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Section	A
Subject	Applied calculus.
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Q1. Find PQ were P is the point in three-dimensional space with coordinates (4, 1, 3) and the point Q with coordinates (1, 2, 4). Find the distance b/w P & Q. Further, find the position vector of the point dividing PQ in the ratio 1:3.

Solution:

Given data.

coordinates of P = (4, 1, 3)

So $\vec{OP} = 4i + 1j + 3k$.

&

Coordinates of Q = (1, 2, 4) ~~(1i+2j+4k)~~

So $\vec{OQ} = 1i + 2j + 4k$.

$PQ = \vec{OQ} - \vec{OP}$.

$= (i + 2j + 4k) - (4i + 1j + 3k)$.

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$PQ = -3i + j + k \rightarrow$ (i)

Now distance b/w P & Q = $|PQ|$

$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$

$= \sqrt{9 + 1 + 1}$

$= \sqrt{11} \rightarrow$ (ii)

Let N be the point which divided PQ in ratio 1:3.
By ratio theorm.

Position vector of N = $\vec{ON}$

$= \dfrac{3(4i + j + 3k) + 1(i + 2j + 4k)}{1+3}$

$= \dfrac{12i + 3j + 9k + i + 2j + 4k}{4}$

$= \dfrac{13i + 5j + 13k}{4} \rightarrow$ (iii)

So eq (i), (ii), (iii) are the required Solution.

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Q.3

Evaluate

Part A. $\int_0^2 x^2 e^x \, dx$.

Sol: Integration by parts

Let $u = x^2$ $\quad v = e^x$

$u' = 2x$ $\quad v' = e^x$.

Sol.

$$\int uv' = uv - \int u'v$$

$$= x^2 e^x - \int 2x e^x dx$$

$$= x^2 e^x - 2\int x e^x \, dx.$$

Now.

Let $u = x$ $\quad v' = e^x$

$u' = 1$ $\quad v' = e^x$.

$$= x^2 e^x - 2\left[x e^x - \int e^x dx\right].$$

$$= x^2 e^x - 2\left[x e^x - e^x\right]\Big|_0^2$$

$$= x^2 e^x - 2x e^x + 2e^x \Big|_0^2$$

$$= \left[(2)^2 e^2 - 2(2)e^2 + 2e^2\right] - (2e^0).$$

$$= \cancel{4e^2} - \cancel{4e^2} + 2e^2 - 2.$$

$$= 2e^2 - 2. \text{ Answer.}$$

Q.3

Part b $\quad \int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx.$

Sol: by Substitution:

let $u = \sqrt{x} \quad \frac{du}{dx} = \frac{1}{2\sqrt{x}}$

so $dx = 2\sqrt{x}\,du$

$= \int_1^2 2\sin u\,du.$

$= 2\int_1^2 \sin u\,du.$

$= -2\cos u \Big|_1^2$

$= u = \sqrt{x}.$

$= -2\cos\sqrt{x}\Big|_1^2$

$= \left[-2\cos\sqrt{2}\right] - \left[-2\cos\sqrt{1}\right].$

$= -2\cos\sqrt{2} + 2\cos\sqrt{1}.$

$= 2\left[\cos\sqrt{1} - \cos\sqrt{2}\right]$

Answer.

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Q.4:
Verify that

$$U(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Satisfies the three-dimentional Laplace's equations.

Sol:

The laplace equation in 3d is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \rightarrow (A).$$

So $U(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$

$$U(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$\frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x\left(\frac{-3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}x) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2} \rightarrow (1)$$

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Now

$$\frac{\partial u}{\partial y} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$$

$$\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[y\left(\frac{-3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}y)+(x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \rightarrow 2$$

$$\frac{\partial u}{\partial z} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$

$$\frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial z^2} = 3z(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \rightarrow 3$$

putting 1, 2, 3 in eq A

$$3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2-(x^2+y^2+z^2)+3y^2-(x^2+y^2+z^2)+3z^2-(x^2+y^2+z^2)\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2-\cancel{x^2}-\cancel{y^2}-\cancel{z^2}+3y^2-\cancel{x^2}-\cancel{y^2}-\cancel{z^2}+3z^2-\cancel{x^2}-\cancel{y^2}-\cancel{z^2}\right]$$

$$= (x^2+y^2+z^2)^{-5/2}(0) = 0$$

So the given $u(x,y,z)$ is solution of laplace equation

Q. 2)

Evaluate:

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx.$$

Sol:

$$\begin{array}{r|l} & 2x-1 \\ \hline 2x+x & 4x^3+10x+4 \\ & \cancel{4x^3} \\ \hline & -2x^2+10x+4 \\ & -2x^2-x \\ & + \quad\;\; + \\ \hline & 11x+4. \end{array}$$

So $2x-1+\dfrac{11x+4}{2x^2+x} = \dfrac{4x^3+10x+4}{2x^2+x}.$

$$\Rightarrow \int \frac{4x^3+10x+4}{2x^2+x} = \int 2x-1+\int \frac{11x+4}{2x^2+2x} \rightarrow (1)$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x-1)}\,dx \rightarrow (2)$$

Now find.

$$\int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \longrightarrow A.$$

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$$\frac{11x+4}{x(2x+1)} = \frac{A(2x+1)+Bx}{x(2x+1)}$$

$$11x+4 = A(2x+1)+Bx \rightarrow (3)$$

put $x = 0$ in (3)

$$\boxed{4 = A}$$

Now put $x = -\frac{1}{2}$ in (3)

$$11\left(-\frac{1}{2}\right)+4 = B\left(-\frac{1}{2}\right)$$

$$\frac{-11}{2}+4 = -\frac{B}{2}$$

$$\frac{-11+8}{\cancel{2}} = -\frac{B}{\cancel{2}}$$

$$-3 = -B \Rightarrow \boxed{B = 3}$$

Putting the value of A & B in (A)

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking integral on b/s

$$\int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

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(9)

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= -4 \ln|x| + \frac{3}{2}\ln|2x+1|$$

putting these values in (2)

$$= x^2 - x + 4\ln|x| + \frac{3}{2}\ln(2x+1)$$

Now put these values in eq (1).

$$\boxed{\int \frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + C}$$

Answer:

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