Name: Rizwan Khan
ID: 17015
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Submitted to Sir Latif Jan
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Question 1 part (a)

Define 2nd order homogenous / non homogeneous differential equation along with example.

Answer:

The ~~ ~~ nonhomogenous differential equation of this type has the form

$$y'' + py' + qy = f(x)$$

When p, q are constant (real or complex) for each equation we can write the relation homogenous. or complementry equation.

Example:

$$y'' + y = 0$$

$y(0) = 3.0$
$y'(0) = -0.5$

Solution:

The function $\cos x$, $\sin x$ and solution of ODES

~~$y = c_1 \sin x + c_2 \cos x$~~ $y = c_1 \cos x + 2 \sin x$

This will turn out to be general solution as defined below.

we need derivative $y' = c_1 \sin x$

From this and initial values are

$$y(0) = c_1 = 3.0$$
$$y'(0) = c_2 = -0.5$$

$$y = 3.0 \cos x - 0.5 \sin x$$

$$y\ c_1 \cos x + c_2 (k \cos x) = c \cos x.$$

where

$c = c_1 + c_2 k$ Answer.

Question 1 (b)

(i) $4(y'') - 6y' + 7y = 0$

Solution:

$4y'' - 6y' + 7y = 0$

Finding root

$4\lambda^2 - 6\lambda + 7 = 0$

$$\lambda = \frac{6 \pm \sqrt{(-6)^2 - 4(4)(7)}}{2(4)}$$

$$\lambda = \frac{6 \pm \sqrt{36 - 112}}{8}$$

$$\lambda = \frac{6}{8} + \frac{2\sqrt{19}\, i}{8}$$

$$\lambda_1 = \frac{3}{4} + \frac{\sqrt{19}\, i}{4}, \quad \lambda_2 = \frac{3}{4} - \frac{\sqrt{19}}{4} i$$

So it has ~~the~~ the complex conjugate roots.

$y_1(x) = e^{\lambda_1 x} \cos \lambda'(x)$

$y_2(x) = e^{\lambda_2 x} \sin \lambda_2 x$

$$y = c_1 e^{3/4x} \cos \frac{\sqrt{19}}{4}(x) + e^{3/4 x} \sin \frac{\sqrt{19}}{4} i(x)$$

Answer.

(ii)

? No Answer.

$y'' - 4(y') - 12y = 3e^{5x}$ X.

Q2 (i) $16y'' - 40y' + 25y = 0$ $\quad y(0) = 3 \quad y'(0) = -9/4$

Solution:

$16y'' - 40y' + 25y$

Characteristic Equation is

$16y^2 - 40y + 25 = ((4y)^2 - 5)^2 = 0$

$y_1 = 5/4, y_2 = 5/4$

General solution:

$y(t) = c_1 e^{5t/4} + c_2 e^{5t/4}$

$y'(t) = 5/4 \, c_1 e^{5t/4} + + 5/4 \, c_2 t e^{5t/4}$

Put it in initial equation

$3 = y(0) = c_1$

$-9/4 = y'(0) = 5/4 \quad c_1 + c_2$

Solution of IVP will be

$y^t = 3e^{5t/4} - 6te^{5t/4}$ Ans.

Q2 Part(ii)

$y'' + 14y' + 49y = 0$ $\quad y(-4) = 1 \quad y'(-4) = 5$

Solution:

$y^2 - 4y - 12 = (y-6)(y+2) = 0 \quad y_1 = (-2),\ y_2 = 6$

General Solution:

$y_c(t) = C_1 e^{-2t} + Ce^{6t}$

$y_c(t) = C_1 e^{-2t} + Ce^{6t}$

Now

$y_c(t) = C_1 e^{-2t} + Ce^{6t}$

and

$y_p(t) = Ae^{5t}$

Putting

$\Rightarrow 25Ae^{5t} - 4(5Ae^{5t}) - 12(Ae^{5t}) - 3e^{5t}$

$-7Ae^{5t} = 3e^{5t}$

$-7A = 3 \Rightarrow A = -\frac{3}{7}$

$y_p(t) = -\frac{3}{7} e^{5t}$ Ans.

Q2 P(iii)

$y'' - 4y' + 9y = 0 \qquad y(0) = 0 \quad y'(0) = -8$

Solution:

$r^2 - 4r + 9 = 0$

$y(t) = C_1 e^{2t} \cos(\sqrt{5}t) + C_2 e^{2t} \sin\sqrt{5}t$

$0 = (y(0) = C_1$

$y(t) = C_2 e^{2t} \sin\sqrt{5}t$

$y(t) = 2C_2 e^{2t} \sin(\sqrt{5}t) + \sqrt{5} C_2 e^{2t} \cos(\sqrt{5}t)$

$-8 = y^{\circ}(0) = \sqrt{5} C_2 \Rightarrow C_2 = \frac{-8}{\sqrt{5}}$

$y(t) = \frac{-8}{\sqrt{5}} e^{2t} \sin(\sqrt{5}t)$

Ans.

Q2

Question 2 e (iv)

$y'' - 8y' + 17y = 0 \qquad y(0) = 4 \quad y'(0) = 1$

Solution:

Let

$r^2 - 8r + 17 = 0$

$y(t) = C_1 e^{4t} \cos(t) + C_2 e^{4t} \sin t$

$y'(t) = 4C_1 e^{4t} \cos t - e^{4t} \sin t)$
$+ 4(C_2 e^{4t} \sin t) + C_2 e^{4t} \cos t$

Initial value

$-4 = y(0) - C_1$

$-1 = C y^2(0) = 4C_1 + C_2$

$y(t) = -4 e^{4t} \cos(t) + 15 e^{4t} \sin t$

Ans.

Question no. 3 (Part

Laplace Transform:

Laplace tranform is an integral transform that converts a function of real variable t (often time) to to a function of complex variable s. The transform has many application in Science and engineering because it is a tool for solving diff equation.

In particular it performs a transform differential equation into Algebric equations and convolution into multiplication.

Example:

Let $f(t) = 1$ when $t \geq 0$, find $F(s)$

Solution:

By integration.

$$\mathcal{L}(t) = \mathcal{L}(1) = \int_0^\infty e^{-st}\,dt$$

$$= -\frac{1}{s} e^{-st} \Big|_0^\infty = \frac{1}{s}$$

Question 3 part A (i)

$f(t) = 6(e^{-5t}) + e^{3t+5} + 5t^3 - 9$

Solution:

$f(t) = 6(e^{-5t}) + e^{3t+5} + 5t^3 - 9$

By Laplace tranform

$$f(t) = \frac{6}{s-(-5)} + \frac{1}{s-3} + 5\frac{3!}{s^{3+1}} - 9\frac{1}{s}$$

$$\Rightarrow \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}$$

(ii)

$g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

Solution:

$g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

By using laplace formulas

$$g(t) = 4\left(\frac{s}{s^2+4^2}\right) - 9\left(\frac{4}{s^2+4^2}\right) + 2\left(\frac{s}{s^2+10^2}\right)$$

$$= \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}$$

Ans.

(iii) $h(t) = e^{3t} + \cos(6t) - e^{3t}\cos(6t)$

Solution:

$h(t) = e^{3t} + \cos(6t) - e^{3t}\cos(6t)$

Laplace formula.

$$h(t) = \frac{1}{s-3} + \frac{s}{s^2+6^2} - \frac{s-3}{(s-3)^2+(6)^2}$$

$$= \frac{1}{s-3} + \frac{s}{s^2+36} - \frac{s-3}{(s-3)^2+36} \text{ Ans}$$

Question 4
Part (i)

Solve the following Laplace transform

$$y'' - 10y' + 9y = 5t \qquad y(0) = -1$$
$$y'(0) = 2$$

Solution:

$$y'' - 10y' + 9y = 5t$$

$$\mathcal{L}\{y''\} - 10\mathcal{L}\{y'\} + 9\{y\} = \mathcal{L}\{5t\}.$$

$$s^2 y(s) - s y(0) - y^2 - 10\{s y(s) - y(0) + 9y(s)\} = \frac{5}{s^2}$$

$$(s^2 - 10s + 9)\, y(s) + s - 12 = \frac{5}{s^2}$$

$$y(s) = \frac{5}{s^2 (s-9)(s-1)} + \frac{12-s}{(s-9)(s-1)}$$

$$y(s) = \frac{5 + 12s^2 - s^2}{s^2 (s-9)(s-1)}$$

$$y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-9} + \frac{D}{s-1}$$

By partial fraction
we get:

$$y(s) = \frac{\frac{50}{81}}{8} + \frac{\frac{5}{9}}{s^2} + \frac{\frac{31}{81}}{s-9} - \frac{2}{s-1}$$

$$y(t) = \frac{50}{81} + \frac{5}{9}t + \frac{31}{81}e^{9t} - 2e^{t}$$ Ans.

Part (ii)

$$y'' + 6y' + 15y = 2\sin(3t) \qquad y(0) = -1$$
$$y'(0) = -4$$

Solution:

$$y'' - 6y' + 15y = 2\sin(3t) \qquad y(0) = -1$$
$$y'(0) = -4$$

$$\mathcal{L}\{y''\} - 6\mathcal{L}\{y'\} + 15\{y\} = 2\sin\mathcal{L}\{3t\}$$

$$= s^2y(s) - sy(0) - y'(0) - 6(sy(s) - y(0)$$
$$+ 15y(3) = 2\frac{3}{s^2+9}$$

$$y(s) = \frac{-s^3 + 2s^2 - 9s + 24}{(s^2+9)(s^2-6s+15}$$

By partial fraction:

$$y(s) = \frac{As + B}{s^2 + 9} + \frac{Cs + D}{s^2 - 6s + 15}$$

$= s^3 + 2s^2 - 9s + 24 = (As + B)(s^2 - 6s + 15) + (Cs + D)(s^2 + 9)$

$= (A + C)s^3 + (-6A + B + D)s^2 + (15A - 6B + 9C)s + 15B + 9D$

$s^3 = A + C = 1$ $\qquad A = 1/10$

$s^2 = -6A + B + D = 2$ $\qquad B = 1/10$

$s^1 = 15A - 6B + 9C = 9$ $\qquad C = -11/10$

$s = 15B + 9D = 24$ $\qquad D = 5/2$

we get.

$$y(t) = \frac{1}{10}\left(\cos(3t) + \frac{1}{3}\sin(3t)\right.$$
$$\left. - 11e^{3t}\cos\sqrt{6}t - \frac{8}{\sqrt{6}} e^{3t}\sin\sqrt{6}t\right)$$

Ans.