Student Details

Name: ______________________ **Student ID:** ____________

alirazakhan12647

Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$$ To the input $x(n)=4^n u(n)$.	Marks 6
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n)=0.6y(n-1)-0.8y(n-2)+x(n)$$	
Q2	(a)	Determine the causal signal x(n) having the z-transform $$x(z)=\frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	Marks 6
	(b)	Determine the partial fraction expansion of the following proper function $$X(z)=\frac{1}{1-1.5z^{-1}+0.5z^{-2}}$$	
Q.3	(a)	A two- pole low pass filter has the system response $$H(z)=\frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition $H(0) = 1$ and $\left\vert H(\frac{\pi}{4})\right\vert^2 = \frac{1}{2}$.	Marks 4

	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.	
Q 4	(c)	A finite duration sequence of Length L is given as $$x(n)=\begin{cases}1, & 0\leq n\leq L-1\\ 0, & otherwise\end{cases}$$ Determine the N- point DFT of this sequence for $N \geq L$	Marks 4
	(d)	Compute the DFT of the four-point sequence $$x(n)=(0\ 1\ 2\ 3)$$	

Q1 (a) 1 Determine the response $y(n), n \geq 0$ of the system describ by 2nd order difference equation.

Ali Raza Khan
12647

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

$$x(n) = 4^n u(n)$$

Sol #

$$y_h(n) = C_1(-1)^n + C_2(4)^n$$

Normally we could assume a solution in form

$$y_p(n) = K(4)^n u(n)$$

we assume that

$$y_p(n) = Kn(4)^n u(n)$$

Put it in given equation

~~y(n) 3y~~

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

(2)

By putting the equation

$$\to Kn(4)^n u(n) - 3K(n-1)(4)^{n-1} u(n-1) - 4K(n-2)(4)^{n-2} u(n-2)$$

$$= (4)^n u(n) + 2(4)^{n-1} u(n-1)$$

for $n=2$ & $K = 6/3$

$$y_p(n) = \frac{6}{3} n (4)^n u(n)$$

So the total solution to the difference equation

$$y(n) = C_1(-1)^n + C_2(4)^n + \frac{6}{3} n (4)^n \quad n \geq 0$$

C_1 and C_2 are determined

$$y(0) = C_1 + C_2$$

$$y(1) = -C_1 + 4C_2 + \frac{24}{5}$$

Computing above by setting

$y(-1) = y(-2) = 0$

So

$$C_1 + C_2 = 1$$

$$-C_1 + 4C_2 + \frac{24}{9} = 9$$

3

Ali Raza Khan
12647

$$y_{zs}(n) = -\frac{1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6}{5}n(4)^n \quad n \geq 0$$

Q! (4) Ali Raza Khan 12647

(b) → Determine the impulsive Response and unit step response of the system described by equation

$$y(n) = 0.6\,y(n-1) - 0.8\,y(n-2) + x(n)$$

Sol #

$$y(z) = \frac{X(z)}{1 - 0.6z^{-1} + 0.8z^{-2}}$$

$$X(z) = 1$$

$$\therefore H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{1}{1 - 0.6z^{-1} + 0.8z^{-2}}$$

$$= \frac{1}{(1 - 1/5\,z^{-1})(1 - \frac{2}{5}z^{-1})}$$

$$H(z) = \frac{1}{1 - 1/5\,z^{-1}} + \frac{2}{1 - 2/5\,z^{-1}}$$

Ali Raza Khan 12647.

5

$$h(n) = \left[-1\left(\tfrac{1}{5}\right)^n + 2\left(\tfrac{2}{5}\right)^n \right] u[n]$$

$\longleftrightarrow X \longleftrightarrow$

⑥

Ali Raza Khan
12817

Q2(B):- Determine the partial-fraction expansion of the proper function.

$$X(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}}$$

Sol
\# first we have to eleminate the negative powers so we will divide and multiply z^2.

$$X(z) = \frac{z^2}{z^2-1.5z^{-1}+0.5z^{-2+2}}$$

$$= \frac{z^2}{z^2-1.5z^{-1}+0.5}$$

$$= \frac{z^2}{z^2-1z^{-1}-0.5z^{-1}+0.5}$$

$$= \frac{z^2}{z(z-1)-0.5(z-1)}$$

$$X(z) = \frac{z^2}{(z-0.5)(z-1)}$$ $P=1,\ P_2=0.5$

7

Rai Razi Khan
12647

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)}$$

$$\frac{z}{(z-1)(z-0.5)} = \frac{A}{(z-1)} + \frac{B}{(z-0.5)} \rightarrow *$$

multiply $(z-1)(z-0.5)$ both sides

$$z = A(z-0.5) + B(z-1)$$

Now for $z=1$

$$1 = A(1-0.5) + B(0)$$

$$1 = A(0.5)$$

$$\boxed{A = 2}$$

For $z=0.5$

$$0.5 = A(z-0.5) + B(0.5-1)$$

$$0.5 = B(-0.5)$$

$$\boxed{B = -1}$$

(8)

Ali Raza Khan
T2647

Put the values of A and B
in eq A

$$\frac{X(z)}{z} = \frac{2}{z-1} - \frac{1}{z-0.5}$$

Scanned with CamScanner

Q3(a): A two pole low pass filter has the system response

$$H(z) = \frac{b_0}{(1 - pz^{-1})^2}$$

Determine the value of b_0 and p such that $H(\omega)$ satisfy the condition $H(0) = 1$ & $|H(\pi/4)|^2 = 1/2$

Sol

At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence

$$b_0 = (1-p)^2$$

At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-p)^2}{(1 - pe^{-j\pi/4})^2}$$

$$H(\pi/4) = \frac{(1-p)^2}{(1 - p\cos(\pi/4) + jp\sin(\pi/4))^2}$$

$$H(\pi/4) = \frac{(1-p)^2}{\left(1 - \frac{p}{\sqrt{2}} + \frac{jp}{\sqrt{2}}\right)^2}$$

Hence

$$\frac{(1-p)^4}{\left[\left(1-\frac{p}{\sqrt{2}}\right)^2 + \frac{p^2}{2}\right]^2}$$

$$= 1/2$$

or

$$\sqrt{2}(1-p)^2 = 1+p^2 - \sqrt{2}p$$

$$\boxed{p = 0.32}$$

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

Ali Raza Khan
12647

(11)

Q3(b):- Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristic at $\omega = 0$ and $\omega = \pi$ and its magnitude response is $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$

Sol
The filter must have poles at

$$P_{1,2} = re^{\pm j\pi/2}$$

and zeros at $z=1$ and $z=-1$

the System function is

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$H(z) = G\frac{z^2-1}{z^2-r^2}$$

The gain factor is determine by evaluating the frequency response $H(\omega)$ at $\omega = \pi/2$:

$$H(\pi/2) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

Ali Raza Khan
12647

(12)

The value of r is determined by evaluating $H(\omega) = 4\pi/9$ Thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = 1/2$$

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

The value of $r^2 = 0.7$ Satisfies this equation therefore the system function for the desired filter is

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

Scanned with CamScanner

(13)

Ali Raza Khan
12647

Q4(a): A finite duration sequence of length L given as

$$x(n) = \begin{cases} 1 & 0 \le n \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the N-point DFT of this sequence for $N \geq L$

Sol The fourier transform of this equation is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}} = \frac{\sin(\omega L/2)}{\sin(\omega/2)} \ldots$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The N-point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally spaced frequencies $\omega_k = 2\pi k/N$ $k=0,1,2 \ldots N-1$

Hence

X

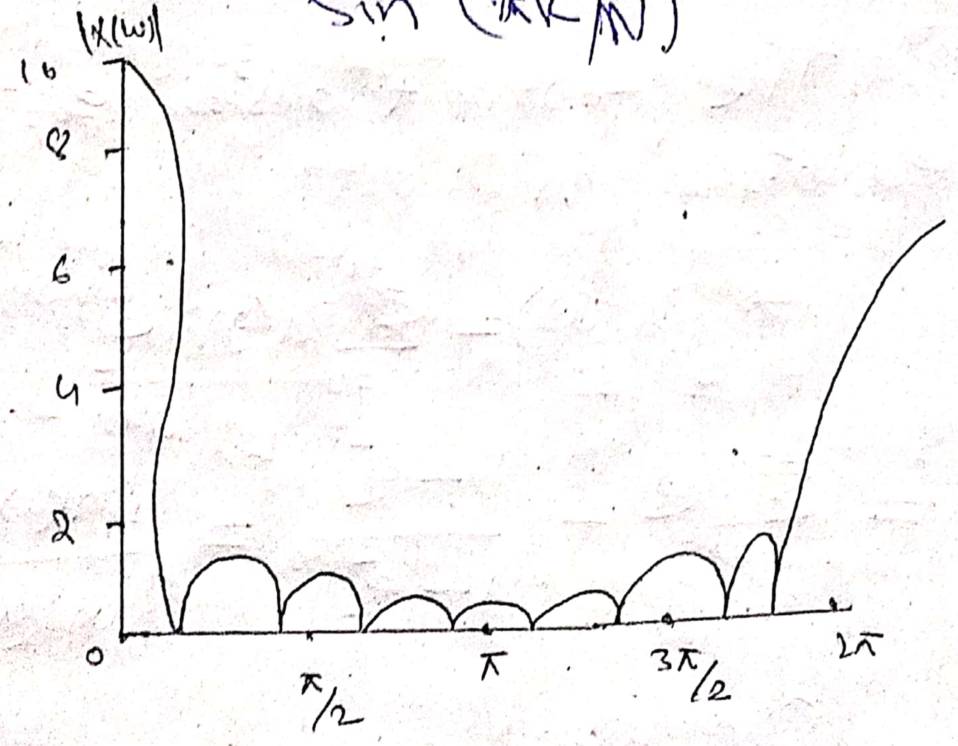
(14)

Ali Raza Khan
12647

Hence

$$X(k) = \frac{1 - e^{-j2\pi kL/N}}{1 - e^{-j2\pi k/N}} \qquad k = 0, 1 \ldots N-1$$

$$= \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$



Ali Raza Khan
12647

Q4(b): Compute the DFT of the four-point sequence

$x(n) = (0\ 1\ 2\ 3)$

Sol

First we have to determine the matrix W_4. By exploiting the periodicity property of W_4 and the symmetry property.

$$W_N^{k+N/2} = W_N^k$$

The matrix may be expressed a

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Scanned with CamScanner

(16)

Ali Raza Khan
12647

then

$$X_4 = W_4 x_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$