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Section	B
Subject	Calculus.

Question 1 :

Solution :

Coordinates of P (4, 1, 3)

$OP = 4i + 1j + 3k$

$OQ = \overrightarrow{OQ} - \overrightarrow{OP}$

$= (i + 2j + 4k) - (4i + 1j + 3k)$

$= -3i + 1j + 1k$ eq ①

Now Distance b/w P & Q = |PQ|

$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$

$= \sqrt{9 + 1 + 1}$

$= \sqrt{11}$ — eq ②

Let M be the point which is divided PQ in ratio 1:3. Then by the ratio theorm position vecter of $M = \overrightarrow{OM}$

$P=2$

$$= \frac{3(4\hat{i} + 1\hat{j} + 3k) + (1)(\hat{i} + 2\hat{j} + 4k)}{1+3}$$

$$= \frac{12\hat{i} + 3\hat{j} + 9k + \hat{i} + 2\hat{j} + 4k}{4}$$

$$= \frac{13\hat{i} + 5\hat{j} + 13k}{4} \quad \text{— eq } (3)$$

This eq 1,2,3 is the required solution.

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Question 2

P=3

Solution:

$$\int \frac{4x^3+10x+4}{2x^2+x}$$

$$\begin{array}{r|l} & 2x-1 \\ \hline 2x^2+x & \cancel{4x^3}+10+4 \\ & \pm\cancel{4x^3} \qquad \pm 2x^2 \\ \hline & -\cancel{2x^2}+10x+4 \\ & \mp\cancel{2x^2} \pm x \\ \hline & 11x+4 \end{array}$$

So $2x-1+\frac{11x+4}{2x^2+x} = \frac{4x^3+10x+4}{2x^2+x}$

$\Rightarrow \int \frac{4x^3+10x+4}{2x^2+x} = \int 2x-1 + \int \frac{11x+4}{2x^2+x}$ — (1)

$\Rightarrow 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$

$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx$ — (2)

Now Find

$$\int \frac{11x+4}{x(2x+1)}\,dx$$

$P = 4$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)}$$

xing ÷ing $[x(2x+1)]$

$$\frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1)+Bx}{\cancel{x(2x+1)}}$$

$$11 + 4 = A(2x+1) + Bx \quad \text{— } 3$$

Put $x = 0$ in eq 3

we get $\boxed{A = 4}$

Now put $x = -1/2$ in eq 3

$$11(-1/2) + 4 = B(-1/2)$$

$$\frac{-11+8}{\cancel{2}} = \frac{-B}{\cancel{2}}$$

$$-3 = -B$$

$\Rightarrow \boxed{B = 3}$

Put the value of A and B in (A)

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking integral on b.H.S

$$\int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4\,dx}{x} + \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= 4\ln|x| + 3/2 \ln|2x+1|$$

Put these value in (2)

$$= x^2 - x + 4\ln|x| + 3/2 \ln|2x+1|$$

Now put these value in (1)

$$\frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x + 4\ln|x| + 3/2 \ln|2x+1| + C$$

Question 3:   P=6

a) $\int_0^2 x^2 e^x \, dx$

Solution

$$\int_0^2 x^2 e^x \, dx$$

Now first Integration

$= \int x^2 e^x \, dx$

$= x^2 \int e^x \, dx - \int \left(\int e^x dx \, \frac{d}{dx} x^2 \right) dx$

$= x^2 e^x - \int e^x (2x) \, dx$

$= x^2 e^x - 2\left[x \int e^x dx - \int \left(\int e^x dx \, \frac{d}{dx} x \right) dx \right]$

$= x^2 e^x - 2\left(x e^x - \int e^x dx \right]$

$= x^2 e^x - 2x e^x + 2e^x$

Now put limits

$= \left| x^2 e^x - 2x e^x + 2e^x \right|_0^2$

$= \left(2^2 e^2 - 2(2) e^2 + 2e^2 - (0 - 0 + 2e^0) \right)$

$= \left(\cancel{4e^2} - \cancel{4e^2} + 2e^2 - 2 \right)$ $\boxed{\Rightarrow 2e^2 - 2 \text{ Ans}}$

b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}} dx$ $P=7$

Solution

First Find Integration

$\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx = ?$

Let $y = \sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$2\,dy = \frac{1}{\sqrt{x}} dx$

$\int \sin(y)(2dy) = 2\int \sin(y)\,dy$

$= 2(-\cos y)$

$= -2\cos y$

Put $y = \sqrt{x}$

$= -2\cos\sqrt{x}$

Put limits

$= -2|\cos\sqrt{x}| = -2(\cos\sqrt{2} - \cos 1)$

$\boxed{= -2\cos\sqrt{2} + 2\cos(1)}$ Ans

Question 4

P=8

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Solution:

By formula

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z} = 0 \quad (A)$$

So

$$u\cancel{t}(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$= x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x(3/2)(x^2+y^2+z)^{-5/2}(2x)\right] + (x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \quad -(1)$$

Now

$$\frac{\partial u}{\partial y} = \frac{-1}{2}(x^2+y^2+z^2)^{-3/2}(2y)$$

$$\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[y(-3/2)(x^2+y^2+z^2)^{-5/2}(2y) + (x^2+y^2+z^2)^{-3/2}\right.$$

$$\frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \quad -(2)$$

$$\frac{\partial u}{\partial z} = \frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$

$$\frac{\partial u}{\partial z} = z(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2) - (x^2+y^2+z^2)^{-3/2} \quad -(3)$$

Now put eq 1, 2, 3 in (A)

$$3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} -$$

$$(x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-3/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2 - (x^2+y^2+z^2) + 3y^2 - (x^2+y^2+z^2) + 3z^2 - (x^2+y^2+z^2)\right]$$

$$(x^2+y^2+z^2)^{-5/2}\left[3x^2 - x^2 - z^2 + 3y^2 - y^2 - z^2 + 3z^2 - x^2 - y^2 - z^2\right]$$

$$= (x^2+y^2+z^2)^{-5/2} \quad (0) = 0$$

So the given $u(x,y,z)$ is solution of laplace eq.