Muhammad Johar Shah

ID: 14124. (SE-07)

Differential Equation.

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Page 1 (D.E)

Q: What is differential Eq?

Ans:

A differential Equation is an equation with a function and one or more of its derivatives

e.g $y = \frac{dy}{dx} = 5x$

$\frac{dy}{dx}$ → differential (derivative)

$5x$ → equation.

Example :- 1

$$\frac{dy}{dx} = x^2 - 3$$

$$\Rightarrow y = \int (x^2 - 3)\, dx$$

$$\Rightarrow y = \frac{x^3}{3} - 3x + k$$

Example 2 :-

$$\theta^2 d\theta = \sin(t + 0.2)\, dt$$

$$\int \theta^2 d\theta = \int \sin(t + 0.2)\, dt$$

$$\frac{\theta^3}{3} = -\cos(t + 0.2) + k$$

Page #2 (D.E)

Q b:- Define Separable Differential Equation?

Ans:-

A first order differential Equation is called separable differential eq (e.g) - $y' = f(x,y)$ if function $f(x,y)$ can be factored into product of two function

Page # 3 (D·E)

Q 1 Intial value product:-

b(i)

$$y' = \frac{xy^3}{\sqrt{1+x^2}} \quad y(0) = -1.$$

$$y^{-3}\,dy = x(1+x^2)^{-\frac{1}{2}}\,dx$$

$$\int y^{-3}\,dy = \int x(1+x^2)^{-\frac{1}{2}}\,dx$$

$$-\frac{1}{2y^2} = \sqrt{1+x^2} + C$$

$$\cancel{-\frac{1}{2y^2} = \sqrt{1+C} \quad , \quad C = \frac{3}{2}}$$

$$-\frac{1}{2} = \sqrt{1} + C \quad , \quad C = -\frac{3}{2}$$

$$-\frac{1}{2y^2} = \sqrt{1+x^2} - \frac{3}{2}$$

Now solve $y(x)$

$$\frac{1}{y^2} = 3 - 2\sqrt{1+x^2}$$

$$y^2 = \frac{1}{3-2\sqrt{1+x^2}}$$

$$y(x) = \pm\frac{1}{\sqrt{3-2\sqrt{1+x^2}}}$$

$$\Rightarrow \quad y(x) = -\frac{1}{\sqrt{3-2\sqrt{1+x^2}}}$$

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Page # 4 (D·E)

Since inner root will be a problem

$$1 + x^2 \geq 0$$

$$3 - 2\sqrt{1 + x^2} > 0$$

$$3 > 2\sqrt{1 + x^2}$$

$$9 > 4(1 + x^2)$$

$$\frac{9}{4} > 1 + x^2$$

$$\frac{5}{4} > x^2$$

So

$$-\frac{\sqrt{5}}{2} < x \quad \text{OR} \quad \frac{\sqrt{5}}{2} > x$$

ANSWER

Q1 b(ii)

$$\frac{dx}{dt} = \frac{t}{x}$$

$$dx \cdot x = dt \cdot t$$

$$\int x\,dx = \int t\,dt$$

$$\frac{1}{2}x^2 + c = \frac{1}{2} \times t^2 + c$$

By xing 2 on b.s

$$\cancel{(2)}\frac{1}{\cancel{(2)}}x^2 + (2)c = \cancel{(2)}\frac{1}{\cancel{(2)}}t^2 + 2c.$$

$$x^2 + c_1 = t^2 + c_1$$

$$x^2 = t^2 + c_1 - c_1$$

$$\sqrt{x^2} = \sqrt{t^2 + c}$$

$$\boxed{x = \sqrt{t^2 + c}}$$

Ans

Pg # 6 = (D.E)

Q2 Explain step of solving linear diff
A eq.

Ans Steps:-
- i) Put the diff eq in correct intial form.

ii) Find the integrating factor, $\mu(t)$,

iii) Multiply everything in the diff eq by that integrating factor $[\mu(t)]$ and verify that the left side becomes the product rule $(\mu(t)y(t))'$ and write it as such.

iv) Integrate b.s, make sure you properly deal with the constant of ~~de~~ integration.

v) Solve the solution $y(t)$.

Pg # 7 (D.E)

Q2 (i) $\cos(x) y' + \sin(x) y = 2\cos^3(x)\sin(x) - 1 \quad 0 \le x < \frac{\pi}{2}$

$y\left(\frac{\pi}{4}\right) = 3\sqrt{2}$

$$y' + \frac{\sin(x)}{\cos(x)} y = 2\cos^2(x)\sin(x) - \frac{1}{\cos(x)}$$

$$y' + \tan(x) y = 2\cos^2(x)\sin(x) - \sec(x)$$

$$\mu(t) = e^{\int \tan(x)\,dx} = e^{\ln \sec(x)} = \sec(x)$$

$$\int \tan(x)\,dx = -\ln|\cos(x)| = \ln|\cos(x)|^{-1} = \ln|\sec(x)|$$

$$e^{\ln f(x)} = f(x)$$

$$\sec(x) y' + \sec(x)\tan(x) y = 2\sec(x)\cos^2(x)\sin(x) - \sec^2(x).$$

$$(\sec(x) y)' = 2\cos(x)\sin(x) - \sec^2 x$$

$$\int (\sec(x) y(x))'\,dx = \int 2\cos x \sin x - \sec^2 x\,dx.$$

$$\sec(x) y(x) = \int \sin(2x) - \sec^2(x)\,dx.$$

$$\sec(x) y(x) = -\frac{1}{2}\cos(2x) - \tan(x) + C$$

$$y(x) = -\frac{1}{2}\cos(x)\cos(2x) - \cos(x)\tan(x) + c\cos(x)$$

$$= -\frac{1}{2}\cos x \cos 2x - \sin x + c\cos x.$$

Pq #8 (D.E)

$$3\sqrt{2} = y\left(\frac{\pi}{4}\right) = -\frac{1}{2}\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{4}\right) + c\cos\left(\frac{\pi}{4}\right)$$

$$3\sqrt{2} = \frac{\sqrt{2}}{2} + c\frac{\sqrt{2}}{2}$$

$$c = 7$$

$$\boxed{y(x) = -\frac{1}{2}\cos(x)\cos(2x) + 7\cos(x)}$$

Ans

Pg # 9 (D.E).

Q2 $x' + 2x = \sin(t)$

(ii)

$$x'(t) + 2x(t) = \sin(t)$$

first order linear diff eq.

$$x'(t) = \sin(t) - 2x(t)$$

$$x'(t) + 2x(t) = \frac{1}{2} ie^{-it} - \frac{1}{2} i.e^{it}$$

$$\cancel{x'(t) + 2x(t) = \frac{1}{2} ie}$$

$$x(t) = c_1 e^{-2t} + \frac{2\sin(t)}{5} - \frac{\cos(t)}{5}$$

Ans

Pg # 10 (D.E)

Q3 (i) $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0, \; y(0) = -3$

$M = 2xy - 9x^2 \quad M_y = 2x$

$N = 2y + x^2 + 1 \quad N_x = 2x$

$\psi_x = M \Rightarrow \psi = \int M\,dx$

$\psi_y = N \quad \psi = \int N\,dy.$

$\psi(x,y) = \int 2xy - 9x^2\,dx = x^2y - 3x^2 + h(y)$

$\psi_y = x^2 + h'(y) = 2y + x^2 + 1 = N$

$h'(y) = 2y + 1$

$h(y) = \int 2y + 1\,dy = y^2 + y + k$

$\psi(x,y) = x^2y - 3x^3 + y^2 + y + y + k = y^2 + (x^2+1)y - 3x^3 + k$

$y^2 + (x^2+1)y - 3x^3 + k = C$

$y^2 + (x^2+1)y - 3x^3 = C - k$

$y^2 + (x^2+1)y - 3x^3 = C$

$(-3)^2 + (0+1)(-3) - 3(0)^3 = C \Rightarrow C = 6$

Pg #11 (D, E)

$$y^2+(x^2+1)y-3x^3-6=0$$

$$y(x)=\frac{-(x^2+1)\pm\sqrt{(x^2+1)^2-4(1)(-3x^3-6)}}{2(1)}$$

$$=\frac{-(x^2+1)\pm\sqrt{x^4+12x^3+2x^2+25}}{2}$$

$$-3=y(0)=\frac{-1\pm\sqrt{25}}{2}=\frac{-1\pm 5}{2}=3,2$$

$$y(x)=\frac{-(x^2+1)-\sqrt{x^4+12x^3+2x^2+25}}{2}$$

$$x^4+12x^3+2x^2+25=0$$

Q3 iii)

$$\frac{2ty}{t^2+1} - 2t - (2 - \ln(t^2+1))y' = 0 \quad y(5)=0$$

$$\frac{2ty}{t^2+1} - 2t + (\ln(t^2+1) - 2)y' = 0$$

$$M = \frac{2ty}{t^2+1} - 2t \qquad M_y = \frac{2t}{t^2+1}$$

$$N = \ln(t^2+1) - 2 \qquad N_t \frac{2t}{t^2+1}$$

$$\psi_y = \ln(t^2+1) + h'(y) = \ln(t^2+1) - 2 = N$$

$$h'(y) = -2 \Rightarrow h(y) = -2y.$$

$$\psi(t,y) = y\ln(t^2+1) - t^2 - 2y$$

$$y\ln(t^2+1) - t^2 - 2y = c$$

$$-25 = c$$

$$y(\ln(t^2+1) - 2) - t^2 = -25$$

$$y(t) = \frac{t^2 - 25}{\ln(t^2+1) - 2}$$

$$\ln(t^2+1) - 2 = 0$$

$$\ln(t^2+1) = 2$$

$$t^2 + 1 = e^2$$

$$t = \pm\sqrt{e^2 - 1}$$

$$-\infty < t < -\sqrt{e^2-1}$$

$$-\sqrt{e^2-1} < t < \sqrt{e^2-1}$$

$$e^2 - 1 < t < \infty$$

Ans