Subject: “Multivariate Calculus”

Name: “Mahab Waqar Khan”

ID#13093

Submitted to: Sir “Daud Khan”

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FINAL TERM

Q1: Find $\frac{\partial^2 z}{\partial x \partial y}$ and $\frac{\partial^2 z}{\partial y \partial x}$ for $z = \arcsin\left[\frac{x}{y}\right]$

$$\frac{\partial}{\partial x}\left(\frac{\partial x}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{-x}{y\sqrt{y^2-x^2}}\right)$$

$$= \frac{-1}{y}\frac{\partial}{\partial x}\left[\frac{+x}{\sqrt{y^2-x^2}}\right]$$

$$= \frac{-1}{y}\left[\frac{1-(\sqrt{y^2-x^2}) - x - \frac{1}{2}(y^2-x^2)^{-1/2} \cdot -2x}{(y^2-x^2)}\right]$$

$$= \frac{-1}{y}\frac{\left[\sqrt{y^2-x^2} + \frac{x^2}{\sqrt{y^2-x^2}}\right]}{y^2-x^2}$$

$$= \frac{-1}{y}\left[\frac{\cancel{y^2}-\cancel{x^2}+\cancel{x^2}}{(y^2-x^2)(y^2-x^2)^{1/2}}\right]$$

$$= \frac{-y}{(y^2-x^2)^{3/2}}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\sin^{-1}\frac{x}{y}\right)\right)$$

$$\frac{\partial}{\partial y}\left(\frac{1}{\sqrt{y^2-x^2}}\right)$$

$$= \frac{0 \cdot \sqrt{y^2-x^2} - 1 \cdot \frac{1}{\cancel{2}}(y^2-x^2)^{1/2} \cdot \cancel{2}y}{(\sqrt{y^2-x^2})^2}$$

$$= \frac{-y}{(y^2-x^2)(y^2-x^2)^{1/2}}$$

$$\frac{-y}{(y^2-x^2)^{3/2}}$$

Q2: $f(x,y) = e^x \sin y + e^y \cos x$

Laplace Equation $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$ — (a)

$f = e^x \sin y + e^y \cos x$

$$\frac{\partial f}{\partial x} = e^x \sin y + e^y(-\sin x)$$
$$= e^x \sin y - e^y \sin x$$

$$\frac{\partial^2 f}{\partial x^2} = e^x \sin y - e^y \cos x$$

$$\frac{\partial f}{\partial y} = e^x \cos y + e^y \cos x$$
$$\frac{\partial^2 f}{\partial y^2} = e^x(-\sin y) + e^y \cos x$$
$$= -e^x \sin y + e^y \cos x$$

Now putting values in (a)

$$(e^x \sin y - e^y \cos x) + (-e^x \sin y + e^y \cos x) = 0$$

$$e^x \cancel{\sin y} - e^y \cancel{\cos x} - e^x \cancel{\sin y} + e^y \cancel{\cos x} = 0$$

Hence satisfied!

Q3) $f(x,y) = x^3 e^{-y} + y\sec x$

$$\frac{\partial f}{\partial x} = 3x^2 e^{-y} + y\sec x\tan x$$

$$\frac{\partial f}{\partial y} = x^3(-1)e^{-y} + \sec x$$

$$= -x^3 e^{-y} + \sec x$$

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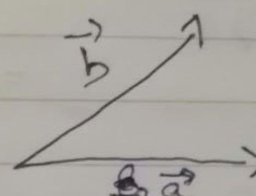
4) $b = 6i + 3i + 2k$ onto $a = \cancel{a}i - 2j - 2k$

$$\text{Proj}_{\vec{a}}\,\vec{b} = \left(\frac{\vec{b}\cdot\vec{a}}{\|\vec{a}^2\|^2}\right)\vec{a}$$



$$= \frac{(6\hat{i} + 3\hat{j} + 2k)\cdot(\hat{i} - 2\hat{j} - 2\hat{k}}{\left(\sqrt{1^2 + (-2)^2 + (-2)^2}\right)^{\cancel{2}}} \cdot (\hat{i} - 2\hat{j} - 2\hat{k})$$

$$= \frac{\cancel{6} - \cancel{6} - 4}{(1 + 4 + 4)} \cdot (\hat{i} - 2\hat{j} - 2\hat{k})$$

$$= \frac{-4}{9}(\hat{i} - 2\hat{j} - 2\hat{k})$$

Q5

a) $f(x,y) = xe^y + \cos(xy)$ at point $(2,0)$

$\vec{a} = 3\hat{i} - 4\hat{j}$

The partial derivation of f at point $(2,0)$ is

$$\frac{\partial f}{\partial x}(x,y) = e^y + (-\sin(xy).y)$$

$$= e^y - y\sin(xy)$$

$$\frac{\partial f}{\partial x}(2,0) = e^0 - 0\sin(2.0)$$

$$= 1$$

$$\frac{\partial f}{\partial y}(x,y) = xe^y + (-\sin(xy).y)$$

$$= xe^y - x\sin xy$$

$$\frac{\partial f}{\partial y}(2,0) = 2e^0 - 2\sin(2.0)$$

$$= 2 - 2.0$$

$$= 2$$

Therefore gradient is

$$\nabla f(2,0) = 1\hat{i} + 2\hat{j} = (1,2) \quad \text{—} \quad (i)$$

The directional derivative at $(2,0)$ in direction of a is

Q5 2/2

$D_u f(2,0) = \nabla f(2,0) \cdot a$ — (ii)

$a = \frac{3\hat{i} - 4\hat{i}}{\sqrt{3^2+(-4)^2}} = \frac{3i-4j}{\sqrt{9+16}} = \frac{3i-4j}{\sqrt{25}}$

$= \frac{3i-4j}{5}$

putting values in (iii)

$D_u f(2,0) = (\hat{i} + 2\hat{j})\left(\frac{3i-4j}{5}\right) = \frac{3-8}{5}$

$= \frac{-5}{5} = -1.$

Q:6: $f(x,y,z) = x^2+y^2+z^2-14$ and point
$(1,-2,3)$

$$f = x^2+y^2+z^2-14$$

$$\vec{n} = \nabla f(1,-2,3) = (f_x, f_y, f_z)$$

$f_x = 2x$ and $f_x = 2(1) = 2$

$f_y = 2y$ and $f_y = 2(-2) = -4$

$f_z = 2z$ and $f_z = 2(3) = 6$

So required equation of tangent

$$f_x(z-z_0)+f_y(y-y_0)+f_z(z-z_0) = 0$$

$$2(x-1)+(-4)(y-(-2))+6(z-3) = 0$$

$$2(x-1)-4(y+2)+6(z-3) = 0$$

$$2x-2-4y-8+6z-18 = 0$$

$$2x-4y+6z-28 = 0$$

$$2x-4y+6z = 28$$

Q7. Evaluate the double Integral

$$= \int_0^1 \int_0^1 xy + y^2 \, dx\,dy$$

$$= \int_0^1 \left[\left(\frac{x^2 y}{2} \Big|_0^1 \right) + y^2 x \Big|_0^1 \right] dy$$

$$= \int_0^1 \left[\left(\frac{1^2 . y^2}{2} - \cancel{\frac{0^2 . y^2}{2}} \right) + \left[y^2(1) - y^2(0) \right] \right] dy$$

$$= \int_0^1 \frac{y^2}{2} + y^2 \, dy$$

$$= \frac{y^3}{2.3} \Big|_0^1 + \frac{y^3}{3} \Big|_0^1$$

$$= \frac{y^3}{6} \Big|_0^1 + \frac{y^3}{3} \Big|_0^1$$

$$= \left[\frac{1^3}{6} - \frac{0^3}{6} \right] + \left[\frac{1}{3} - \frac{0}{3} \right]$$

$$= \frac{1}{6} + \frac{1}{3} = \frac{1+2}{6} = \frac{3}{6} = \frac{1}{2}$$