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Subject : Advanced fluid Mechanics

Submitted to

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30/9/020.

Question No 1

Ans (a)

Drag: A body which is wholly immersed in a homogenous fluid may be subjected to two kind of forces arising from relative motion between body and fluid. These forces are termed as drag and lift. If the forces parallel to the motion then it is termed as drag force.

There are two components.

(1) → **Pressure Drag (F_p)**

It is equal to integration of components in direction of motion of all pressure forces exerted on surface of body.

$$F_p = C_p \rho \frac{v^2}{2} A$$

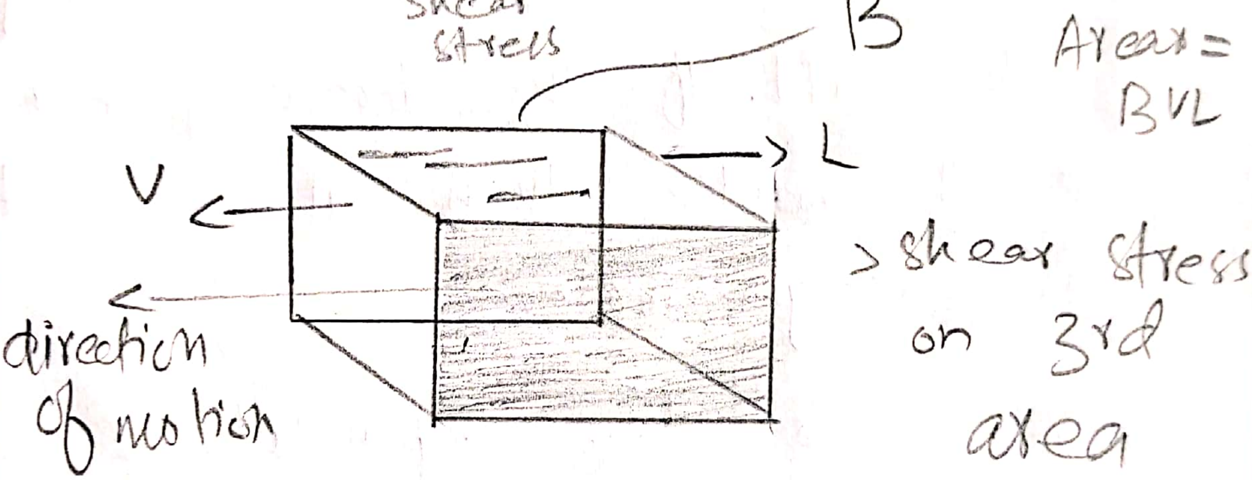
where C_p depends on shape

(2) Friction Drag :- (F_f)

It is equal to integration of components of shear stress along surface of body in direction of motion

$$F_f = \underbrace{C_f \, \rho \, \frac{V^2}{2}}_{\text{Shear stress}} BL$$

fig



→ Friction Drag of Boundry layer

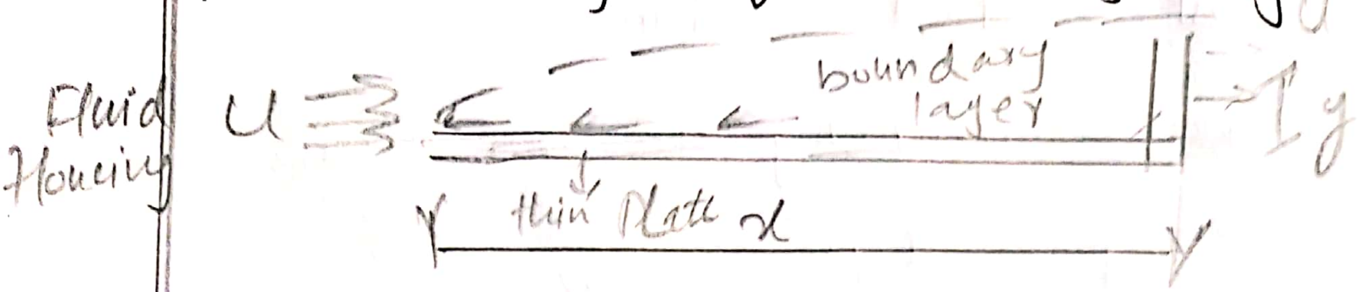
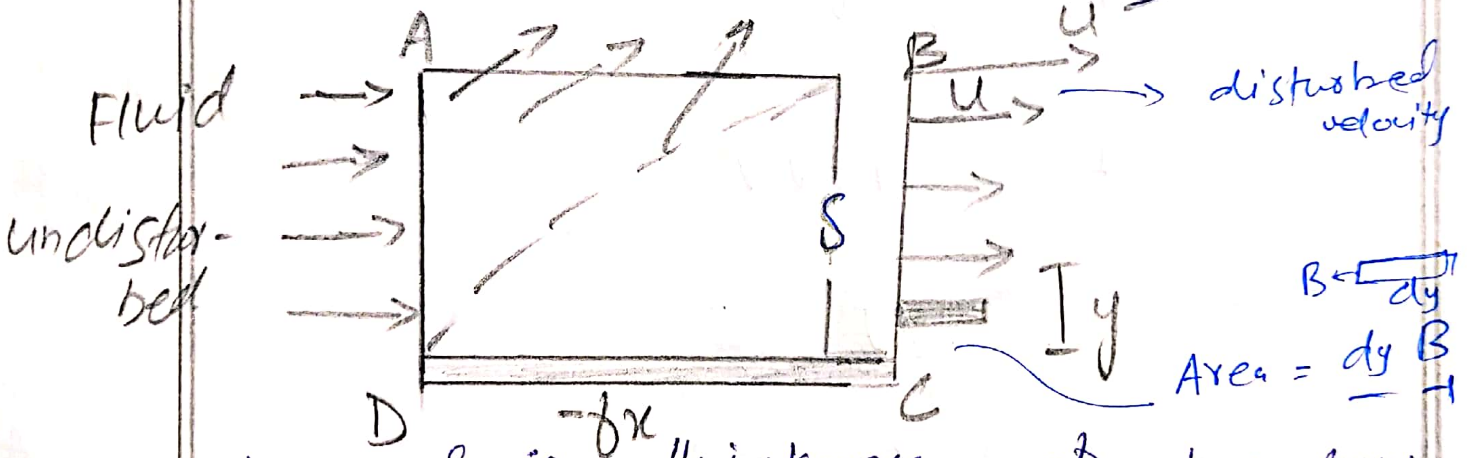


Fig shows growth of boundry layer along one site of smooth plate inside the fluid.

Now conside a contool volume



where δ is thickness of boundary layer and U is undisturbed velocity.

Thus $-F_x$ = drag = (rate in momentum in x-direction)

leaving through BC + rate of momentum through AB) - rate of momentum entering through DA)

$$\Delta P = P_{out} - P_{in}$$

Thus accourding to momentum

$$\Sigma F = \frac{d(P)}{dt} = \frac{dmv}{dt}$$

where $\frac{dm}{dt} = \rho Q$ thus

$$F = \rho Q V$$

or

$$F = \rho A . v . V$$

$$F = \rho A V^2$$

$DA \rightarrow \rho U (UB\delta)$

$BC \rightarrow \rho B \int_0^\delta u^2 . dy$

$AB \rightarrow \rho U \left(UB\delta - B \int_0^\delta u . dy \right)$

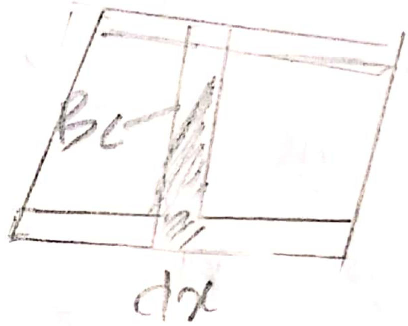
Putting value.

$$F_X = \rho B \int_0^\delta u (U - u) \, dy$$

Solving it $F_x = \rho B u^2 \delta \alpha$ where α is function of boundary layer

Now to find local wall shear stress

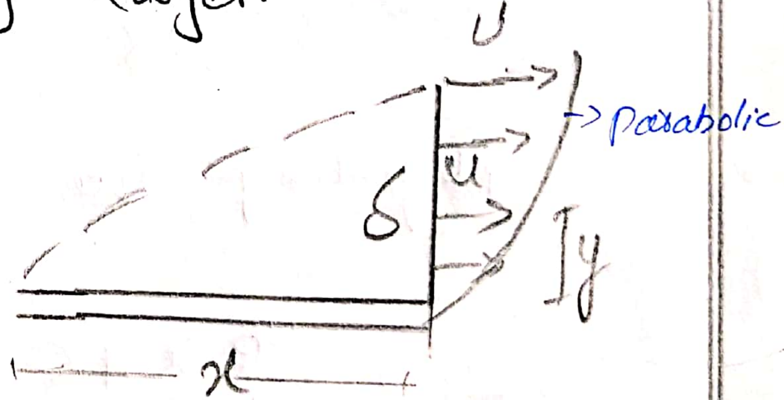
$$\tau_0 = \frac{dF_x}{B \cdot dx} \text{ — area}$$



$$F_x = \rho B u^2 \delta \alpha$$

$$\tau_0 = \rho u^2 \alpha \frac{d\delta}{dx}$$ in general equation of shear stress

→ Laminar boundry layer :-



$$\frac{u}{U} = F\left(\frac{y}{\delta}\right)$$

Assume

$$\eta = \frac{y}{\delta} \text{ or } y = \eta\delta$$

thus $\frac{u}{U} = f(\eta)$ or $u = Uf(\eta)$

In case of laminar flow

$$\tau_0 = \mu\left(\frac{du}{dy}\right)$$

$$= \frac{\mu}{\delta}\left(\frac{du}{dn}\right) = \frac{\mu U}{\delta}\left[\frac{df(n)}{dn}\right]$$

Solving the equation

$$\tau_0 = \frac{\mu U \beta}{\delta} \quad \text{— (1)}$$

As general equation is $\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$

Equating both equation

$$\frac{\mu U \beta}{\delta} = \rho U^2 \alpha \frac{d\delta}{dx}$$

or

$$\delta \, d\delta = \frac{\mu \beta}{\rho U \alpha} dx$$

Integrating the equation

$$\frac{\delta^2}{2} = \frac{\mu \beta}{\rho U \alpha} x + C$$

Now at $x = 0$, $\delta = 0$ Thus $C = 0$

$$\frac{\delta^2}{2} = \frac{\mu \beta}{\rho U \alpha} x$$

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or

$$\delta = \sqrt{\frac{2\mu B}{\rho U \alpha}\, x} \quad \text{or} \quad \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}}$$

$\times$ing and $\div$ing by "x"

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}} \cdot \frac{x}{\sqrt{x} \cdot \sqrt{x}}$$

where $\alpha = 0.135$

$B = 1.63$

$$R_x = \frac{\rho U x}{\mu}$$

$\cancel{\delta = \sqrt{\frac{2B}{\alpha}}}$ $$\delta = \frac{4.91}{\sqrt{R_x}} \cdot x \quad \text{or} \quad \frac{\delta}{x} = \frac{4.91}{\sqrt{R_x}}$$

Now

$$\tau_0 = \frac{\mu U B}{\delta}$$

thus putting value

$$\boxed{\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}}$$

where R_x is local Reynold number

Now

$$F_D = B \int_0^L \underbrace{\tau_0\, dx}_{\text{stress}}$$

putting values

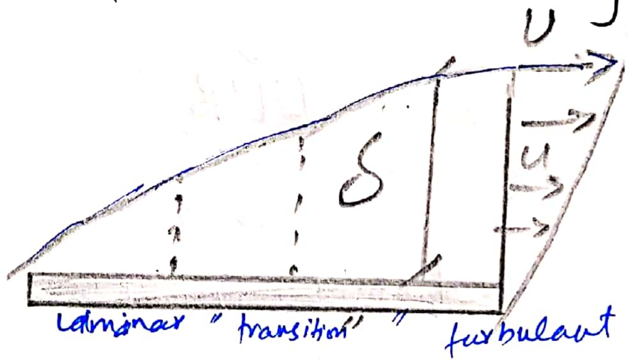
$$F_D = 0.664 B \sqrt{\mu U L U^3}$$

As general equation is

$$F_D = C_f \frac{\rho U^2}{2} BL$$ → equating both equation

$$C_f = 1.328 \sqrt{\frac{\mu}{\rho L U}} = \frac{1.328}{\sqrt{R}}$$

Turbulant boundary layer



resistance is less so curve become straight

fig show that velocity distribution in turbulant boundary layer shows a much steeper gradient near wall and flatter through out reamining layer.

The shear stress is greater in turbulant than in laminar layer. As we have

$$\tau_0 = f \frac{\rho v^2}{8}$$

wher v denotes average velocity of pipe

now we have obtained an approximate relation between V and U by using pipe factor equation of

$$\frac{V}{U_{max}} = \frac{1}{1+1.33\sqrt{f}}$$

Using friction factor of 0.028 from chart which is middle critical value

so

$$\boxed{U = 1.235V}$$

Now we have

$$\tau_0 = f \rho \frac{v^2}{8}$$

As we know

$$f = \frac{0.316}{R^{0.25}}$$

thus $$\tau_0 = \frac{0.316}{\left(\frac{DV}{\nu}\right)^{1/4}} \quad \frac{\rho V^2}{8}$$

where $V = \frac{U}{1.235}$ thus

$$\tau_0 = \frac{0.316}{\left(\frac{D}{\nu}\left(\frac{U}{1.235}\right)\right)^{1/4}} \cdot \frac{\rho}{8}\left(\frac{U}{1.235}\right)^2$$

& $D = 2\delta$

thus

$$\tau_0 = \frac{0.023\,\rho U^2}{\left(\frac{\delta U}{\nu}\right)^{1/4}}$$

As we have

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$

Equating both and integrating For boundary condition of

$x = 0 \quad , \delta = 0$

thus

$$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\frac{\nu}{Ux}\right)^{1/5} x$$

For $\alpha = 0.0972$

$$\boxed{\frac{\delta}{x} = \frac{0.377}{(R_x)^{1/5}}}$$

Putting values in equation

$$\tau_0 = 0.0587 \, \rho \frac{U^2}{2} \left(\frac{\nu}{Ux}\right)^{1/5}$$

Now $F_f = B \int_0^L \tau_0 \, dx$

$$F_f = 0.0735 \, \rho \, \frac{U^2}{2} \left(\frac{\nu}{UL}\right) BL$$

As $F_f = c_f \, \rho \frac{U^2}{2} BL$

equating both

$$\boxed{c_f = \frac{0.0735}{R^{1/5}}}$$

R is less than 10^7 for $500{,}000 < R < 10^7$

For $R > 10^7$

$$\boxed{C_f = \frac{0.455}{(\log R)^{2.58}}}$$

(b) Ans

Question no 1

As specific energy, $E = y + \frac{V^2}{2g}$

The flow Q per unit width b can be expressed as

$$q = \frac{Q}{b}$$

now average velocity will be

$$V = Q/A = \frac{qb}{by} = q/y$$

Thus

$$E = y + \frac{V^2}{2g} \Rightarrow y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

$$(E-y) = \frac{1}{2g}\left(\frac{q^2}{y^2}\right) \text{ or } (E-y)y^2 = \frac{q^2}{2g}$$

Thus plot of E vs y will be parabolic. For particular q, there will be two kind of possible value of y, for a given E

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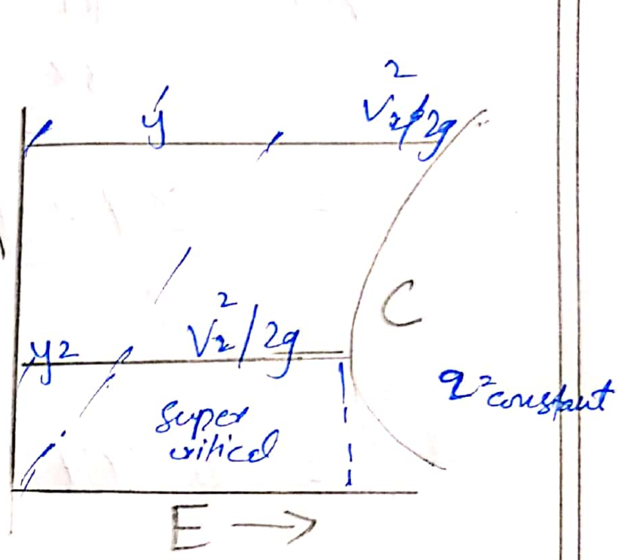
(2)

The equation is cubic with three roots, with third root being negative Point C represent dividing point between two regime of flow thus for given q_1 E_1 value of E is minimum and flow at that point is critical flow depth of flow at that point is critical ~~vecl~~ velocity V_c.



Thus

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

For minimum specific energy

$$\frac{dE}{dy} = 0$$

Thus

$$\frac{dE}{dy} = 1 - \frac{\cancel{2}}{\cancel{2}g}\left(\frac{q^2}{y^3}\right) = 0$$

$$\Rightarrow \frac{q^2}{gy^3} = 1 \Rightarrow q^2 = gy^3$$

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$$\frac{q^2}{g} = y^3 \Rightarrow \boxed{\left(\frac{q^2}{g}\right)^{1/3} = y_{cr}}$$

Now

$$q^2 = gy^3$$

&

$$q = Vy \Rightarrow V^2y^2 = gy^3$$

or

$$V^2 = gy_{cr}$$

or

$$V_{cr} = \sqrt{gy_{cr}}$$

Problem No 2

Given data :

Water flows at rate, $Q = 3.5 m^3/s$

Bed slope, $S_o = 0.0008$

$n = 0.0219$

width of bed in student ID = 7769 mm

Required :

Depth of rectangular channel = ?

Critical depth, $y_c = ?$

Critical velocity, $V_c = ?$

Flow is critical or sub-critical = ?

Sol :

Manning Equation

$$Q = \left(\frac{1}{n} R_n^{2/3} S_o^{1/2} \right) A \quad \text{—} ①$$

Area $= 7.769 \times d$

Parameter $= d + 7.769 + d$

Hydraulic Radius $R_n = \dfrac{\text{Area}}{\text{Parameter}}$

$$= \frac{7.769 d}{2d + 7.769}$$

Put value in eq ①

$$Q = \left(\frac{1}{n} R_n^{2/3} S_o^{1/2}\right) A$$

$$3.5 = \frac{1}{0.0219} \times \left(\frac{7.769d}{2d * 7.769}\right)^{2/3} \times (0.0008)^{1/2} \times 7.769d$$

$$\frac{3.5 \times 0.0219}{(0.0008)^{1/2}} = \left(\frac{7.769d}{2d + 7.769}\right)^{2/3} \times 7.769d$$

$$\left(\frac{3.5 \times 0.0219}{\sqrt{0.0008}}\right)^{3/2} = \frac{60.35d^2}{2d + 7.769}$$

$$4.461(2d + 7.769) = 60.35d^2$$

$$8.92d + 34.65 = 60.35d^2$$

$$60.35d^2 - 8.92d - 34.65 = 0$$

$$\boxed{d = 0.835}$$

So the depth of channel is ~~0.835m~~ 0.835m

Now As q = discharge per unit width $q = \frac{Q}{b}$

$$= \frac{3.5}{7.769}$$

$$q = 0.450$$

=> Critical Depth y_{cr}

Using equation

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

$$= \left(\frac{(0.450)^2}{9.81}\right)^{1/3}$$

$$y_{cr} = 0.274\,m$$

=> Critical Velocity, V_{cr}

$V_{cr} = \sqrt{g y_{cr}}$

$V_{cr} = \sqrt{(9.81)(0.274)}$

$V_{cr} = 1.63 \, m/s$

$V = Q/A = \dfrac{3.5}{7.769 \times 0.835}$

$V = 0.539 \, m/s$

$y = 0.835$

$y_{cr} = 0.274 m$

$V_{cr} = 1.63 \, m/s$

As $y > y_{cr}$

and

$V < V_{cr}$

So flow is Sub critical flow

Problem NO 3

Given data ::

Width of smooth plate,

$B = 200mm = 0.2m$

Length of smooth plate, $L = 800mm$
$= 0.8m$

Oil with specific gravity = $S = 0.89$

Undisturbed velocity, $U = 5 m/s$

Kinematic viscosity, $\nu = 0.93 \times 10^{-4} m^2/s$

Required data ::

Fraction drag on one side of a smooth plate, $F_f = ?$

Sol :: Check the flow

As $\nu = 0.93 \times 10^{-4} m^2/s$

$$R = \frac{LU}{\nu} = \frac{(0.8)(5)}{0.93 \times 10^{-4}}$$

$$R = 43010.75 < 500,000$$

thus Flow is laminar

Now

$$C_f = \frac{1.328}{\sqrt{R}} \Rightarrow \frac{1.328}{\sqrt{43010.75}}$$

$$C_f = 6.403 \times 10^{-3}$$

$$C_f = 0.0064$$

$$\Rightarrow F_f = C_f \, \rho \, \frac{V^2}{2} \, BL$$

$$= (0.0064)\,(\text{Soil} \times \gamma_{\text{water}}) \times \frac{(5)^2}{2} \times (0.2)(0.8)$$

$$= \left((0.0064)\,(0.89 \times 1000) \times \frac{5^2}{2} \times (0.2)(0.8)\right)$$

$$\boxed{F_f = 11.392\,N}$$