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Section: A

Subject:- Advanced Fluid Mechanics

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Question No.1:

part (a):
Define Drag with its components. Write down the equations for friction Drag Coefficient both in laminar and turbulant boundary layer.

Answer:
FORCES ON IMMERSED BODIES: A body which is wholly immersed in a homogenous fluid may be subjected to two kind of forces arising from relative motion between body & fluid. These forces are termed as drag & lift, depending on forces either parallel or right angle to motion.

Drag force on submersed body can have 2 components.

(1) Pressure Drag (Fp):- It is equal to the integration of component in the direction of motion of all of forces exerted on surface of body.

$$F_p = C_p \cdot \frac{\rho v^2}{2} \cdot A$$

where C_p depend on shape

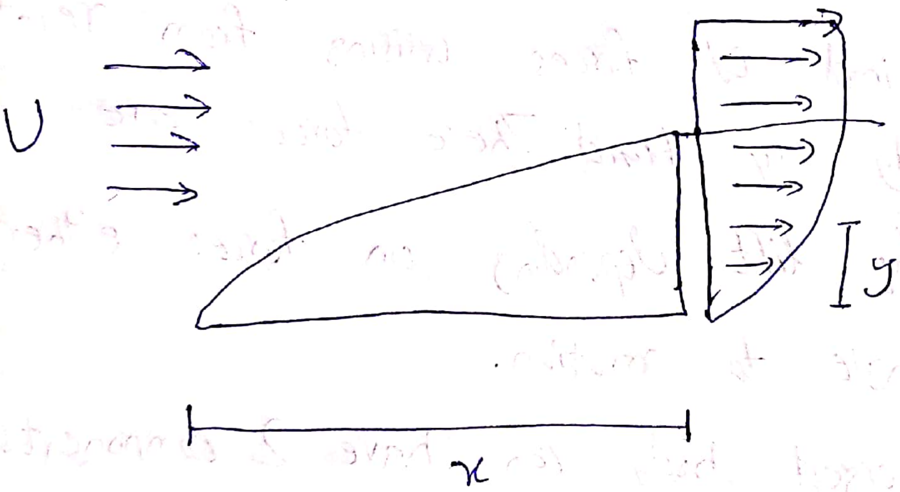
(2) Friction Drag:-

It is equal to the integration of component of all shear stresses along the surface in the direction of motion.

$$F_f = C_f \cdot \rho \cdot \frac{v^2}{2} (BL)$$

"C_f" depends on ~~velocity~~ viscosity

Friction Drag on Boundary layer:-



$$\tau_o = \frac{\mu U \, df(\eta)}{\delta \, d(\eta)}$$

$$\tau_o = \frac{\mu U B}{\delta} \quad \text{—— (x)}$$

As we have $Z_o = \rho v^2 \times \frac{d\delta}{du}$

Compare with

$$\rho U^2 \alpha \frac{d\delta}{dx} = \frac{\mu U B}{\delta}$$

$$\delta \, d\delta = \frac{\mu B \, dx}{\rho U \alpha}$$

Integrating on b.s

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} . x + C \qquad \because C = 0$$

$$\delta = \sqrt{\frac{2B}{\alpha}} . \sqrt{\frac{\mu u}{\rho U}}$$

$$B = 1.63 \, , \quad \alpha = 0.135$$

$$\delta = \frac{4.91 \, x}{\sqrt{R_x}} \quad \text{——— (6)}$$

where (R_n) is local Reynold number.

As we have,

$$\tau_0 = \frac{\mu U B}{\delta}$$

$$F_x = \rho B u^2 \delta \alpha$$

where α is a function at boundary layer velocity distribution.

Now to find shear stresses.

$$\tau = \frac{F_x}{A} = \frac{dF_x}{B\,dx} = \frac{dF_x}{B\,dx}$$

$$\bar{\tau}_0 = \frac{\rho B u^2 \alpha \, d\delta}{B\,dx} = \frac{\rho u^2 \alpha \, d\delta}{dx}$$

LAMINAR BOUNDARY LAYER:-

In case of laminar flow,

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0} = \frac{\mu}{\delta}\left(\frac{du}{dx}\right)$$

$$= \frac{\mu U}{\delta}\left[\frac{df(h)}{dh}\right]$$

By solving,

$$\tau_0 = \frac{\mu U \beta}{\delta} \quad — (1)$$

Equating $\Rightarrow$ $\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$

$$\delta\, d\delta = \frac{\mu \beta}{\rho U \alpha}\, dx$$

Solving it

$$\frac{\delta^2}{2} = \frac{\mu\beta}{\rho U \alpha} x + C$$

At $x = 0,\ \delta = 0,\ C = 0$

$$\delta = \sqrt{\frac{2\mu\beta x}{\rho U \alpha}} = \sqrt{\frac{2\beta}{\alpha}} \cdot \frac{x}{\sqrt{R_x}}$$

Experimentally $\beta = 1.63,\ \alpha = 0.135,$

Putting values in (i)

$$\frac{\delta}{x} = \sqrt{\frac{2 \times 1.63}{0.135}} \times \frac{1}{\sqrt{R_x}} = \frac{4.91}{\sqrt{R_x}}$$

Where R_x may be called the local Reynold number.

It should be noted that R_x increases linearly in down stream direction.

Now,

$$F_f = B \int_0^2 \tau_0 \, dx \Rightarrow$$

$$\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

$$R_x = \frac{x U \rho}{\mu}$$

Thus. $F_f = 0.664\, B \sqrt{\rho \mu L U^3}$

where, $F_f = C_f \rho \frac{V^2}{2} BL$

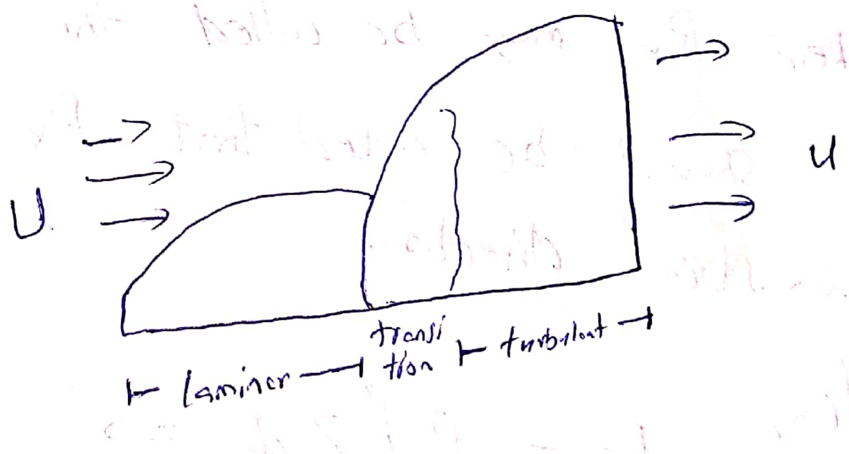
equating both

$$C_f = 1.328 \sqrt{\frac{\mu}{\rho L U}} = \frac{1.328}{\sqrt{R}}$$

For laminar $R \leq 500{,}000$.

Turbulent Boundary Layer:

This fig. show the velocity distribution of boundary layer which is steeper near walls & flatter through out layer.



The shear stress is greater in turbulent than in lam

Thus, $\tau_0 = \rho f \frac{v^2}{8}$

Where v is the average velocity.

To obtain relation b/w average & max. we have.

$$\frac{V}{U_{max}} = \frac{1}{1+1.33\sqrt{f}} \qquad \because f = 0.028$$

$$\frac{V}{U_{max}} = \frac{1}{1+1.33\sqrt{0.028}}$$

$$U = 1.235\ V$$

$$\boxed{V = \frac{U}{1.235}}$$

Eq

$$f = \frac{0.316}{(R_h)^{3/4}}$$

$$\tau_0 = f \rho \frac{V^2}{8}$$

$$\tau_0 = \frac{0.316}{\left((P/\nu)\left(\frac{U}{1.235}\right)\right)^{1/4}} \cdot \frac{1}{8}\left(\frac{U}{1.238}\right)^2$$

$$\tau_0 = \frac{0.23\ \rho v^2}{\left(\frac{2\delta}{\nu}\right)^{1/4}}$$

As we have general Eq.

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$

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Equation (1) & (2)

$x = 0, \ \delta = 0$

$$\boxed{\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\frac{v}{Ux}\right)^{1/5} . x}$$

$d = 0.0972$

$$\boxed{\delta = \frac{0.377}{(R_n)^{1/5}} . x} \quad —(3)$$

$$\boxed{Z_o = 0.0587) \frac{v2}{2} \left(\frac{v}{Ux}\right)^{1/5}}$$

Now

$$F_f = B \int_0^L Z_o \, dx$$

$$\boxed{F_f = 0.0735 \, \rho \frac{v^2}{2} \left(\frac{v}{UL}\right)^{1/5} . BL}$$

$$F_f = C_f . \rho . \frac{v^2}{2} B.L$$

$$C_f = \frac{0.0735}{(R)^{1/5}}$$

For $R > 10^7$

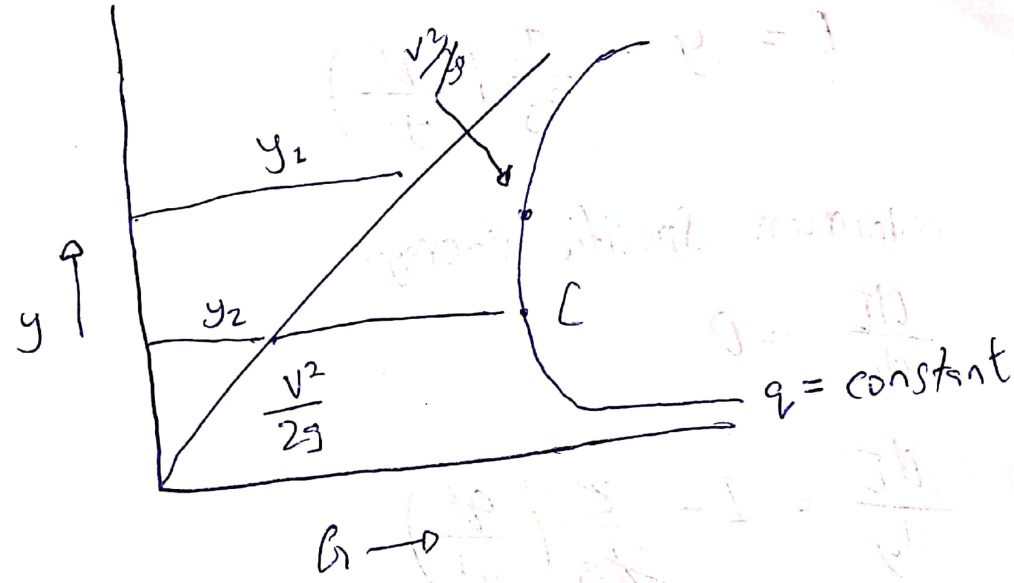
$$\boxed{C_f = \frac{0.455}{(\log F)^{2.57}}}$$

Question No.1

Part (b):

Derive equation for critical depth, critical velocity of rectangular section of a channel.

Answer:



This is specific energy equation.(4)

For particular q, there will be two kind of possib value of y for given E. The eq. is cubic with three roots with third being negative giving no values. This two alternative depths represents two totally different flow regimes - slow & deep on upper portion & fast & shallow on lower portion.

Point represent dividing point between two regions of flow. Thus for given "q", value of E is minimum & flow at this point is critical flow. Depth of flow at this point is critical depth y_c & velocity at this point is critical velocity.

Thus relation of critical depth can be found as

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

For minimum specific energy.

$$\frac{dE}{dy} = 0$$

$$\frac{dE}{dy} = 1 - \frac{2}{2g}\left(\frac{q^2}{y^3}\right)$$

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3}$$

$$1 = \frac{q^2}{gy^3} \Rightarrow q^2 = gy^3$$

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3} \quad \text{critical depth}$$

As $q = vy$, $v_c^2 = gy^3$

OR $\boxed{V_c = \sqrt{gy_c}}$ critical velocity

$$y_c = \frac{V_c^2}{g}$$

Now.

$$\frac{y_c}{2} = \frac{V_c^2}{2g}$$

$$E_{min} \Rightarrow y_c + \frac{V_c^2}{2g} = y_c + \frac{y_c}{2}$$

$$\frac{3}{2} y_c \quad \text{OR} \quad y_{cr} = \frac{2}{3}$$

	Subcritical	Critical	Supercritical
Depth of flow	$y > y_c$	$y = y_c$	$y < y_c$
Velocity Slop	$V < V_c$ mild slope $S_0 < S_c$	$V = V_c$ Critical slope	$V > V_c$

ID # 7820

Question No. 2

Given:

Depth of Rectangular Channel $(d) = ?$

Flow rate $(Q) = 3.5\ m^3/sec$

Slope of bed $(S_o) = 0.0008$

$n = 0.0219$

width of Bed $= 7820\ mm$

$= 7.820$

Critical depth $= ?$

Flow Subcritical or Supercritical $= ?$

Solution:

Area $= 7.820 \times d$

$= 7.820d$

Perimeter $= d + 7.820 + d$

$= 7.820 + 2d$

Hydraulic Radius $(R_h) = A/P$

$$= \frac{7.820\,d}{7.820 + 2d}$$

By using Manning Equation.

$$Q = \frac{1}{n} A R_h^{2/3} \cdot S_o^{1/2}$$

Putting values.

$$3.5 = \frac{1}{0.0129} \times 7.820d \times \left(\frac{7.820d}{2d + 7.820}\right)^{2/3} + (0.0008)^{\frac{1}{2}}$$

$$\boxed{d = 0.558\,m}$$

$$\text{Area} = 7.820\,(0.558)$$

$$\boxed{= 4.36\ m^2}$$

$$\text{Perimeter} = 7.820 + 2\,(0.558)$$

$$\boxed{= 8.94\ m}$$

$$\text{Hydraulic Radius } (R_h) = \frac{4.36}{8.94}$$

$$\boxed{= 0.487\ m}$$

Finding Critical depth:-

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

As, $q = Q/B$

$$= \frac{3.5}{7.820}$$

$$= 0.447 \text{ m}^2/\text{sec}$$

$$\Rightarrow \quad y_{cr} = \left(\frac{(0.447)^2}{9.81}\right)^{1/3}$$

= [redacted]

$$= 0.281$$

As $y > y_{cr}$

$$0.5580 > 0.281$$

So Flow is sub-critical.

Question No.3

Data:

Friction Drag $(F_D) = ?$

Width $(B) = 200\,mm = 0.2\,m$

Length $(L) = 800\,mm = 0.8\,mm$

Specific Gravity $(S) = 0.89$

Undisturbed velocity $(V) = 5\,m/sec$

Kinematic viscosity $(\nu) = 0.93 \times 10^{-4}\,m^2/sec$

Solution:

Checking whether flow is Laminar or not By Reynold Number,

$$R = \frac{DV}{\nu}$$

For Smooth flat plate.

$$D = L \quad , \quad V = U$$

So

$$R = \frac{LU}{\nu}$$

$$= \frac{0.8 \times 5}{0.93 \times 10^{-4}} = 43010$$

$43010 \leq 500{,}000 \rightarrow$ Laminar

By using formula;

$$F_f = C_f \cdot \rho \cdot \frac{V^2}{2} \cdot BL$$

where $C_f = \frac{1.328}{\sqrt{R}} = \frac{1.328}{\sqrt{43010}} = 0.0064$

$$S = \frac{\rho_{oil}}{\rho_{water}} \Rightarrow 0.89 = \frac{\rho_{oil}}{1000}$$

$$\rho_{oil} = 0.89 \times 1000$$

$$\rho_{oil} = 890 \ kg/m^3$$

$$\Rightarrow F_f = C_f \cdot \rho \cdot \frac{V^2}{2} \cdot BL$$

$$= 0.0064 \times 890 \times \frac{(5)^2}{2} \times 0.2 \times 0.8$$

$$\boxed{F_f = 11.39 \ N}$$