Q1) $\frac{dy}{dt} = e^{y-t} \sec(y)(1+t^2)$; $y(0)=0$

Sol) $dy = e^{y} e^{-t} \sec(y)(1+t^2) \cdot dt$

$\Rightarrow \frac{dy}{e^{y}\sec y} = e^{-t}(1+t^2)\,dt$

$\Rightarrow$ Taking integral on both sides

$= \int \frac{dy}{e^{y}\sec y} = \int e^{-t}(1+t^2)\,dt$

$= \int e^{-y}\frac{dy}{\sec y} = \int e^{-t}(1+t^2)\,dt$

$= \frac{1}{2}e^{-y}(-\cos y + \sin y) = e^{-t}(-3-2t-t^2)+C$ — (A)

putting initial condition we get

$\Rightarrow \frac{1}{2}e^{-0}(-\cos 0 + \sin 0) = e^{-0}(-3-2(0)-0)+C$

$\Rightarrow \frac{1}{2}(-1) = -3+C$

$-\frac{1}{2} = -3+C$

$\Rightarrow -\frac{1}{2}+3 = C$ $\Rightarrow C = 5/2$ put in (A)

$\Rightarrow \frac{1}{2}e^{-y}(-\cos y + \sin y) = e^{-t}(-3-2t-t^2)+\frac{5}{2}$ Ans.

This should be (+) Mistake.

(Q2) $\sqrt{x+y}+\sqrt{x-y}\,dx \; (-) \; (\sqrt{x+y}-\sqrt{x-y})\,dy=0$

Sol let

$$M=\sqrt{x+y}+\sqrt{x-y}$$

$$N=-\sqrt{x+y}+\sqrt{x-y}$$

$$\frac{\partial M}{\partial y}=\frac{d}{dy}\left(\sqrt{x+y}+\sqrt{x-y}\right)$$

$$=\frac{dM}{dy}=\frac{1}{2\sqrt{x+y}}-\frac{1}{2\sqrt{x-y}} \Rightarrow \frac{\sqrt{x-y}-\sqrt{x+y}}{2(\sqrt{x-y})(\sqrt{x+y})} \quad - ★1$$

Now

$$\frac{\partial N}{\partial x}=-\frac{1}{2\sqrt{x+y}}+\frac{1}{2\sqrt{x-y}}=\frac{-\sqrt{x-y}+\sqrt{x+y}}{2(\sqrt{x-y})(\sqrt{x+y})} \rightarrow ★$$

★1 should be equal to ★2

but there is a mistake of minus

This is a exact differential equation they should be equal such that ★1 must be equal to ★2.

Q3 $(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$

Sol

$$\frac{d^4y}{dx} + \frac{d^2y}{dx} = 3x^2 + 4\sin x - 2\cos x$$

Since $y = y_c + y_p$ — ①

y_c is associated with homogenous part

That is $m^4 + m^2 = 0 \Rightarrow (A.E)$

$\Rightarrow m^2(m^2+1) = 0$

$\Rightarrow m_1 = 0, m_2 = 0, m_3 = 2, m_4 = -1$

Thus

$$y_c = c_1 + c_2 x + c_3 \cos x + c_4 \sin x$$

Also to find y_p = we use undetermined co-efficients approach

$y_p = (Ax^2 + Bx + C) + Dx\cos x + Ex\sin x$ — ⓐ

$y'(p) = 2Ax + B + Dx(-\sin x) + (D\cos x + Ex\cos x + E\sin x)$ ⓑ

$y''(p) = Dx\sin x + D\cancel{\cos x} - D\cos x - D\cos x - E\,x\cos x +$

$E\cancel{\sin x} - E\cancel{\sin x} - E\sin x$

$y'''(p) = Dx\sin x - D\cos x - Ex\cos x - E\sin x.$

$y''''(p) = Dx\cos x + \sin x + D\sin x + Ex\sin x - E\cos x - E\cos$

Putting values in original D.E and comparing coefficient

$Dx\cancel{\cos x} + D\cancel{\sin x} + Ex\cancel{\sin x} - E\cancel{\cos x} - E\cancel{\cos x} - 2A - Dx\cancel{\cos x} -$

$2D\cancel{\sin x} - Ex\cancel{\sin x}$

$= 3x^2 + 4\sin x - 2\cos x.$

$2A = 1$

$A = \frac{1}{2}$

$B = C = D = E = 0$

So $y_p = \frac{1}{2} x^2$

$y = y_c + y_p$

$y = c_1 + c_2 x + c_3 \cos x + c_4 \sin x + \frac{x^2}{2}$. Ans.