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Degree \* \* \* \* \* BSc

Section \* \* \* \* \* 1

Paper \* \* \* \* \* Linear Algebra

Q No 1.

Ans:

$$\left[ \begin{array}{ccc|ccc} 1 & 3 & 0 & 5 & & \\ 0 & 1 & -5 & 0 & 7 & \\ 0 & 0 & 1 & 0 & -6 & \\ 0 & 0 & 0 & 1 & 0 & \end{array} \right]$$

Multiplying  $R_3$   $-3$  and the  
add to  $R_1$ .

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 3 & -3 & 0 & 23 \\ 0 & 1 & -5 & 0 & 7 & \\ 0 & 0 & 1 & 0 & -6 & \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 13 \\ 0 & 1 & -5 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

Multiply  $R_3$  by 5 and then add to  $R_2$

$$5R_3 + R_2$$

$$\left[ \begin{array}{ccccccc} 1 & 0 & 0 & 0 & 0 & 23 & \\ 0 & 1 & -5t & 0 & 0 & -23 & \\ 0 & 0 & 1 & 0 & 0 & -6 & \\ 0 & 0 & 0 & 0 & 1 & 0 & \end{array} \right] \quad \begin{array}{l} \\ 5R_3 + R_2 \\ \\ \end{array}$$

$$\left[ \begin{array}{ccccccc} 1 & 0 & 0 & 0 & 0 & 23 & \\ 0 & 1 & 0 & 0 & 0 & -23 & \\ 0 & 0 & 1 & 0 & 0 & -6 & \\ 0 & 0 & 0 & 0 & 1 & 0 & \end{array} \right]$$

So this is the final linear equation system.

$$x_1 = 23$$

$$x_2 = -23$$

$$x_3 = -6$$

$$x_4 = 0$$

Verification

$$23 + 3(-6) = 5$$

$$23 - 18 = 5$$

$$5 = 5 \rightarrow \text{true}$$

$$-23 - 5(-6) = 7$$

$$-23 + 30 = 7$$

$$7 = 7 \text{ true.}$$

Q2 :-

Ans:- Part (A)

Matrix 1

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Matrix 1 + Matrix 2

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

$R_3 - 2R_2$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & -5 & -9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix} \rightarrow \text{This now matrix (2)}$$

Matrix 2 to matrix (1)

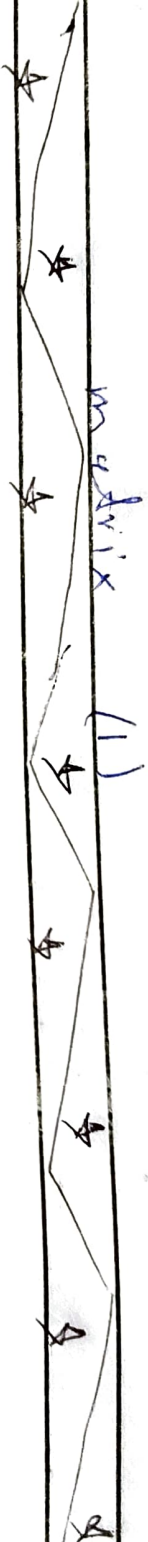
$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$R_3 + 2R_2$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0+2 & 3+(-8) & -5+4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & 1 \end{bmatrix}$$

so this



Part (b)

$$(a) \begin{bmatrix} e & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & 0 & 0 & e \end{bmatrix} \text{ is in echelon form}$$

$$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & 35 \\ 0 & 8 & -34 \\ 4 & 0 & 29 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_3 + 2R_1$

$$\begin{bmatrix} -2 & 0 & 35 \\ 0 & 8 & -34 \\ 0 & 0 & 29 \\ 0 & 0 & 0 \end{bmatrix}$$

This is in echelon form.



$$R_3 \leftrightarrow R_4$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 1 & -4 & 15 \\ 0 & 0 & 0 \end{bmatrix}$$

$$4R_1 - 3R_2$$

$$\begin{bmatrix} -2 & 0 & 35 \\ 2 & 8 & -1 \\ 1 & -4 & 15 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_3 \leftrightarrow R_4$$

$$4R_1 - 3R_2$$

$$\begin{bmatrix} -2 & 0 & 35 \\ 2 & 8 & -1 \\ 1 & -4 & 15 \\ 0 & 0 & 0 \end{bmatrix}$$

$$4R_1 - 3R_2$$

$$2R_3 + R_2$$

$$\begin{bmatrix} -2 & 0 & 35 \\ 2 & 8 & -1 \\ 4 & 0 & 29 \\ 0 & 0 & 0 \end{bmatrix}$$

$$2R_3 + R_2$$

$$R_3 + R_1$$

$$\begin{bmatrix} -2 & 0 & 35 \\ 2 & 8 & -1 \\ 2 & 8 & -1 + 35 \end{bmatrix}$$

$$R_2 + R_1$$

Reduced Row (column) Echelon Form: A matrix is said to be in reduced row (column) echelon form when it satisfies the following conditions

Row (column) Echelon Form:

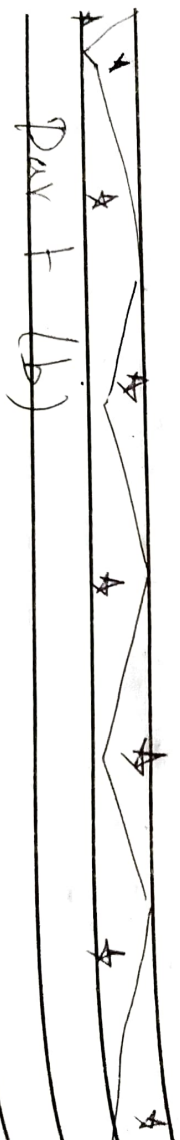
A matrix is said to be in row (column) echelon form when it satisfies the following condition

1- The first non-zero element in each row (column) called the leading entry, is 1.

2- Each leading entry is in a column (row) to the right of the leading entry in the previous row (column).

For example:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Part (b)

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 0 & 0 & 0 \\ 1 & -4 & 15 \end{bmatrix}$$



Q3

Part (A)

Ans:-

Reduced Row (column) Echelon Form:

A matrix is said to be the reduced row (column) echelon form when it satisfies the following conditions:

1- The matrix satisfies condition for a row (column) echelon form.

2- The leading entry in each row (column) is the only non-zero entry in its column (row).

For example:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$