

Quiz

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Subject : Differential Equations

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* Solve the following system of equations

$$x + 3y + 5z + 2t = 2$$

$$-y + 3z + 4t = 0$$

$$2x + y + 9z + 6t = -3$$

$$3x + 2y + 4z + 8t = -1$$

Sol; Using Gaussian Elimination to find solution of the above system of equations.

Augmented Matrix;

$$\left[\begin{array}{cccc|c} 1 & 3 & 5 & 2 & 2 \\ 0 & -1 & 3 & 4 & 0 \\ 2 & 1 & 9 & 6 & -3 \\ 3 & 2 & 4 & 8 & -1 \end{array} \right]$$

Using Elementary Row operation.

$R_3 - 2R_1$

$R_4 - 3R_1$

$$\left[\begin{array}{cccc|c} 1 & 3 & 5 & 2 & 2 \\ 0 & -1 & 3 & 4 & 0 \\ 0 & -5 & -1 & 2 & -7 \\ 0 & -7 & -11 & 2 & -7 \end{array} \right]$$

$R_3 + 5R_2$

$R_4 - 7R_2$

$$\left[\begin{array}{cccc|c} 1 & 3 & 5 & 2 & 2 \\ 0 & -1 & 3 & 4 & 0 \\ 0 & 0 & -16 & -18 & -7 \\ 0 & 0 & -32 & -26 & -7 \end{array} \right]$$

CHECK

We have equation;

$$-y + 3z + 4t = 0$$

By putting values;

$$-\left(\frac{7}{4}\right) + 3\left(-\frac{7}{20}\right) + 4\left(\frac{7}{10}\right) = 0$$

$$-\frac{7}{4} - \frac{21}{20} + \frac{28}{10} = 0$$

Taking L.C.M;

$$\frac{-35 - 21 + 56}{20} = 0$$

$$\frac{-56 + 56}{20} = 0$$

$$\frac{0}{20} = 0$$

$$\boxed{0 = 0}$$

Okay

Now; eq "iii" \Rightarrow

$$z + \frac{9}{8} \left(\frac{7}{10} \right) = \frac{7}{16}$$

$$z + \frac{63}{80} = \frac{7}{16}$$

$$z = \frac{7}{16} - \frac{63}{80}$$

$$= \frac{35 - 63}{80} \quad (\text{L.C.M})$$

$$= -\frac{28}{80}$$

$$\boxed{z = -\frac{7}{20}}$$

Now; eq "ii" \Rightarrow

$$y - 3 \left(-\frac{7}{20} \right) - \frac{14}{5} \left(\frac{7}{10} \right) = 0$$

$$y + \frac{21}{20} - \frac{14}{5} = 0 \quad \rightarrow (\text{L.C.M})$$

$$y = \frac{14}{5} - \frac{21}{20} = \frac{56 - 21}{20} = \frac{35}{20}$$

$$\boxed{y = \frac{7}{4}}$$

Now; eq "i" \Rightarrow

$$-x + \frac{7}{4} + 3 \left(-\frac{7}{20} \right) = 0 \quad \rightarrow (\text{L.C.M})$$

$$x = -\frac{21}{20} + \frac{7}{4} = \frac{-21 + 35}{20} = \frac{14}{20}$$

$$\boxed{x = \frac{7}{10}}$$

"Required Solution"

Thus $(x, y, z, t) = \left(\frac{7}{10}, \frac{7}{4}, -\frac{7}{20}, \frac{7}{10} \right)$

$$\left[\begin{array}{cccc|c} 1 & 3 & 5 & 2 & 2 \\ 0 & -1 & 3 & 4 & 0 \\ 0 & 0 & -16 & -18 & -7 \\ 0 & 0 & -32 & -26 & -7 \end{array} \right]$$

$$R_4 - 2R_3 \quad \left[\begin{array}{cccc|c} 1 & 3 & 5 & 2 & 2 \\ 0 & -1 & 3 & 4 & 0 \\ 0 & 0 & -16 & -18 & -7 \\ 0 & 0 & 0 & 10 & 7 \end{array} \right]$$

$$\begin{array}{l} R_1 - R_2 \\ -R_2 \\ -\frac{1}{16}R_3 \\ \frac{1}{10}R_4 \end{array} \quad \left[\begin{array}{cccc|c} -1 & 1 & 3 & 0 & 0 \\ 0 & 1 & -3 & -4 & 0 \\ 0 & 0 & 1 & 9/8 & 7/16 \\ 0 & 0 & 0 & 1 & 7/10 \end{array} \right]$$

Now we have the equations;

$$-x + y + 3z = 0 \quad \text{--- "i"}$$

$$y - 3z - 4t = 0 \quad \text{--- "ii"}$$

$$z + \frac{9}{8}t = \frac{7}{16} \quad \text{--- "iii"}$$

$$\boxed{t = \frac{7}{10}}$$