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Question No. 1 :-

$$\frac{dy}{dt} = e^{y-t} \sec(y)(1+t^2) \qquad y(0)=(0)$$

By seperating variable

$$\int e^{-y} \cos y \, dy = \int (1+t^2) e^{-t} \, dt$$

Integrating by part (i)

$u = e^{-y} \quad dv = \cos y \, dy$

$du = -e^{-y} dy \quad v = \sin y$

Integrating by parts (ii)

$u = e^{-y} \quad dv = \sin y \, dy$

$du = -e^{-y} dy \quad v = -\cos y$

By integrating 1^{st} part

$$L.H.S = e^{-y} \sin y + \int e^{-y} \sin y \, dy$$

By integrating part (2)

$$= e^{-y} \sin y - e^{-y} \cos y - \int e^{-y} \cos y \, dy$$

Since The last integral is same as L.H.S

$$L.H.S = e^{-y}(\sin y - \cos y)$$

by adding L.H.S

$$2(L.H.S) = e^{-y}(\sin y - \cos y)$$

÷ing by 2

$$L.H.S = \frac{e^{-y}}{2}(\sin y - \cos y)$$

Integrating Part (3)

$$u = 1+t^2 \quad dv = e^{-t}dt$$

$$du = 2t\,dt \quad v = e^{-t}$$

Integrating by Part (4)

$$U = 2t \quad dv = e^{-t}dt$$

$$dU = 2dt \quad v = -e^{-t}$$

Let us evaluate R.H.S

By integrating Part (3)

$$R.H.S = -(1+t^2)e^{-t} + \int 2te^{-t}dt$$

By integrating Part (4)

$$= -(1+t^2) - 2te^{-t} + \int 2e^{-t}\,dt$$

$$= -(t^2+2t+1)e^{-t} - 2e^{-t} + C$$

$$= -(t^2+2t+3)e^{-t} + C$$

Comparing L-H-S and R-H-S

$$\frac{e^{-y}}{2}(\sin y - \cos y) = -(t^2+2t+3)e^{-t} + C$$

Since $y(0) = 0$ we have

$$\tfrac{1}{2}(0-1) = -3 + C$$

$$C = 5/2$$

Hence the solution is implicity expressed as

$$\frac{e^{-y}}{2}(\sin y - \cos y) = -(t^2+2t+3)e^{-t} + 5/2$$

Question No 2 ::

$$(\sqrt{x+y}+\sqrt{x-y})\,dx-(\sqrt{x+y}-\sqrt{x-y})\,dy=0$$

$$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}\longrightarrow (1)$$

This is Homogeneous differential eq in x and y to Solve this put

$$y=vx$$

$$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$$

Thus eq (1) becomes

$$v+x\frac{dv}{dx}=\frac{\sqrt{x+vx}+\sqrt{x-vx}}{\sqrt{x+vx}-\sqrt{x-vx}}$$

$$v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

$$v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}\times\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}+\sqrt{1-v}}$$

$$v+x\frac{dv}{dx}=\frac{\cancel{2}(1+\sqrt{1-v^2})}{\cancel{2}v}$$

$$v+\frac{x\,dv}{dx}=\frac{1+\sqrt{1-v^2}}{v}$$

$$\frac{x\,dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$\frac{x\,dv}{dx} = \frac{1+\sqrt{1-v^2}-v^2}{v}$$

$$\frac{x\,dv}{dx} = \frac{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{-v^2}\right)} = \frac{dx}{x}$$

taking Integrals on b/s

$$\int \frac{x\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \int \frac{dx}{x}$$

put $1+\sqrt{1-v^2} = t$

$$\Rightarrow \tfrac{1}{2}\left(1-v^2\right)^{-1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int -\frac{dt}{t} = \int \frac{dx}{x}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln\left(1+\sqrt{1-v^2}\right) = \ln cx$$

$$\ln\left(1+\sqrt{1-v^2}\right) = -\ln cx$$

$$\ln\left(1+\sqrt{1-v^2}\right) = \ln(cx)^{-1}$$

$$1+\sqrt{1-v^2} = 1/cx$$

$$1+\sqrt{1-y^2/x^2} = 1/cx$$

$$1+\sqrt{\frac{x^2-y^2}{x^2}} = 1/cx$$

$$x+\sqrt{x^2-y^2} = 1/c$$

$$x+\sqrt{x^2-y^2} = c_1 \qquad \because 1/c = c_1$$

which is a Required Solution.

Question No. 3 :

$$(D^4+D^2)y = 3x^2+4\sin x-2\cos x$$

Solution: $(D^4+D^2)y = 3x^2+4\sin x-2\cos x$

$$Z(D)y = Z(x)$$

As it is non-homogeneous linear equation so solution will be

$$y = y_c + y_p \text{ —— (i)}$$

$$D^4+D^2=0 \implies D^2(D^2+1)=0$$

Either $D^2=0 \implies \boxed{D=0}$

$$D^2+1=0 \implies D^2=-1$$

$$D=\sqrt{-1} \implies \boxed{D=i} \text{ or } D=\boxed{0+i}$$

Roots are real and Complex

$$y_c = C_1e^{0x}+e^{0x}(C_2\cos x+C_3\sin x)$$

$$y_c = C_1+C_2\cos x+C_3\sin x$$

$$y_p = \frac{1}{Z(D)} F(x)$$

$$y_p = \frac{1}{D^4 + D^2} (3x^2 + 4\sin x - 2\cos x)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$$Z(D) = D^4 + D^2$$

at $D = 0 \Rightarrow Z(D) = 0$

So

$$Z'(D) = 4D^3 + 2D$$

Now also for $D = 0 \Rightarrow Z'(D) = 0$

again differentiating

$$Z''(D) = 12D + 2$$

So for $D = 0$

$$Z''(0) = 12(0) + 2 = 2$$

So replacing $\frac{1}{Z(D)}$ with $\frac{x^2}{Z''(D)}$

$$\Rightarrow y_p = \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2}{12D+2} \cdot 2\cos x$$

Putting $D=0$ in all

$$y_p = \frac{x^2 \cdot 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2} - \frac{2x^2 \cos x}{12(0)+2}$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2 \cos x}{2}$$

$$= \frac{3}{2}x^4 + 2x^2 \sin x - x^2 \cos x$$

So

putting in Equation (i)

$$y = C_1 + C_2 \cos x + C_3 \cos x + \frac{3}{2} + 2x^2 \sin x - x^2 \cos x$$

$$\boxed{y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2)\sin x + \frac{3}{2}x^4}$$