Final Term Assignment
Intro to dynamic structural

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7736

Section - B

Where "k" is lb/ft and "m" is in lb sec/ft^2

General Solution to the EOM for undamped free vibration is,

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$u(0) = 1/2''$, $\frac{1}{2\times12} = \frac{1}{24}$ ft and $\dot{u}(0) = 0$

$$u(t) = \left(\frac{1}{24}\right) \times \cos(19.48t) + 0$$

Equivalent Static force at any time "t" is

$$f_s(t) = k.u(t) = \frac{90625 \times \cos(19.48t)}{24}$$

$$f_s(t) = 3776\,(19.48t)$$

Amplitude of dynamic displacement, u_0 for ~~undar~~ undamped free vibration is

$$u_0 = \sqrt{\left[(u(0))^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2\right]} = \sqrt{\left(\frac{1}{24}\right)^2 + 0} \qquad \dot{u}(0) = 0$$

$$\boxed{u_0 = \frac{1}{24}\text{ ft}}$$

Amplitude of equivalent Static force, f_{s0}

$$ku_0 = 90625 \times \frac{1}{24} = 3776\text{ lb}$$

QNO(2):-

PROBLEM:-

GIVEN DATA:-

$E = 29000 \text{ ksi}$

$I = 150 \text{ in}^4$

$\delta_{st} = 7695 \text{ lb}$

Take $\zeta = 2.5\%$

Solution:-

E.O.M for damped free vibration;

$ku + c\dot{u} + m\ddot{u} = 0 \longrightarrow (1)$

it is known from problem NO.1 that;

$k = 90625 \text{ lb/ft}$ and

$m = 238.97 \text{ lb.sec}^2/\text{ft}$

$c = \zeta \times 2 m \omega_n$

$c = 0.025 \times 2(238.97)(20.04)$

$c = 0.025 \times 9577.9176$

$c = 239.447 \text{ lb.sec/ft}$

$\Rightarrow$

By subsitituting values of K, c and m in eq(1):

$$ku + c\dot{u} + m\ddot{u} = 0$$

$$90625u + 229.447\dot{u} + 238.97\ddot{u} = 0$$

Solution to the E.O.M for damped free vibration is:

$$u(t) = e^{-\zeta w_n t}\left[u(0)\cos(w_D t) + \frac{1}{w_D}\left[u(0)\zeta w_n\right]. \sin(w_D t)\right.$$

$$\Rightarrow w_D = \sqrt{\frac{k}{m}} = \sqrt{\frac{90625}{238.97}}$$

$$w_D = 19.47 \text{ rad/sec}$$

$$\rightarrow u(t) = e^{-0.025\times 19.47t}\left[\frac{1}{24}\times\cos(19.47t) + \frac{1}{19.47}\times\left[0+\frac{1}{24}\times 0.025\times 19.47 - \times\sin(19.47t)\right]\right.$$

$$u(t) = e^{-0.48675}\left[0.041667\times\cos(19.47t) + 0.05136\times 0.02028\sin(19.47t)\right)$$

$$u(t) = e^{-0.48675}\left[0.041667\times\cos(19.47t) + 0.001041\sin(19.47t)\right)$$

$$\Rightarrow f_s(t) = K.u(t) = 90625\times u(t)$$

$f_s(t) =$ [redacted] $e^{-0.48675}\left[8776\cos(19.47) + 94.34\sin(19.47t\right.$

Given Data:

Force = 60 kips

Displacement of tank = $\left(\frac{10}{1000}\right)''$ = $\left(\frac{7683}{1000}\right)''$ = 7.683''

cycles = 7

Time taken to complete 7 cycles = 3.57 sec

amplitude of displacement = 2.286 cm
= 0.9''

Required Data:.

(a) Damping ratio

(b) Natural period of un-damped vibration

(c) stiffness of structures

(d) weight of tank.

(e) Damping co-efficient

(f) Number of cycles to reduce the displacement amplitude to 0.5''.

Solution:-

→ Displacement of tank, u_1 = 7.683''

→ After 7 cycles, ~~i.e After j=~~

i.e, After j=7, $u_{j+1} = u_8 = 0.9''$

(2)

(a) Damping ratio = ζ = ?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{1+1}}\right]$$

$$7 = \frac{1}{2\pi\zeta} \ln\left[\frac{7.683}{0.9}\right]$$

$$\zeta = 0.0488 = 4.88\%$$

(b) Natural Period of undamped vibration = Tn = ?

As, the 7 cycles of vibrations are completed in 3.57sec

$\Rightarrow$ Time required to complete one cycle, $T_D = \frac{3.57}{7} = 0.51 sec$

$$T_D = 0.51 \text{ sec}$$

Now,

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{(\omega_n\sqrt{1-\zeta^2})}$$

$$\Rightarrow T_D = \frac{T_n}{(1-\zeta)^2}$$

$\Rightarrow T_n = T_D \times \sqrt{1-\zeta^2}$

$\Rightarrow T_n = 0.51 \times \sqrt{1-(0.0488)^2}$

$T_n = 0.5094 = 0.51$ sec

$\boxed{T_n = 0.51 \text{ sec}}$

(c) Stiffness of structure, k = ? :-

$$k = \frac{60 \times \cos 60^\circ}{7.683} = 3.91 \text{ k/in}$$

$\boxed{k = 3.91 \text{ k/in} = 46920 \text{ lb/ft}}$

(d) weight of tank, W = ?

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{\frac{W}{g}}} = \sqrt{\frac{k \cdot g}{W}}$$

$$\Rightarrow \omega_n^2 = \frac{k \times g}{W}$$

$$W = \frac{k \times g}{\omega_n^2}$$

(4)

Also, $\omega_n = \frac{2\pi}{T_n}$

$$W = \frac{kg}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$= kg \times \frac{T_n^2}{4\pi^2}$$

~~$$W = \frac{\frac{46920\,lb}{ft} \times \frac{32.2\,ft}{sec^2} \times (0.51\,sec)^2}{4\pi^2}$$~~

$$W = \left[\frac{46920\,lb}{ft} \times \frac{32.2\,ft}{sec^2}\right] \times \frac{(0.51\,sec)^2}{4\pi^2}$$

~~$= [1510824]$~~

$= 9953.93\,lb = 9.95k$

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(c) Damping co-efficient, c = ?

It is known that,

$$\zeta = \frac{c}{2m\omega_n}$$

$$\Rightarrow c = \zeta \times 2m\omega_n$$

$$= \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$\Rightarrow c = \frac{(0.0488) \times 4 \times \pi \times \left(\frac{9953.93}{32.2}\right)}{0.51}$$

$$c = 371.71 \text{ lb.sec/ft.}$$

(b) Number of cycles to reduce the displacement amplitude to 0.5', J = ?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$\Rightarrow j = \frac{1}{2\pi \times 0.0488} \ln\left[\frac{7.683}{0.5}\right]$$

$$j = 8.91 \text{ or } 9 \text{ cycles}$$