

IQRA NATIONAL UNIVERSITY

NAME : NOUMAN TAHIR SHAH

ID : 7735

SECTION : A

MODULE : 8TH SEMESTER

$$(\sigma_1)$$

Sol: $\sigma_x = -40 \text{ MPa}$

$$\sigma_y = 70 \text{ MPa}$$

$$\tau_{xy} = -50 \text{ MPa}$$

we need to find σ_{x_1} , σ_{y_1} , and $\tau_{x_1 y_1}$

Now, substitute numerical values to transformation

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} (\cos 2\theta + \tau_{xy} \sin 2\theta)$$

$$\text{So } \sigma_{x_1} = \frac{-40 + 70}{2} + \frac{(-40 - 70)}{2} (\cos 2\theta + \tau_{xy} \sin 2\theta)$$

$$\sigma_{x_1} = \frac{-40 + 70}{2} + \frac{(-40 - 70)}{2} (\cos 2(-15^\circ) + (-50) \sin 2(-15^\circ))$$

$$\sigma_{x_1} = -7.63 \text{ MPa}$$

Now $\sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} (\cos 2\theta - \tau_{xy} \sin 2\theta)$

$$\sigma_{y_1} = \frac{-40 + 70}{2} - \frac{(-40 - 70)}{2} \cos 2(-15) - (-50) \sin 2(-15)$$

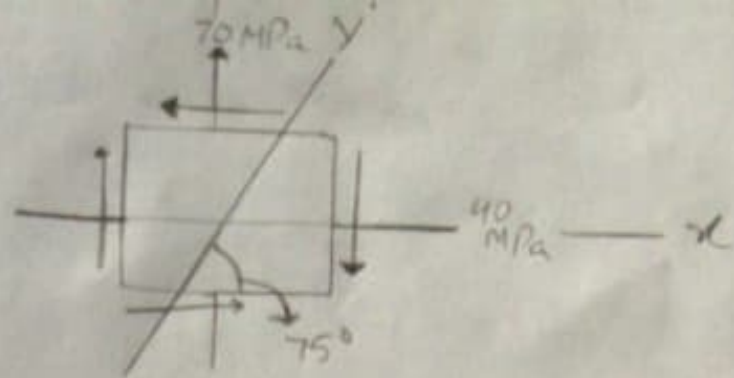
$$\sigma_{y_1} = 37.63 \text{ MPa}$$

$$\tau_{x_1 y_1} = -\frac{(\sigma_x - \sigma_y)}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\tau_{x_1 y_1} = -\frac{(-40 - 70)}{2} \sin 2(-15) + (-50) \cos 2(-15)$$

$$\tau_{x_1 y_1} = -70.80 \text{ MPa}$$





QUESTION #02 (a)

SOLUTION: First we find principle plane:

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}, \quad \tan 2\theta_p = \frac{2(-50)}{-40 - 70} =$$

$$\tan \times 2\theta_p = 0.9091$$

$$2\theta_{p_2} = \tan^{-1}(0.9091) = 42.27$$

$$\theta_{p_2} = \frac{42.27}{2}$$

$$\boxed{\theta_{p_2} = 21.14^\circ} \text{ (for x-axis)}$$

Also, $2\theta_{p_1} = 42.27 + 180$

$$\theta_{p_1} = \frac{222.27}{2}$$

$$\boxed{\theta_{p_1} = 111.135^\circ} \text{ (for y-axis)}$$

Now we find principal stress.

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{-40+70}{2} + \sqrt{\left(\frac{-40-70}{2}\right)^2 + (-50)^2}$$

$$\sigma_1 = 89.33 \text{ MPa}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_2 = \frac{-40+70}{2} - \sqrt{\left(\frac{-40-70}{2}\right)^2 + (-50)^2}$$

$$\sigma_2 = -59.33 \text{ MPa}$$

We must check which angle goes with which principal stress.

$$\begin{aligned}\sigma_{x_1} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{-40+70}{2} + \left(\frac{-40-70}{2}\right) \cos 2(21.14) + (-50) \sin(21.14)\end{aligned}$$

$$\sigma_{x_1} = -59.33 \text{ MPa}$$

Hence,

$$\sigma_1 = 89.33 \text{ MPa with } \theta_{P_1} = 111.135^\circ$$

$$\sigma_2 = -59.33 \text{ MPa with } \theta_{P_2} = 21.14^\circ$$

part b

Given data

The state of stresses which are given as

$$\sigma_x = -40 \text{ MPa}$$

$$\sigma_y = 70 \text{ MPa}$$

$$\tau_{xy} = -50 \text{ MPa}$$

Required data

Maximum In-plane shear stress = ?

Maximum shear plane ?

Solution :-

first we find Max in-plane shear stress.

$$|\tau_{\max \text{ in plane}}| = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-40 - 70}{2}\right)^2 + (-50)^2}$$

$$|\tau_{\max \text{ in plane}}| = 74.3 \text{ MPa}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-40 + 70}{2}$$

$$\boxed{\sigma_{avg} = 15}$$

Now we have to find Max. Shear plan

$$\begin{aligned}\tan 2\phi_s &= \frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} \\ &= \frac{(-40 - 70)/2}{-50}\end{aligned}$$

$$\tan 2\phi_s = -1.1$$

$$\phi_s = \frac{1}{2} \tan^{-1}(-1.1)$$

$$\boxed{\phi_s = -23.90}$$