Iqra National university, Peshwar

Department of Electrical engineering

Assignment

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Course code: MTH102

Course Title: Calculus and analytic geoemtry

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Module: 2, program BEE

Total marks: 30

Q1(a) Identify $\lim_{h\to 0} \dfrac{\sqrt{2+h}-\sqrt{2}}{h}$

Solution: $\lim_{h\to 0} \dfrac{\sqrt{2+h}-\sqrt{2}}{h}$ $\left(\dfrac{0}{0}\right)$

x-ing and ÷-ing both side by $\sqrt{2+h}+\sqrt{2}$

$$\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h} \times \frac{\sqrt{2+h}+\sqrt{2}}{\sqrt{2+h}+\sqrt{2}}$$

$$\lim_{h\to 0} \frac{\sqrt{(2+h)^2}-(\sqrt{2})^2}{h(\sqrt{2+h}+\sqrt{2})}$$

$$\lim_{h\to 0} \frac{2+h-2}{h(\sqrt{2+h}+\sqrt{2})}$$

$$\lim_{h\to 0} \frac{h}{h(\sqrt{2+h}+\sqrt{2})}$$

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$$\lim_{h \to 0} \frac{1}{\sqrt{2+h} + \sqrt{2}}$$

Apply limit

$$= \frac{1}{\sqrt{2+0} + \sqrt{2}}$$

$$= \frac{1}{\sqrt{2} + \sqrt{2}}$$

$$= \frac{1}{\sqrt{4}}$$

$$\boxed{= \frac{1}{2\sqrt{2}}}$$

Q1(b): Find the first order derivatives of the function $y=(x+\frac{1}{x})(x-\frac{1}{x}+1)$

Solution:

$$y = x+\frac{1}{x})(x-\frac{1}{x}+1)$$

$$= \frac{dy}{dx} = \frac{d}{dx}(x+\frac{1}{x})(x-\frac{1}{x}+1)$$

$$= (x+x^{-1})\frac{d}{dx}(x-x^{-1}+1)+(x-x^{-1}+1)$$

$$\frac{d}{dx}(x+x^{-1})$$

$$= (x+x^{-1})(1+x^{-2})+(x-x^{-1}+1)(1-x^{-2})$$

$$= \left(x+\frac{1}{x}\right)\left(1+\frac{1}{x^2}\right)+\left(x-\frac{1}{x}\right)\left(1-\frac{1}{x^2}\right)$$

$$= x+x\frac{1}{x^2}+\frac{1}{x}+\frac{1}{x^3}+x-x\frac{1}{x^2}-\frac{1}{x}+\frac{1}{x^3}+1$$

$$= 2x+2\frac{1}{x^3}+1-\frac{1}{x^2}$$

$$= 2x + 1 - \frac{1}{x^2} + \frac{1}{x^3} \rightarrow Ans$$

Q2(a): A dynamite blast blow up a heavy rock with Lanch velocity of 160m/se reaches a hight of $S = 160t - 16t^2$ ft after sec.

(i) How high does the rock go

(ii) Find the velocity and speed of the rock when it 256ft above the ground on the way up and down

(iii) Find the acceleration of the rock at time 5 sec.

Solution

Given $S = 160t - 16t^2$

At any time 't' the velocity is

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$$v = \frac{ds}{dt} = \frac{d}{dt}(160t - 16t^2)$$

$$\boxed{v = 160 - 32t}$$

At maximum hight $v = 0$

So $160 - 32 \Rightarrow 0$

$$t = \frac{160}{32} = 5 \text{ sec}$$

$$\boxed{t = 5 \text{ sec}}$$

Hence

$$S_{max} = S(5) = 160(5) - 16(5)$$

$$S_{max} = 800 - 400$$

$$\boxed{= 400 \text{ ft}}$$

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(b) Given that $S = 256ft$

Then $= 160t - 16t^2 + 256 = 0$

$= 16(t^2 - 10t + 16) = 0$

$= t^2 - 10t + 16 = 0$

~~$t(t$~~

$= t^2 - 8t - 2t + 16 = 0$

$= t(t-8) - 2(t-8) = 0$

$= (t-8)(t-2) = 0$

$= t - 8 = 0, \quad t - 2 = 0$

$= \boxed{t_2 = 8 sec}, \quad \boxed{t_1 = 2 sec}$

Since $U = 160 - 32t$

$t_1 = 2 sec, \quad U(2) = 160 - 32(2) = 96 \ m/sec$

At $t_2 = 8 sec, \quad U(2) = 160 - 32(8) = -96 \ m/sec$

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c since $v = 160 - 32t$

t = Time(t) given = 5 sec

So

$v = 160 - 32(5)$

$v = 160 - 160$

$v = 0$

So to find acceleration,

$a = \frac{dv}{dt} = \frac{d}{dt} 0$

$a = 0$

$\boxed{a = 0 \ m/s^2 \text{ ans}}$

Q3a: Does the curve, $y = x^4 - 2x^2 + 2$ have any horizental tengent, if so where

Solution: $y = x^4 - 2x^2 + 2$

$$\frac{dy}{dx} = \frac{d}{dx}(x^4 - 2x^2 + 2)$$

$$\frac{d}{dx}(x^4) - 2\frac{d}{dx}(x^2) + \frac{d}{dx}(2)$$

$$\frac{dy}{dx} = 4x^3 - 4x + 0$$

$$\frac{dy}{dx} = 4x^3 - 4x$$

If the tangent horizontal then

$$\frac{dy}{dx} = 0$$

So

$$4x^3 - 4x = 0$$

$$4x(x^2 - 1) = 0$$

$$4x = 0 \;,\; x^2 - 1 = 0$$

$$x = 0 \;,\; x^2 = 1$$

$$\sqrt{x^2} = \pm\sqrt{1}$$

$$x = \pm 1$$

$$x = 0 \;,\; 1 \;,\; -1$$

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The coresponding point in

$$y = x^4 - 2x^2 + 2$$

For

$$x = 0,\ y = 0 - 0 + 2 \Rightarrow y = 2$$

For

$$x = 1,\ y = 1 - 2 + 2 = 1$$

For

$$x = -1,\ y = -1 - 2 + 2 = 1$$

Hence

$$(0, 2), (1, 1) \text{ and } (-1, 1)$$

End