Anees Sher 11743 (1)

Q1 a Part

Estimate $\int \theta \sqrt[4]{1-\theta^2}\, d\theta$.

Sol:

$$\int \theta \sqrt[4]{1-\theta^2}\, d\theta$$

Applying u. substitution: $u = 1-\theta^2$

$$= \int -\frac{\sqrt[4]{u}}{2}\, du$$

Take constant out.

$$= -\frac{1}{2}\int \sqrt[4]{u}\, du.$$

Applying Radial rule.

$$= \frac{-1}{2} \cdot \int u^{1/4}\, du$$

Applying Power rule.

$$= -\frac{1}{2} \frac{u^{\frac{1}{4}+1}}{\frac{1}{4}+1}$$

$$\Rightarrow \frac{-1}{2} \frac{(1-\theta^2)^{\frac{1}{4}+1}}{\frac{1}{4}+1}, \quad \frac{-2}{5}(-\theta^2+1)^{5/4}$$

$$= -\frac{2}{5}(-\theta^2+1)^{\frac{5}{4}} + c \quad \text{Ans}$$

Q1 Part b

Estimate $\int_0^1 x^3(1+x^4)^3\, dx$ using substitution method.

Sol:

$$\int_0^1 x^3(1+x^4)^3\, dx.$$

Expand it: $x^3(1+x^4)^3, \; x^3 + 3x^7 + 3x^{11} + x^{15}$

$$= \int_0^1 x^3 + 3x^7 + 3x^{11} + x^{13} \, dx$$

Applying sum rule.

$$= \int_0^1 x^3 dx + \int_0^1 3x^7 dx + \int_0^1 3x^{11} dx + \int_0^1 x^{15} dx$$

$$\Rightarrow \left[\frac{x^{3+1}}{3+1}\right]_0^1 + 3\left[\frac{x^{7+1}}{7+1}\right]_0^1 + 3\left[\frac{x^{11+1}}{11+1}\right]_0^1 + \left[\frac{x^{15+1}}{15+1}\right]_0^1$$

$$\Rightarrow \left[\frac{x^4}{4}\right]_0^1 + 3\left[\frac{x^8}{8}\right]_0^1 + 3\left[\frac{x^{12}}{12}\right]_0^1 + \left[\frac{x^{16}}{16}\right]_0^1$$

$$\Rightarrow \frac{1}{4} + 3\left(\frac{1}{8}\right) + 3\left(\frac{1}{12}\right) + \left[\frac{1}{16}\right]$$

$$\Rightarrow \frac{1}{4} + \frac{3}{8} + \frac{\cancel{3}\,1}{\cancel{12}\,4} + \frac{1}{16}$$

$$\Rightarrow \frac{1}{4} + \frac{3}{8} + \frac{1}{4} + \frac{1}{16}$$

$$\Rightarrow \frac{4+6+4+1}{16}$$

$$\Rightarrow \frac{15}{10} \text{ Ans}$$

Aneesgher 11743

Q2 Part (a)

Illustrate The centre & radius of The sphere
$x^2 + y^2 + z^2 + 3x - 4z + 1$

Sol:

$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$

$(x^2 + 3x) + y^2 + z^2 - 4z + 1 = 0$

$(x^2 + 3x + (\frac{3}{2})) + (y-0)^2 + (z^2 - 4z + (\frac{-4}{2})^2)$

$= -1 + (\frac{3}{2})^2 + (\frac{-4}{2})^2$

$= (x + \frac{3}{2})^2 + (y^0)^2 + (z-2)^2 = \frac{21}{4}$

So,

$(x_0, y_0, z_0) = $ centre

$= (\frac{-3}{2}, 0, 2)$

land Radius $a = \sqrt{\frac{21}{4}}$ //

Q2 Part b

$y = \sqrt{x} \quad 0 \le x \le 4$

Sol:

Given That $y = \sqrt{x}$.

$0 \le x \le 4 \Rightarrow a \le x \le b$.

As $V = \int_a^b \pi y^2 \, dx$.

$V = \int_0^4 \pi (\sqrt{x})^2 \, dx$.

$V = \pi \int_0^4 x \, dx = \pi \frac{x^2}{2} \Big|_0^4$

Ans $\Rightarrow V = \frac{\pi}{2} [(4)^2 - 0] = 8\pi$ //

11743 Anees Sher (4)

Q3

If $A = 2\hat{i} - 4j + \sqrt{5}k.$

$B = -2\hat{i} + 4j - \sqrt{5}k.$

Sol.

$B \cdot A = (-2j + 4j - \sqrt{5}k) \cdot (2\hat{i} - 4j + \sqrt{5}k)$

$B \cdot A = -4i - 16j - 5k$

$\boxed{B \cdot A = -25}$

$A \cdot A = (2\hat{i} - 4j + 5k) \cdot (2\hat{i} - 4j + \sqrt{5}k)$

$= 4 + 16 + 5$

$= \boxed{25}$

$Proj_A B = \left(\frac{B \cdot A}{A \cdot A}\right) A$

$= \left(\frac{-\cancel{25}\,1}{\cancel{25}}\right)(2\hat{i} - 4j + \sqrt{5}k)$

$= -1(2\hat{i} - 4j + \sqrt{5}k)$

$= \boxed{-2\hat{i} + 4j - \sqrt{5}k}$

//

11743 Anees Sher

Q4

Find the area of the region between the Graph & the x-axis.

$y = -x^2 + 5x - 4, \ [0, 2]$

Sol: $y = f(x) = -x^2 + 5x - 4$

$[a, b] = [0, 2]$

$$A = \int_a^b f(x)\,dx$$

$$A = \int_0^2 (-x^2 + 5x - 4)\,dx$$

$$A = \left(\frac{-x^3}{3} + \frac{5x^2}{2} - 4x - 0\right|_0^3$$

$$A = \frac{(3)^3}{3} + \frac{5(3)^2}{2} - 4(3) - 0$$

$$A = \frac{-\cancel{27}^{\,9}}{\cancel{3}} + \frac{5(9)}{2} - 12.$$

$$A = -9 + \frac{45}{2} - 12$$

$$A = -9 + 22.5 - 12.$$

$$A = 13.5 - 12$$

$$\boxed{A = 1.5}$$

11743 Anees Sher

Q5) a Estimate the angle between $A = i - 2j - 2k$ &

$B = 6i + 3j + 2k$

Sol:

$A = i - 2j - 2k$

$|A| = \sqrt{(1)^2 + (2)^2 + (2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$

$B = 6i + 3j + 2k$

$|B| = \sqrt{(6)^2 + (3)^2 + (2)^2}$

$|B| = \sqrt{36 + 9 + 4}$

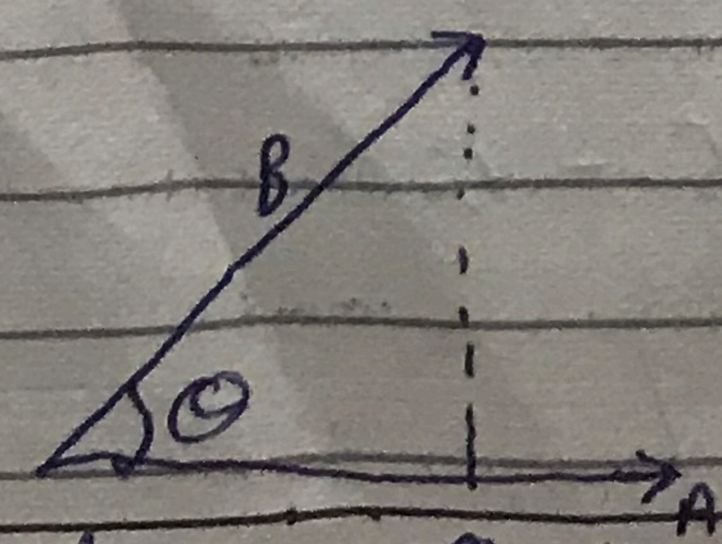
$|B| = \sqrt{49} = 7$

$\theta = \cos^{-1}\left(\dfrac{A \cdot B}{|A|\,|B|}\right)$

$\theta = \cos^{-1}\left(\dfrac{(i - 2j - 2k) \cdot (6i + 3j + 2k)}{3 \times 7}\right)$

$\theta = \cos^{-1}\left(\dfrac{(1)(6) + (-2)(3) + (-2)(2)}{21}\right)$

$\theta = \cos^{-1}\left(\dfrac{-4}{21}\right)$



Q 5 b part

Change into a spherical coordinate equation for the sphere $x^2+y^2+(z-1)^2=1$

Sol:

$$x^2+y^2+(z-1)^2=1$$

$$=(\rho\sin\phi\cos\theta)^2+(\rho\sin\phi\sin\theta)^2+(\rho\cos\phi-1)^2=1$$

$$=\rho^2\sin^2\phi\cos\theta^2+\rho^2\sin^2\phi\sin^2\theta+\rho^2\cos\phi^2+1-2\rho\cos\phi=1$$

$$=\rho^2\sin^2\phi(\cos\theta^2+\sin^2\theta)+\rho^2\cos\phi^2+1-2\rho\cos\theta=1-1$$

$$=\rho^2(\sin^2\phi)+\rho^2\cos\phi^2+2\rho\cos\phi$$

$$=\rho^2(\sin^2\phi+\cos^2\phi)-2\rho\cos\phi=0$$

$$\rho^2=2\rho\cos\phi$$

$$\rho=2\cos\phi$$