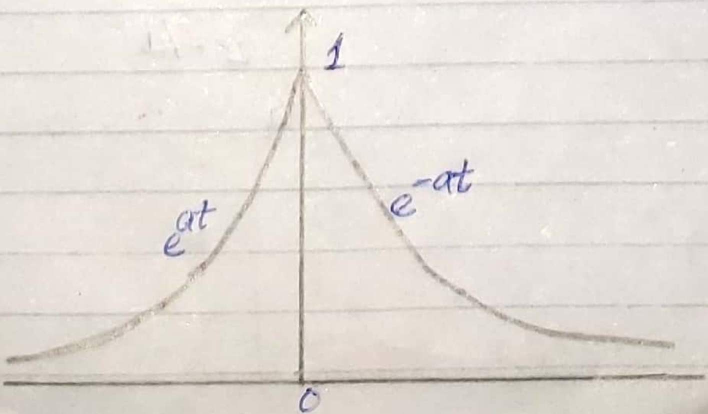
The transform of the given function $x(t)$ is given by

$$x(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt$$

Note:-

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{for } t < 0 \end{cases}$$



$\therefore$

$$X(j\omega) = \int_{-\infty}^{0} e^{at} e^{-j\omega} dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t} dt$$

$$\Rightarrow \int_{-\infty}^{0} e^{(a-j\omega)t} dt + \int_{0}^{\infty} e^{-(a+j\omega)t} dt$$

$$= \frac{e^{(a-j\omega)t}}{a-j\omega}\Bigg|_{-\infty}^{0} + \frac{e^{-(a+j\omega)}}{-(a+j\omega)}\Bigg|_{0}^{\infty}$$

$$= \frac{1}{(a-j\omega)}\left[e^{0} - e^{-\infty}\right] - \frac{1}{(a+j\omega)}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{1}{a-j\omega}[1-0] - \frac{1}{(a+j\omega)}[0-1]$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$= \frac{a+\cancel{j\omega}+a-\cancel{j\omega}}{a^2-(j\omega)^2}$$

$$= \frac{2a}{a^2+\omega^2}$$