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Section = A

Subject = Hydraulic Engineering

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Q.No (01)

A prototype gate valve which will control the flow in a pipe system conveying paraffin is to be studied in a model. List the significant variables on which the pressure drop across the valve would depend. Perform dimensional analysis to obtain the relevant non-dimensional groups.

A 1/15 scale model is built to determine the pressure drop across the valve with water as the working fluid.

- (a) For a particular opening, when the velocity of paraffin in the prototype is 3.0 m/s what should be the velocity of water in the model for dynamic similarity?
- (b) What is the ratio of the quantities of flow in prototype and model?
- (c) Find the pressure drop in the prototype if it is 60 kPa in the model.

(The density and viscosity of paraffin are 800 kg/m^3 and 0.020 kg/m s respectively. Take the kinematic viscosity of water as $1.0 \times 10^{-6} \text{ m}^2/\text{s}$.)
The pressure drop Δp is expected to depend upon the gate opening h , the overall depth d , the velocity v , density ρ , and viscosity μ .

Solution:

The pressure drop Δp is expected to depend upon the gate opening h , the overall depth d , the velocity v , density ρ , and viscosity μ .

(3)

List the relevant variables

$$\Delta p, h, d, V, \rho, \mu$$

Write down dimensions:

$$\Delta p = ML^{-1}T^{-2}$$

$$h = L$$

$$d = L$$

$$V = LT^{-1}$$

$$\rho = ML^{-3}$$

$$\mu = ML^{-1}T^{-1}$$

Number of Variable $n=6$

Number of independent dimensions $m=3$
(M, L and T)

Number of non-dimensional groups: $n-m=3$

Chosen $m (=3)$ scaling variable

geometric (d): isinematic / time dependent (1):
dynamic / mass-dependent (2)

Form dimensionless group by non-dimensionalizing
the remaining variable: $\Delta p, h$ and μ .

$$\Pi_1 = \Delta p d^a V^b \rho^c$$

(4)

$$M^0 L^0 T^0 = (M^{-1} T^{-2}) (L)^a (L T^{-1})^b (M^{-3})^c$$

$$= M^{1+c} L^{-1+a+b-3c} T^{-2-b}$$

$$M: 0 = 1+c \Rightarrow c = -1$$

$$T: 0 = -2-b \Rightarrow b = -2$$

$$L: 0 = -1+a+b-3c \Rightarrow a = 1+3c-b = 0$$

$$\Rightarrow \Pi_1 = \Delta p V^{-2} \rho^{-1} = \frac{\Delta p}{\rho V^2}$$

$$\Pi_2 = \frac{h}{d} \quad (\text{by inspection, since } h \text{ is a length})$$

$$\Pi_3 = \mu d a V^b g^c \quad (\text{probably obvious by now but here goes anyway...})$$

$$M^0 L^0 T^0 = (M L^{-1} T^{-1}) (L)^a (L T^{-1})^b (M^{-3})^c$$

$$= M^{1+c} L^{-1+a+b-3c} T^{-1-b}$$

$$M: 0 = 1+c \Rightarrow c = -1$$

$$T: 0 = -1-b+0 \Rightarrow b = -1$$

$$L: 0 = -1+a+b-3c \Rightarrow 0 = 1+3c-b = -1$$

$$\Rightarrow \Pi_3 = \mu d^{-1} V^{-1} g^{-1} = \frac{\mu}{\rho V d}$$

Recognition of the Reynolds number suggests that we replace Π_3 by

$$\Pi_3 = (\Pi_3)^{-1} = \frac{\rho V d}{\mu}$$

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Hence dimensional analysis yields

$$\Pi_3 = \left(\frac{\rho v d}{\mu} \right) = \rho d \left(\frac{\rho v d}{\mu} \right)_m$$

From the last, we have a velocity ratio

$$\frac{V_p}{V_m} = \frac{(\mu/\rho)_p}{(\mu/\rho)_m} \frac{d_m}{d_p} = \frac{0.002/1800 \times 1}{1.0 \times 10^{-6} \times 5} = 0.5$$

Hence

$$V_m = \frac{V_p}{0.5} = \frac{3.0}{0.5} = \boxed{6.0 \text{ m/s}}$$

(b) The ratio of the quantities of flow is

$$\frac{Q_p}{Q_m} = \frac{(\text{Velocity} \times \text{area})_p}{(\text{Velocity} \times \text{area})_m} = \frac{V_p (d_p)^2}{V_m (d_m)^2} = 0.5 \times 5^2 = 12.5$$

○ Finally for the pressure drop

$$\Pi_2 = \left(\frac{\Delta P}{\rho v^2} \right)_p = \left(\frac{\Delta P}{\rho v^2} \right)_m \Rightarrow \frac{(\Delta P)_p}{(\Delta P)_m} = \frac{\rho_p}{\rho_m} \left(\frac{V_p}{V_m} \right)^2$$

$$\Pi_2 = \frac{800 \times 0.5^2}{1000} = \boxed{0.2}$$

Hence

$$\Delta P_p = 0.2 \times \Delta P_m = 0.2 \times 60 = \boxed{12.0 \text{ kPa}}$$

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QND (02)

Design a practical profile of gravity dam with the following data:

- ① Maximum Depth of water in the reservoir is (your first two digits of R)
- ② Specific gravity of dam material is G (can be of your own choice)
- ③ Allowable compressive strength for the dam masonry is (your first three digits of R T/m^2)
- ④ Height of wave is H_w (can be your own choice)
- ⑤ G and H_w is of your own choice but should be differ from one another.

Given Data:

- Maximum Depth of water in the reservoir is $H = 77$
- Specific gravity of dam material is $G = 2.11$
- Allowable compressive strength for the dam masonry is $\sigma_c = 779$
- Height of wave $H_w = 8.11$
- $u = 0.7$
- $cu = 0$

Solution:

$$H_{\text{mining}} = \frac{\sigma_c}{\gamma_w (G - (u + 1))}$$
$$= \frac{779 \times 1000}{1000 (2.11 - 0 + 1)}$$

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$$\frac{779000}{(3.11) \times 1000} = 250.48 \text{ m} > H_w = 77$$

So it is low gravity dam.

(2) Top width = 'a'

$$\text{Free board} = 1.5 H_w$$

$$F-B = 1.5 \times 77$$

$$F-B = 115.5$$

$$\text{Height of Dam} = H_D = H_w + F-B$$

$$H_D = 77 + 115.5$$

$$H_D = 192.5$$

Now

$$a = 14\% \text{ of } H_D$$

$$a = 0.14 \times 192.5$$

$$a = 27.05 \text{ m}$$

(3) Base width 'b' (with an offset)

(i) For no sliding criteria

$$b = \frac{H_w}{u_c} = \frac{77}{0.7 \times 3.11} = 32.13$$

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$$b' = 53 \text{ m}$$

ii) For no tension criteria

$$b' = \frac{Hw}{\sqrt{g}} = \frac{77}{\sqrt{9.81}}$$

$$b' = 53.0 \text{ m}$$

iii) Depth of vertical portion on US side

$$h' = 2a \sqrt{g - cu}$$

$$h' = 2(12.48) \sqrt{(9.81 - 0)}$$

$$h' = 36.2 \text{ m}$$

iv) Upstream offset:

$$\text{Upstream offset} = \frac{q}{16} = \frac{12.48}{16} = 0.78 \text{ m}$$

v) Depth below the water level to the end of inclined portion in

$$u/s = 3.14 a \sqrt{g}$$

$$u/s = 3.14 \times 12.48 \sqrt{9.81}$$

$$u/s = 56.92 \text{ m}$$

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① Total width of the base of the dam

$$b = \frac{b' + a}{16}$$

$$b = \frac{53 + 12.48}{16}$$

$$b = 53.78 \text{ m}$$

$$\textcircled{2} \tan \phi = \frac{b'}{H}$$

$$\tan \phi = \frac{53}{77}$$

$$\phi = \tan^{-1}(0.688)$$

$$\phi = 34.52^\circ$$

③ Depth of vertical portion on D/S (from WL on L/S side)

$$\tan \phi = \frac{a}{d'}$$

$$\tan \phi = \frac{12.48}{d'}$$

$$\tan \phi \times d' = 12.48$$

$$\tan(34.52) \times d' = 12.48$$

$$d' = \frac{12.48}{0.687}$$

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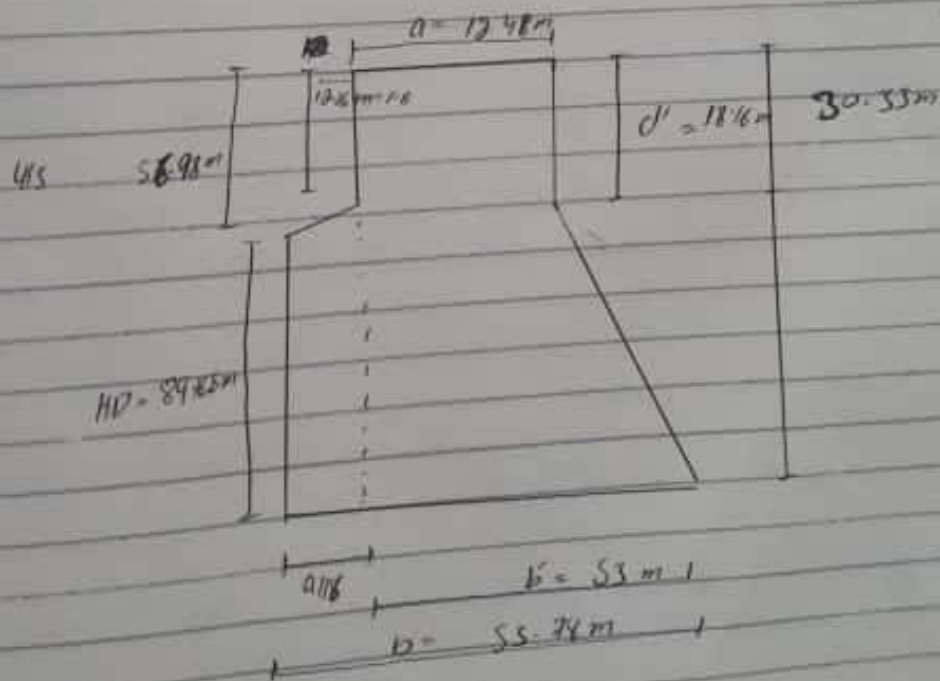
$$d' = 18.165 \text{ m}$$

depth of vehicle portion

$$d = d' + F-B$$

$$d = 18.165 + 12.165$$

$$d = 30.33 \text{ m}$$



Q No (04)

What will be the effect of sediment particle diameter, particle density, particle concentration, particle shape, viscosity of water, turbulence of water flowing in reservoir on fall velocity? Explain in detail.

Ans

Sediment particle diameter:

The sedimentation diameter of a particle is the diameter of a sphere that has the same specific gravity and has the same terminal uniform settling velocity as the given particle in the same sedimentation fluid.

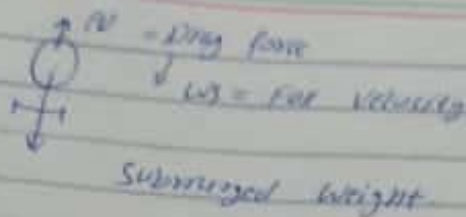
Fall Velocity:

When a grain falls down in still water it obtains a constant velocity when the upward fluid drag force on the grain is equal to the downward submerged weight of the grain. This constant velocity is defined as the fall velocity of the grain. This is also called settling velocity.

Fall velocity depends on

- ① particle diameter
- ② particle density
- ③ particle concentration
- ④ particle shape
- ⑤ viscosity of water temperature
- ⑥ Turbulance

(13)



The force balance b/w the drag force and the submerged weight gives

$$F_D = \text{submerged weight}$$

$$\frac{1}{2} \rho_c \frac{\pi d^2 w_s^2}{4} = (\rho_s - \rho) g \frac{\pi d^3}{6}$$

$$A = \frac{\pi d^2}{4} = \text{Projected Area}$$

C_D = Drag coefficient

$$w_s = \text{Fall velocity of sediment} = \sqrt{\frac{4g d (\rho_s - \rho)}{5 C_D}}$$

ρ = Density of water

ρ_s = Density of sediment particle

① Particle diameter:

$$d = \frac{D^3 \Delta \rho}{18 \eta}$$

Where D is the diameter of the particle and η is the absolute viscosity of the surrounding fluid. $\Delta \rho$ is the density difference between that of the particle and its surrounding fluid. If $\Delta \rho$ is positive the particle will settle or if negative than float.

Particle Shape:-

Particle having regular shape tends to be effected more than irregular shape since regular shape particles have even surface which offers very little or no friction which while particles with irregular shape offers more friction as with smaller surface area are more likely to be effected due to their less resistance.

Particle Density:- Density of the particle is directly proportion to the rate of fall velocity since particle with high density tends to settle down early compared with the particle of less density.

Particle Concentration:-

When the suspended concentration of sediment increases, the setting velocity of each particle decreases due to the modification of the flow induced by previous particles.

Viscosity :-

Fluid velocity through porous media is approximated as inversely proportional to the kinematic viscosity. A decrease in viscosity therefore increase the velocity of a compound through porous media.

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Turbulence :-

The turbulence of water effect the fall velocity of water in reservoirs because the non-linearity and zig zag water and cause the variation in flow.

QNo (03)

Using any hydraulic model and explain the concept of Dimensional analysis and similitude. Each student should have separate model analysis.

Ans

Dimensional analysis:-

Dimensional analysis is a mathematical technique making use of study of dimensions.

* It deals with the dimension of physical quantities involved in the phenomena.

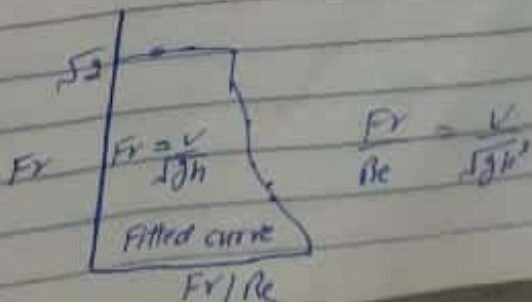
* In dimensional analysis, one first predicts the physical parameters that will influence the flow and then by grouping these parameters in dimensionless combination a better understanding of the flow phenomena is made possible.

Advantages:-

$$Fr = \frac{V}{\sqrt{gh}} = \frac{Fr}{Re} = \frac{V}{\sqrt{gh}}$$

Experiment become expensive

Dimensional analysis shows



Conclusion

One figure alone is enough as opposed to many figure of dimensional analysis

Similitude

For pressure loss study $(F_r = W / \rho V^2)$

$$\text{if } \frac{V}{\sqrt{\rho H^3}} = \frac{V}{\sqrt{\rho H^3}}$$

$$F_{r, \text{model}} = F_r$$

For any model let suppose a turbine certain fluid mechanical phenomena is governed by

$$f(\pi_1, \pi_2, \dots, \pi_n) = 0$$

Where π_i is non dimensioned where is model is similar to the prototype.

- ↳ Geometry similarity
- ↳ Kinematics "
- ↳ Dynamic "

Geometric similarity: A model and prototype are geometrically similar if all corresponding dimensions or their characteristics have the same ratio scale ratio

Kinematic similarity: A model and prototype are kinematically similar if homologous points are at homologous positions at homologous time

Kinematic similarity requires geometric similarity

Dynamic similarity: A model and prototype are dynamically similar if ratio of any two forces are same for model and prototype

↳ We are now interested to conduct a model analysis in a wind tunnel to know the drag the prototype

$n=5$ $k=3$ $m=2$

Repeating

L, U, ρ

$\Pi_1 = F(L)^a (U)^b (\rho)^c$

$\Pi_2 = \mu (L)^a (U)^b (\rho)^c$

We will find values for a, b, c and then we see that it similar on both sides.