ID - 13748

NAME - HAMZA ISHAQ

DIGITAL SIGNAL PROCESSING

(1)

Q. 1

PART. A

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

The Characteristics equation is

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2 \quad \text{Hence}$$

$$y_h(n) = c_1 2^n + c_2 n 2^n$$

The particular solution is

$$y_p(n) = K(-1)^n u(n)$$

Substituting this solution into the difference equation we obtain

ID - 13748
NAME - HAMZA ISHAQ
DIGITAL SIGNAL PROCESSING

$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^n$
$(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$

for $n = 2$, $k(1+4+4) = 2 \Rightarrow k = \frac{2}{9}$

The total solution is

$$y(n) = [c_1 2^n + c_2 n^2 + \frac{2}{9}(-1)^n] u(n)$$

from the initial conditions.
we obtain $y(0) = 1$ $y(1) = 2$

Then

$$c_1 + \frac{2}{9} = 1$$

$$\Rightarrow c_1 = \frac{7}{9}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 2$$

$$\Rightarrow c_2 = \frac{1}{3}$$

ID - 13748
NAME - HAMZA ISHAQ
DIGITAL SIGNAL PROCESSING

Q. 1

PART B

Sol:-

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - 2x(n-2)$$

The characteristics equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = \frac{1}{2}, \frac{1}{5}$ Hence

$$y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n$$

with $x(n) = \delta(n)$, we have

$$y(0) = 2$$

$$y(1) - 0.7y(0) = 0 \Rightarrow y(1) = 1.4$$

Hence, $c_1 + c_2 = 2$ and

$$\frac{1}{2}c_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$$

(4)

ID - 13748

NAME - HAMZA ISHAQ

- DIGITAL SIGNAL PROCESSING

$\Rightarrow C_1 + \frac{2}{5} C_2 = \frac{14}{5}$

These equation yield

$C_1 = \frac{10}{3}, \ C_2 = \frac{-4}{3}$

$$h(n) = \left[\frac{10}{3} \left(\frac{1}{2}\right)^n - \frac{4}{3} \left(\frac{1}{5}\right)^n \right] u(n$$

The step response is

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$= \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

Scanned with CamScanner

ID- 13748

NAME- HAMZA ISHAQ

DIGITAL SIGNAL PROCESSING

(5)

$$= \frac{10}{3} \left(\frac{1}{2}\right)^n \sum_{K=0}^{n} 2K - \frac{4}{3} \left(\frac{1}{5}\right)^n \sum_{K=0}^{n} 5K$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^n \left(2^{n+1} - 1\right) u(n) - \frac{1}{3}$$

$$\left(\frac{1}{5}\right)^n \left(5^{n+1} - 1\right) u(n)$$

ID - 13748
NAME-HAMZA ISHAQ
DIGITAL SIGNAL PROCESSING

Q.2

PART-A

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$= \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4$, $B = -3$, $C = -1$

Hence, $x(n) = [4(2)^n - 3 - n]u(n)$

ID - 13748 (7)

NAME - HAMZA ISHAQ

DIGITAL SIGNAL PROCESSING

Q.2

PART-B

We have

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{z^{n-1}}{1-az^{-1}} dz = \frac{1}{2\pi j} \oint_c \frac{z^n dz}{z-a}$$

where C is a circle at radius greater than |a|. we shall evalute this integrate (3.4.2) with $f(z) = z^n$. we distinguish two cases.

The z-Transform and its Application to the Analysis to LTI Systems.

1) If $n \geq 0$, $f(z)$ has only zeros and hence on poles inside C is $z = a$.

Hence

$$x(n) = f(z_0) = a^n \quad n \geq 0$$

2) If $n < 0$, $f(z) = z^n$ has an nth order

ID- 13748
NAME - HAMZA ISHAQ
DIGITAL SIGNAL PROCESSING

pole at $z=0$. which is also inside C Thus there are contribution from both poles. For $n=-1$ ~~who~~ we have

$$x(-1) = \frac{1}{2\pi j} \oint_c \frac{1}{z(z-a)} dz = \frac{1}{z-a}\Big|_{z=0} + \frac{1}{z}\Big|_{z=a} = 0$$

If $n=-2$, wehave

$$x(-2) = \frac{1}{2\pi j} \oint_c \frac{1}{z^2 (z-a)} dz = \frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=a} = 0$$

By continuing in the same way we can show that $x(n)=0$ for $n<0$. Thus $x(n) = a^n u(n)$.

The Inverse z-Transform by Power Series Expansion

The basic in this method is the following Given a z-transform $X(z)$ with

ID - 13748
NAME - HAMZA ISHAQ
DIGITAL SIGNAL PROCESSING

its Corresponding ROC. we can expand X(z) into a power series of the form

$$X(z) = \sum_{n=-\infty}^{\infty} C_n z^{-n}$$

which converges in the given ROC. Then by the uniqueness of the z-transform $x(n) = c_n$. when X(z) is rational the expansion can be performed by long devision.
To illustrate this technique, we will also serve to emphasize again the importance of the ROC in dealing with z-transforms.

ID - 13748

NAME - HAMZA ISHAQ

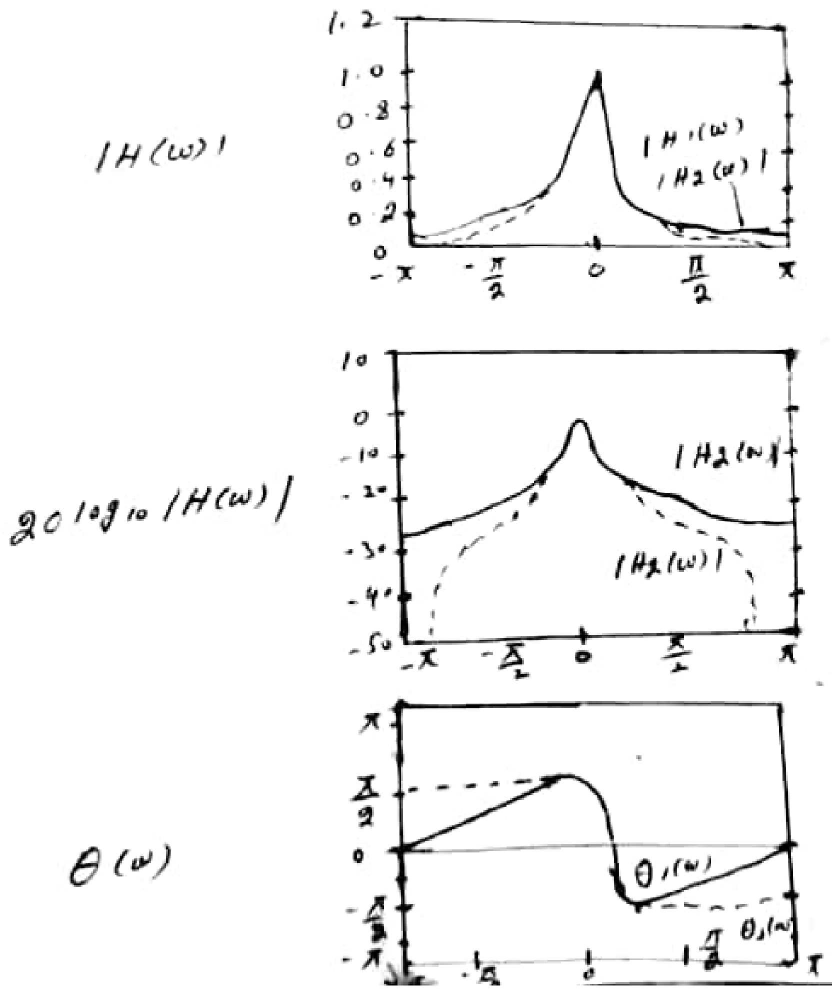
DIGITAL SIGNAL PROCESSING

Q. 3

PART : A

A two pole lowpass filter has the system function

$$H(z) = \frac{b_0}{(1 - pz^{-1})^2}$$



ID - 13748

NAME - HAMZA ISHAQ

DIGITAL PROCESSING

At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{(1-Pe^{-i\pi/4})^2}$$

$$= \frac{(1-P)^2}{(1-P\cos(\pi/4)+iP\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2}+iP/\sqrt{2})^2}$$

Hence

$$\frac{(1-P)^2}{[(1-P)\sqrt{2})^2+P^2/2]^2} = \frac{1}{2}$$

ID - 13748
NAME - HAMZA ISHAG
DIGITAL SIGNA PROCESSING

(12)

Q. 3

PART - B

Sol:-
Clearly, the filter must have poles at $p_{1,2} = re^{\pm j\pi/2}$

and zeros at $= 1$ and $z = -1$
Consequently the System function is

$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G \frac{z^2-1}{z^2+r^2}$$

The gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$. Thus we have

ID - 13748
NAME - HAMZA ISHAQ
DIGITAL SIGNAL PROCESSING

(13)

$$H\left(\frac{\pi}{2}\right) = G \frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega) = 4\pi/9$

Thus we have

$$\left| H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \quad \frac{2-2\cos(8\pi/9)}{1+r^4+2r^4\cos(8\pi/9} =$$

or equivalently.

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

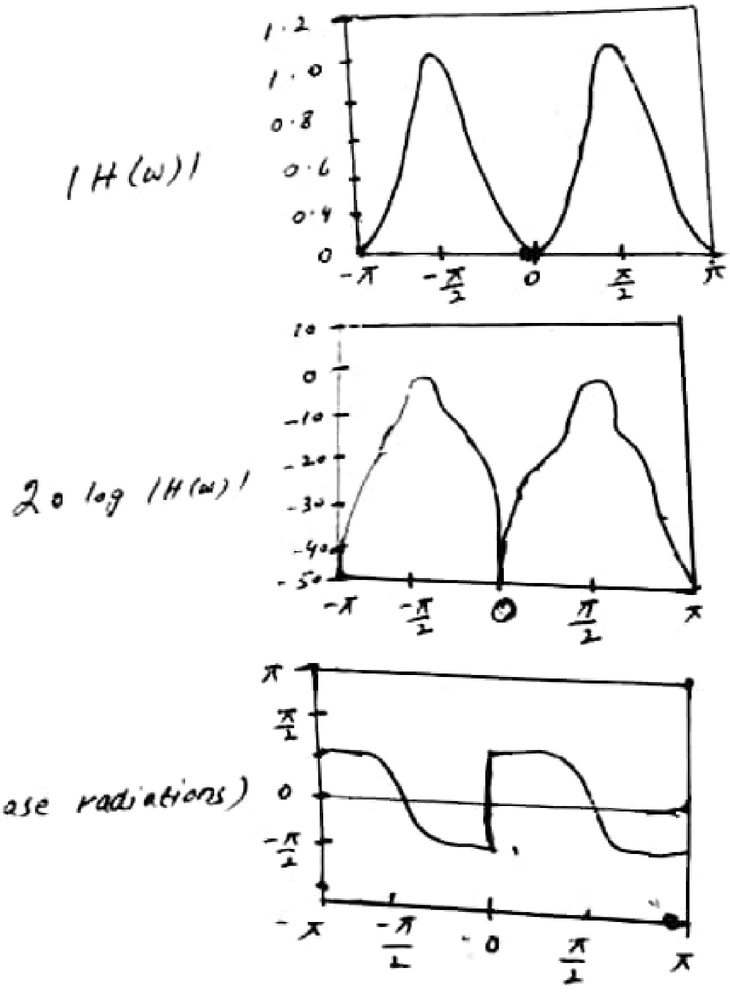
The value of $r^2 = 0.7$ satisfies this equation.

Therefore the system function

ID - 13748
NAME - HAMZA ISHAQ
DIGITAL SIGNAL PROCESSING

for the desired filter is

$$H(z) = 0.15 \frac{1 - z^{-2}}{1 + 0.7z^{-2}}.$$



it should be emphasized that the main purpose of the methodlogy for designing simple digital

ID - 13748 (15)
NAME - HAMZA ISHAQ
DIGITAL SIGNAL PROCESSING

filter by pole- zero placement is
to provide insight into the effect
That poles and zeros have on the
frequency response characteristics of
systems. The methodlogy is not intended
as a good method for desingning
digital filters with well- specified
passband and stopband characteristics.
Method for the design of stop-
histicated digital filters for practical
applications are discussed in chapter 8.

ID : 13748 (16)
Name: HAMZA ISHAQ
DIGITAL SIGNAL PROCESSING

Q.4

PART:- A

Sol:-

The fourier transform of this sequence is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-i\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-2\omega n} = \frac{1 - e^{-2\omega L}}{1 - e^{-2\omega}} = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-2\omega(L-1/2}$$

The magnitude and phase of $X(\omega)$ are illustrate in Fig 5 5 for $L = 10$ The N. point DFT of $x(n)$ is simply $X(\omega)$ evaluted at the set of N equally spaced frequencies $\omega_2 = 2\pi k/N. k = 0.1 \ldots N-1$.

Hence

Scanned with CamScanner

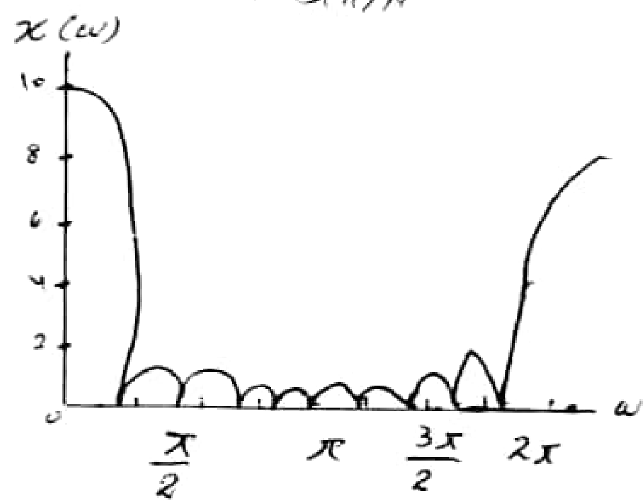
1D - 13748
NAME - HAMZA ISHAQ

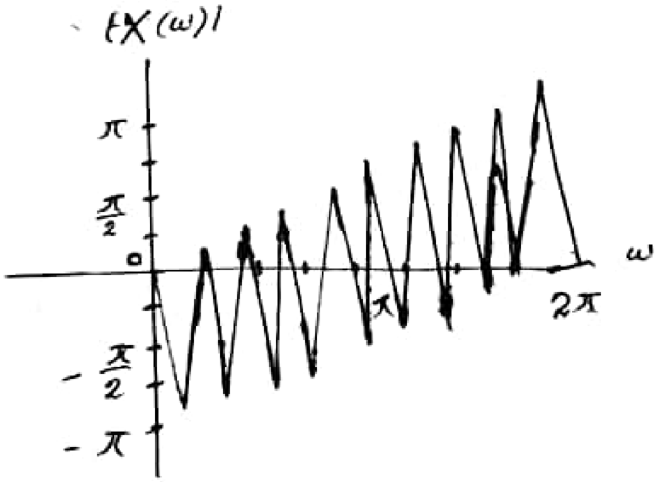
DIGITAL SIGNAL PROCESSING

(17)

$$x(K) = \frac{1 - e^{-j2\pi K L/N}}{1 - e^{-j2\pi K/N}} \qquad K = 0, 1 \ldots N-1$$

$$= \frac{\sin(\pi K L/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$





Scanned with CamScanner

ID- 13748
NAME- HAMZA ISHAQ
DIGITAL SIGNAL PROCESSING

(18)

Q.4

PART-B

Sol:-

Each Sequence Consists of four nonzero points. for the purposes of illustrating the operation involved in circular convolution
Thus the Sequences $x_1(n)$ and $x_2(n)$ are graphed as wise direction on a circle.
This establises the reference direction in rotating one of the sequences relative to the other

Now, $x_3(m)$ is obtained by circulary convolving $x_1(n)$ with $x_2(n)$ as specified

ID - 13748
NAME - HAMZA ISHAQ
DIGITAL SIGNAL PROCESSING

by (5.2 39). Beginning with m = 0 we have

$$x_3(0) = \sum_{n=0}^{3} x_1(n) x_2(-n))N$$

$x_2((-2))_4$ is simply the sequence $x_2(n)$ folded and graphed on a circle as illustrated in fig 5.8(b).

The product sequence is obtained by multiplying $x_1(n)$ with $x_2(-n)_4$ point by point

Finally. we sum the values in the product sequence to obtain

$$x_3(0) = 14$$

for m=1 wehave

$$x_3(1) = \sum_{n=0}^{3} x_1(n) x_2(1-n))_4$$

ID 13748
NAME HAMZA ISHAQ
DIGITAL SIGNAL PROCESSING

20

It is easily verified that $x_2(1-n)_4$ is simply the sequence $x_2(-n))_4$ rotated counter clock wise by one unit in time as illustrated in figure 5 8(c) This rotated sequence multiples $x_1(n)$ to yield the product sequence.

we sum the values in the product sequence to obtain $x_3(1)$

$$x_3(1) = 16$$

for $m=2$ we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2(2-n)_4$$

Now $x_2(2-n)_4$ is the folded sequence in figure rotated two units of time in the counter clock wise direction

ID- 13748

NAME - HAMZA ISHAQ

DIGITAL SIGNAL PROCESSING