DATE

ASSIGNMENT:

DIFFERENTIAL EQUATION:
NO (3)
first after mid.

NAME:

MALIK AIMAL KHAN.

ID:

7968

SECTION:

B

SUBMITTED TO:

MAM SHUMAILA MAZHAR.

Q NO1 :-

$$x^3y''' + 2x^2y' + 2y = 10x + 10/x$$

SOLUTION:

$$x^3\frac{d^3y}{dx^3} + 2x^2\frac{dy}{dx} + 2y = 10x + 10x^{-1}$$

$$x^3D^3y + 2x^2D^2 + 2y = 10x + 10x^{-1}$$

let $x = e^t$

$t = \ln x$

$$xD = D$$

$$x^2D^2 = D(D-1) - D^2 - D$$

$$x^3D^3 = D(D-1)(D-2)$$

Now from this we get.

$$(D^3 - 3D^2 + 2D + 2(D^2 - D) + 2)y = 10x + \frac{10}{x}$$

$$(D^3 - 3D^2 + 2D + 2D^2 - 2D + 2)y = 10x + 10x^{-1}$$

$$(D^3 - D^2 + 2)y = 10x + 10x^{-1}$$

$D = m$

$x = e^t$

Putting in equation

$$(m^3 - m^2 + 2)y = 10e^t + \frac{10}{e^t}$$

By synthetic division.

-1	1	-1	0	2
		-1	2	-2
	1	-2	2	$\boxed{0}$

So we have

$$D^2 - 2D + 2 = 0$$

using Quadratic formula.

$a = 1 \quad b = -2 \quad c = 2$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-(-2) \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)}$$

$$D = \frac{2 \pm \sqrt{-4}}{2}$$

$$D = \frac{2 \pm \sqrt{-1}\,\sqrt{4}}{2}$$

$$D = \frac{2 \pm \sqrt{-1}\,(2)}{2}$$

$\sqrt{-1} = i$

$$D = \frac{2 \pm i\,(2)}{2}$$

$$D = \frac{2 \pm 2i}{2}$$

$$D = \frac{\cancel{2}(1+i)}{\cancel{2}}$$

$$D = 1 + i$$

Now Particular integral.

$$y_p = \frac{1}{D^3 - D^2 + 2} \cdot 10e^t + \frac{1}{D^3 - D^2 + 2} \cdot \frac{10}{e^t}$$

$$y_p = \frac{10e^t}{D^3 - D^2 + 2} + \frac{10/e^t}{D^3 - D^2 + 2}$$

DATE ....

5

$$y_p = \frac{10e^t}{(1)^3 - (1)^2 + 2} + \frac{10e^{-t}}{(1)^3 - (1)^2 + 2}$$

$$y_p = \frac{10e^t}{2} + \frac{10e^{-t}}{2}$$

$$y_p = 5e^t + 5e^{-t}$$

General solution.

$$y = e^{-x}(c_1 \cos t + c_2 \sin t) + 5e^t + 5e^{-t}$$

Put $e^t = x$ and $t = \ln x$

$$y = e^{-x}(c_1 \ln x + c_2 \sin x) + 5e^x + 5e^{-x}$$

$$y = e^{-x}(c_1 \ln x + c_2 \sin x) + 5e^x + \frac{5}{e^x}$$

Q no 2:

$$x^3 y''' + 4x^2 y'' - 5xy' - 15y = x^4$$

SOLUTION:

$$x^3 y''' + 4x^2 y'' - 5xy - 15y$$

$$y = x^m \qquad y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$y''' = m(m-1)(m-2)\,x^{m-3}$$

$$= x^3 m(m-1)(m-2)x^{m-3} + 4x^2 m(m-1)x^{m-2} - 5xmx^{m-1}$$
$$-15x^m = 0$$

$$= m(m-1)(m-2)x^m + 4m(m-1)x^m - 5mx^m - 15x^m = 0$$

$$x^m\left[[(m^2-m)](m-2) + 4(m^2-m) - 5m - 15\right] = 0$$

$$m^3 - 2m^2 - m^2 + 2m + 4m^2 - 4m - 5m - 15 = 0$$

$$m^3 + m^2 - 7m - 15 = 0$$

Putting values.

$$(-1)^3 + (-1)^2 - 7(-1) - 15 = 0$$

$$-1 + 1 + 7 - 15 = 0$$

$m = 2$

$8 + 4 - 14 - 15 = (-17)$

$m = -2$

$-8 + 4 + 14 - 15$

$-4 - 1 = -5$

$m = 3$

$m = -2 + i$

$m = -2 - i$

Q no 3: $y'' + 2/x\, y' - 6/x^2\, y = 10$

SOLUTION:

$f(x) = 10$

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} C_1 x^{-3} & C_2 x \\ -3C_1 x^{-4} & 2C_2 x \end{vmatrix}$$

$$= 2C_1 C_2 x^{-2} + 3C_1 C_2 x^{-2} = 5C_1 C_2 x^{-2}$$

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & C_2 x^2 \\ 10 & 2C_2 x \end{vmatrix} = -10 C_2 x^2$$

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} = \begin{vmatrix} C_1 x^{-3} & 0 \\ -3C_1 x^{-4} & 10 \end{vmatrix} = 10 C_1 x^3$$

DATE ....

$$u_1' = \frac{W_1}{W} = \frac{-10c_2x^2}{5c_1c_2x^{-2}} = \frac{-2}{c_1}x^4$$

$$u_2' = \frac{W_2}{W} = \frac{10c_1x^{-3}}{5c_1c_2x^{-2}} = \frac{2}{c_2}x^{-1}$$

So

$$u_1' = \frac{-2}{c_1}x^4$$

$$u_2' = \frac{2}{c_2}x^{-1}$$

For u_1' we have.

$$u_1 = \frac{-2}{c_1}\int x^4\,dx$$

$$u_1 = \frac{-2}{5c_1}x^5$$

$$u_2 = \frac{2}{c_2}\int \frac{1}{x}\,dx$$

$$u_2 = \frac{2}{c_2}\ln x$$

Now Particular integral.

$$y_p = \frac{-2}{5c_1} x^5 c_1 x^3 + \frac{2}{c_2} \ln x \, c_2 x^2$$

$$y_p = -2x^2 + 2\ln(x)\, x^2$$

$$y = c_1 x^{-3} + c_2 x^2 - 2x^2 + 2\ln x \,(x^2)$$

Q No 4:

$$x^2 y'' + 7xy' + 5y = x^5$$

$$y(0) = 2 \quad \text{and} \quad y'(1) = 2$$

Solution:

$$x^2 \frac{dy^2}{dx^2} + 7x \frac{dy}{dx} + 5y = x^5$$

$$\left(x^2 \frac{d^2}{x^2} + 7x \frac{dxy}{dx} \right) = x^5$$

Put $xD = D$

$$x^2 D^2 = D^2 - D$$

$$= (D^2 + D + 7D + 5)y = e^{5t}$$

$= (D^2 - D + 7D + 5)y = e^{5t}$

$= (D^2 + 6D + 5)y = e^{5t}.$

By quadratic formula.

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-6 \pm \sqrt{36 - 20}}{2}$$

$$D = \frac{-6 \pm \sqrt{16}}{2}$$

$$D = \frac{-6 \pm \sqrt{(4)^2}}{2}$$

$D = -3 \pm 2$

The following are the roots.

$y_c = C_1 e^{-5t} + C_2 e^{-t}$

As $y_p = \frac{1}{D^2 + 6D + 5} e^{5t}.$

$$y_p = \frac{1}{(5)^2 + 6(5) + 2} e^{5t}.$$

$$y_p = \frac{1}{60} \cdot e^{5t}.$$

General solution.

$y = y_c + y_p$

$$y = C_1 x^{-5} + C_2 x^{-1} + \frac{1}{60} e^{5t}.$$

$$y = C_1 x^{-5} + C_2 x^{-1} + \frac{1}{60} x^5$$

Now for $x = 0$

$\therefore e^0 = 1$

$y(0) = 2 \qquad y = 2$

$$2 = C_1(2)^{-5} + C_2(2)^{-1} + \frac{1}{60}(2)^5$$

$$2 = -32 C_1 - 2 C_2 + \frac{8}{15}$$

$$2 - \frac{8}{15} = -32C_1 - 2C_2$$

$$\frac{22}{15} = -3C_1 - 2C_2$$

Now differentiate the other equation $y'(1) = 2X = 2$

$$2 = -5C_1 C_2^{-6} - C_2(2)^{-2} + \frac{1}{12}(2)^4$$

$$2 = -5C_1(-64) - C_2(4) + \frac{16}{12}$$

DATE ....

$$2 = 320C_1 + 4C_2 + \frac{4}{3}$$

$$2 - \frac{4}{3} = 320C_1 + 4C_2$$

xliy eq with (2) and then integration it.

$$\frac{-44}{15} = 64$$

$$\frac{2}{3} = \pm 320C_1 + 4C_2$$

$$\frac{34}{15} = -256C_1$$

$$C_1 = \frac{34}{15} \times 256$$

$$C_1 = 580$$

Putting values of eq (c)

$$\frac{22}{15} = -32(580) - 2C_2$$

$$\frac{22}{15} + 18560 = -2C_2$$

$$C_2 = \frac{18561}{-2}$$

$$C_2 = -9280$$

Putting values of C_1 and C_2.

$$y = 580x^{-5} - 9280x^{-1} + \frac{1}{60}x^5$$

$$y = \frac{580}{x^5} - \frac{9280}{x} + \frac{1}{60}x^5$$

$$y = \frac{580}{x^5} - \frac{9280}{x} + \frac{1}{60}x^5 .$$

Q NO 5:

$$(x+1)^2 y'' - 3(x+1)y' + 4y = x^2$$

$$y = x^m \qquad y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

Solution:

$$y = \cancel{x}^m$$

$$y' = mx^{m-1} \qquad y'' = m(m-1)x^{m-2}$$

$$= (x^2+1+2x)\,m(m-1)x^{m-2} - 3(x+1)mx^{m-1} + 4x^m = 0$$

$$= x^2 m(m-1)x^{m-2} + m(m-1)x^{m-2} + 2x\,m(m-1)x^{m-2}$$

$$-3(xmx^{m-1} + mx^{m-1}) + 4x^m = 0$$

$$m(m-1)x^m - 3mx^m + 4x^m + 2m(m-1)x^{m-1} + m(m-1)x^{m-2} = 0$$

$$(m(m-1) - 3m + 4)x^m + (2m(m-1)x^{m-1}) + m(m-1)x^{m-2} = 0$$

DATE

$$m(m-1)-3m+4=0$$

$$m^2-m-3m+4=0$$

$$m^2-4m+4=0$$

$$a=1 \quad b=-4 \quad c=4$$

$$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$m=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(4)}}{2(1)}$$

$$m=\frac{4\pm\sqrt{0}}{2}=\frac{4}{2}=2$$

$m=2$ $\qquad y=$

$$y=C_1x^2+C_2x^2\ln x.$$

$$y_1=C_1x^2 \qquad y_2=C_2x^2\ln x$$

$$y''-\frac{3}{(x+1)}y'+\frac{4}{(x+1)^2}y=\frac{x^2}{(x+1)^2}=\left(\frac{x}{x+1}\right)^2$$

$$f(x)=\frac{x^2}{(x+1)^2}$$

$$W = \begin{vmatrix} 4x^2 & c_2 x^2 \ln x \\ 2c_1 x & c_2 x + c_2 2x \ln x \end{vmatrix}$$

$$W_1 = \begin{vmatrix} 0 & c_2 x^2 \ln x \\ \frac{x^2}{(x+1)^2} & c_2 x + c_2 2x \ln x \end{vmatrix}$$

$$W_1 = \frac{-x^2}{(x+1)^2} c_2 x^2 \ln x .$$

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix}$$

$$W_2 = \begin{vmatrix} c_1 x^2 & 0 \\ 2c_1 x & \frac{x^2}{(x+1)^2} \end{vmatrix}$$

$$W_2 = \frac{c_1 x^4}{(x+1)^2}$$

$$u_1' = \frac{W_1}{W}$$

$$u_2' = \frac{W_2}{W}$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$y = y_h + y_p$$